# Topological (in) Hegel

## Topological Notions of Qualitative quantity and Multiplicity in Hegel's Fourfold of Infinities

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I, Borislav G. Dimitrov, hereby declare that this Ph.D. thesis entitled “ Topological (in) Hegel - Topological Notions of Qualitative quantity and Multiplicity in Hegel’s Fourfold of Infinities” was carried out by me for the degree of Doctor of Philosophy in English under the guidance and supervision of Prof.Dr.Ph.Sc. Maria Dimitrova, PhD Program in Philosophy Taught in English, Faculty of Philosophy, University “St. Kliment Ohridski”, Sofia, Bulgaria.

The interpretations put forth are based on my reading and understanding of the original texts. This thesis contains no material that has been submitted previously, in whole or in part, for the award of any other academic degree or diploma. Except where otherwise acknowledged at the respective place in the text, this thesis is my own work.

Sofia, Bulgaria
October, 2015

Borislav G. Dimitrov
Signature

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**Preface**

Even a cursory reading of Hegel's works is sufficient to convince one that the categories of quality and quantity, the concepts of space, time, place are basic to the system. In contrast with Aristotle and Kant, Hegel's speculative logic exhibits a presuppositionless derivation of the categories. Hegel regards his categories, concepts and notions not as completed and frozen. For him, the categories must be derived, not from presupposed forms of judgement, or from our presuppositions, but from what he calls the sheer 'simplicity of thinking', from the indeterminate being of thought and from the indeterminate thought of being.

Hegel's categories of quality and quantity, in particular 'qualitative quantity' in relation with his concepts of space, time, place are subject to which much attention has not been paid by Hegel's expositors and commentators and yet the importance of it cannot be denied. (Haldar,1932).

Topological reading of 'qualitative quantity' within Hegel's fourfold (manifold) of multiplicity is subject that lack attention at all from Hegel's commentators. This conceptual gap, ontological, epistemological, phenomenological is widening within the 'linguistic turn', 'spatial turn' or 'topological turn' and emerging topological approaches to various fields of social science and their need for deep conceptualization based on the philosophical categories, notions and concepts. As Richard Ek establishes in Theorizing the Earth (2010), "The spatial turn is actually a 'philosophical turn' repackaged and promoted as a fundamental concern with question of space, place and polity in the social sciences and humanities." (Ek, Richard, Mekonnen, T. 2010:49-66)

John WP Phillips, who critically examines the recent arguments asserting a 'topological turn' in culture, the range of topologically informed interventions in social and cultural theory, remarkes in his paper On Topology (2013)[1], that such contemporary fashionable notions of 'topological approaches' and 'becoming topological of culture' "demands a greater critical reflection than the notion of a 'topological turn' suggests."

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[1] Phillips, John WP. (2013), On Topology, Theory, Culture and Society, 9/2013; 30(5):122-152: http://www.researchgate.net/profile/John_Phillips20/publications [accessed Mar 21, 2015].

I believe that such demand of critical reflection shall be based on Hegel's categories of logic, notions and concept, and following my assertion that 'topological' in intuitively presented by Hegel in his logic, dialectic and method, with the present thesis I argue how Hegel's categories of qualitative and quantitative, the concept of space, time and place can provide contemporary researcher with the power and methodology of such a greater critical reflection, and ability to readdress in the new and enhanced mode the variety of these toplogical approaches. The methodology of an applied philosophical topology shall be based on the topological re-reading of Hegel and topological hermeneutics.

The main objective of the present thesis is to demonstrate how Hegel's categories, concepts, language, syntax and semantics, his use of rethorical power exibit topological notions and thus the reading of topological (in) Hegel is open for conceptualization.

Perhaps it would not be an exaggeration to say that there is no system of thought more intimately bound up with one fundamental principle than is the system of Hegel. My assertion with the present thesis is that topological reading of Hegel reviels true topological system, thus there are reasonable grounds for us to see the doctrine of Hegel, in particular his Science of Logic and Philosophy of Nature as Hegel's Analysis Situs.

A correct interpretation of Hegel's system depends upon a thorough comprehension of the categories of quality and quantity, the concepts of space, time, and place. If these categories and concepts are neglected, the system must remain a sealed book. The aim of the present thesis is to unseal the topological character of Hegel's though, that influenced the formation and presence of topological thinking within the tradition of later continental philosophy and directs toward the horizon of new philosophical topology.

The concern of the present thesis is to set forth topological reading and interpretation of such categories as quality and quantiny, in particular the qualitative quantity within Hegel's forfold of multiplicity and such concepts as space, time, place, to emphasize on the importance of topological (in) Hegel for a theory of knowledge, and, in the light of it, and to give some insight toward the current philosophical topology.

The aim of the thesis is to critically examine whether it is methodologically possible to combine mathematical rigor – topology with a systematic dialectical methodology in Hegel, and if so, to provide as result of my interpretation the outline of Hegel's Analysis Situs, also with the proposed models (build on the topological manifold, cobordism, topological data analysis, persistent homology, simplicial complexes and graph theory, to provide an indication of how the merger of Hegel's dialectical logic and topology may be instrumental to a systematic logician and of how a systematic dialectical logic perspective may help mathematical model builders.

**Abstract**

The present study is interdisciplinary, involving the interrelations between philosophy and topology, where topology is understood in both meanings as mathematical discipline and rhetorical notions, and culminating to what I regard as topological philosophy or philosophical topology, based in particular on Hegel's notion of multiplicity, implemented in his logic unfolding the true topological fourfold of infinities, where qualitative and quantitative, spatial and temporal, as well as rhetorical notions such as the four basic tropes of rhetoric: 'metaphor', 'metonymy', 'synecdoche', 'ironi' are presented in Hegel's manifold (Mannigfaltigkeit) of infinities, quality and quantity, time and space. Not only these four basic tropes of rhetoric are equally presented in Hegel's philosophical narratives, but also the emphasis is on the metonymy seen as metonymy of metonymy or 'metalepsis', is strongly presented in Hegel's logic, in particular in his 'topological' notion of Qualitative quantity.

The study investigate in particular the categories of 'quality', 'quantity' and 'measures' within the notions of 'multiplicity in Hegel's dialectical logic, with an emphasis on the topological notion of qualitative quantity - the category that remained 'inapparent' within the well know dialectics of transformation of quality to quality, where the new quality appears as leap (nodal line) exhibiting abrupt changes and discontinuous transformation leading to the new measure. The exhibit form of Hegel's qulitative quantity is related with continuous changes and smooth, gradual, topological transformations. The gradualness of such transformations demonstrates topological homeomorphism as exhibit form of the category of qualitative quantity, which can be successfully implemented in mathematical, indeed topological models and methods, such as topological manifold, cobordism, topological data analysis, persistent homology, simplicial complexes and graph theory.

The topological notions of multiplicity in Hegel's fourfold of infinities is discussed: (1) the bad qualitative infinity; (2) the good qualitative infinity; (3) the bad quantitative infinity; (4) the good quantitative infinity, related with the fourfold interplay of the two pair of Ancient Greek categories of 'time' and 'space' as fourfold constructed of 'Chronos' and 'Kairos', 'Chora' and 'Topos' as follow: (1) chronochora, chronotopos, kairochora and kairotopos - where the four categorical models implements the following: 'quantitative quantity' – 'quantitative quality' - 'qualitative quantity' – 'qualitative quality'. The claim supported is

that ‘quality’ is in the core of ‘quantitative infinity’, ‘infinity’ is a quality of quantity – ‘Qualitative quantity’.

There is homology between Hegel’s fourfold of infinities (multiplicity) and the fourfold of Hegel’s judgments (Judgment of Existence; Judgment of Reflection; Judgment of Necessity; Judgment of the Notion). The true topological character of Hegel’s logic is revealed as coherence between Hegel’s fourfold of infinities (multiplicity) and the fourfold of Hegel’s judgments, between method and subject matter, and this topological character is presented in Hegel through the double negation, where the Understanding and its negation, Dialectical Reason, and the Negation of the Negation – Speculative Reason, can be seen in the fourfold of Hegel’s judgments. In the very last chapter of Hegel’s Science of Logic, method and subject matter supposedly conjoin. [2] (Carlson 2005).

Following William Lawvere’s assertion (Lawvere, 1996) [3] that a significant fraction of Hegel’s Logic can be modeled mathematically through the use of “cylinders” (diagrams of shape Δ) in a category, wherein the two identical subobjects (united by the third map in the diagram) are “opposite”.about the use of “cylinders” (diagrams of shape Δ), my interpretation of Hegel’s Objective Logic offer methodological modeling of the categories, based on cylinders - the ‘cobordism’, and on the shape Δ - the ‘simplicial complex’ (simplicial complexes).

Hegel’s fourfold of infinities is build on the fourfold of quality and quantity ratio – the notions and ratios of *quantitative quantity; quantitative quality; qualitative quantity; qualitative quality*. The fourfold of the qualitative and quantitative ratios, relate to the fourfold of the measure. All measures are ratios of two other measures. Measure is twofold, divided into two —the external and the internal measure. This is the dialectical real of the real measure. Measure is the unity of quality and quantity, yet in *the center* of the series of measures is a *master signifier* that organizes everything, even while escaping measurement. This ‘master signifier’, the empty center *between* quality and quantity is like the *hole of the torus*, the nothing, the void, and Hegel names it *‘substrate.’* The substrate is *discontinuous* within the

[2] David Gray Carlson, 2005, Why Are There Four Hegelian Judgments?, p.114:125, in Hegel’s theory of the subjects, David G. Carlson, ed. 2005, Palgrave Macmillan 2005

[3] F. William Lawvere, 1996, Unity and identity of opposites in calculus and physics, Applied Categorical Structures, June 1996, Volume 4, Issue 2, pp 167-174

series of measure *and continuous* at the same time. The substrate is what Hegel calls a *true infinity*. Substrate can be organized in a series of measures. Substrate is abstract measureless. What is important to see at this point in relation to measure is entailed in a duality between the nodal relation of quantity and quality, on the one side, and substrate, on the other side. The first side is measure as such—quantity and quality. The second side—the substrate—is something deeper than quantity and quality.

The model of Cobordism (Rene Thom) is proposed as topological model of Hegel's fourfold of infinities within his concept of multiplicity. For Hegel multiplicity means the quantification of quality and the qualification of quantity, multiplicity of the double-entendre implemented in the inevitable double-meaning of Qualitative quantity. For Hegel, the inability to read history as two-sided, as both continuous and discrete, to co-board between, into, within this twofold, to see beings as double, as both qualitative and quantitative, is the road to the destruction of spirit. Trough the 'Topological', Hegel's logic and dialectics are seen in the current. Today, it is Hegel who give us the true, current, topological reply on the question - What does it mean to think multiplicity as both quality and quantity?

*Topological* is subject to philosophical[4] systematization, as well as the *philosophical* is subject to topological (mathematical) formalization. In the development of *philosophy* and *mathematics* (mathematical and philosophical Mind) there is mathesis universalis and some "problematic situations" that have a common "categorial" structure to study them and the "method" of the study can not be other than "mathematical (topological) modeling" or "explication" of philosophical "categories" and together with it – categorial (philosophical) "interpretation" of the mathematical (topological) "structures". *Philosophical*-topological (mathematical) Mind and *mathematical (topological)*-philosophical Mind have *one and the same* "subject", to whom study apply with the two "polar-opposite" of its "form" *methods,* or studying the same "subject" in the two *polar-opposite* "forms" through "method", which is *inherently "the same*".

[4] ontological, epistemological, logical, *dialectical, hermeneutical, linguistic, psychological, ethical, anthropological, sociological, semiotic, legal, economical and etc.*

Novel ‘topological’ reading of Hegel’s notions of spatial and temporal, qualitative and quantitative, is proposed as fundamental in re-thinking of the evolution of hierarchical systems.

The evolution of hierarchical systems is approached topologically as specific ‘problem situation’ for mathematical and philosophical mind, in the term of Philosophical Analisis Situs, where philosophical aspect is present through the categories of Hegel’s multiplicity – the dialectics and logic of ‘qualitative’ and ‘quantitative’, spatial and temporal, and mathematical aspect is presented through the models, such as Cantor Set, logistic map, bifurcation diagrams and topological notion of Cobordism.

In addition to the Hierarchical (vertical evolution) relations, an emphasis on the topological notion of qualitative quantity in Hegel, reveals the role of Heterarchy and heteronomy in Evolution (horizontal evolution), since the exhibit form of Qualitative quantity is related with continuous changes and smooth, gradual, topological transformations. The gradualness of such transformations demonstrates topological homeomorphism as exhibit form of the category of Qualitative quantity, which could be successfully implemented in mathematical, indeed topological models and methods.

Hegel’s notions of manifold presented in the model of Rene Thom’s cobordism is discussed and implemented through the logistic map of Feigenbaum bifurcation Diagram accepted as universal scenario of development, change and evolution.

The proposed outcome demonstrate that within the interval of chaos, marked in the Feigenbaum Diagram, where the parameter ‘a’ increase over the value of 3.0 in the higher octaves of 4.33 , the ‘voices’ of Hegel and Cantor are present within the region of chaos known as Cantor dust. In the zone of chaos and Cantor dust, Hegel’s multiplicity and four measures works in progress and logic breaths the thin air of being retrieving itself in metaphysic. Beyond the octaves of the values 4.33 in Feigenbaum bifurcation Diagram, within the chaos, there are the heads of Canto comets or the divergence diagrams, where can be found the seeds of the new orders and multiplicity (manifolds) of new bifurcations and Feigenbaum diagrams. The visible ‘white’ corridors of homeostasis are windows open for new hierarchies of order and possibilities of development. Based on the said proposition, the focus of the present paper is on the evolutionary scenario, where in addition to the currently accepted paradigm of hierarchy of evolutionary systems where the core of representation is through the phylogenetic tree structure (vertical evolution), the thesis of reticulate (horizontal evolution) is asserted and discussed as exhibiting the heterarchy and heteronomy of evolutionary systems.

The standard evolutionary representation, the phylogenetic tree, and the notion of hierarchy of evolutionary systems, faithfully represents the vertical evolution, but cannot capture horizontal, or reticulate, events, which occur when distinct clades merge together to form a new hybrid lineage. Both hierarchy and heterarchy of the tree structure and the structure of reticulate events could be mathematically investigated, modeled and represented through the field of algebraic topology known as Topological Data Analysis, where the primary mathematical tool considered is a homology theory for point-cloud data sets—persistent homology—and a novel representation of this algebraic characterization— simplicial complex and barcodes.

The persistent homology in evolution, which characterizes global properties of a geometric object that are invariant to continuous deformation, such as stretching or bending without tearing or gluing any single part of it, and the properties that includes such notions as connectedness, is the current method of implementation of Hegel' topological logic of multiplicity and Qualitative quantity.

## Introduction

Back in 1989, as a Research Associate at Institute for Philosophical Research at Bulgarian Academy of Science, I have established Hegel's category and notion of 'qualitative quantity' (Dimitrov, 1989a) as vocamen of my research interests in philosophy, in particular within the area that can be defined as philosophical topology. With my first publications (Dimitrov, 1989, Dimitrov, 1990), applicable to 'dialectics and the problem of novelty', I introduced the claim about the topological notion of Hegel's category of 'qualitative quantity', where 'topological' implies gradual transformation and continuous change without leap or abrupt changes, thus exhibit form of this category is topological homeomorphism.

I have supported my argument about topological homeomorphism as exhibit form of Hegel's Qualitative quantity with exploration of D'Arcy W. Thompson's "Growth and Form" (1917), and Hermann Haken (Haken, 1983) discussion on D'Arcy W. Thompson's transformation, concluding that Haken's finding of structural stability and homology exhibited by such transformation of the forms, explicitly state the notion of qualitative quantity.

In his book "Synergetics: Introduction and Advanced Topics", 41 in the Chapter 1.13. "Qualitative Changes: General approach", Hermann Haken explores and illustrate the structural stability with an example /figure 1.13, p.434 in Haken/ given by the Scottish biologist, mathematician and classics scholar D'Arcy W. Thompson, the author of the book, On Growth and Form, /1917/. My assertion is that Hegel's category of qualitative quantity is illustrated with Herman Haken's citation of D'Arcy W. Thompson.

Homology is related with the works of D'Arch Thompson, especially his "On Growth and Form" (1917). In the last chapter of "On Growth and Form", D'Arcy Thompson's illustrates his "cartesian transformations" of animal forms. Thompson's mappings are referred to as "rubber sheet" mappings. D'Arch Thompson suggested that one should study the change from one biological form to another by examining the unique mathematical object that maps between them in accord with biological homologies.

The example illustrated this transformation actually is a good example of homeomorphism. Two objects are homeomorphic if they can be transformed /or deformed/ into each other by a continuous inverible mapping, continuous one-to-one and having continuous inverse. The two fish are two objects with the same topological properties. They are said to be homeomorphic. There are properties that are not destroyed by stretching and distorting an object.

The claim I made with my first two publications (Dimitrov, 1989, Dimitrov, 1990), contradicted the well established, until the time of 'persetroika', paradigm of dialectical materialism, the paradigm that follow from Engels's "law" for the transformation of quantity into quality and vice versa, states that continuous quantitative development results in qualitative "leaps" in nature whereby a completely new form or entity is produced.

In Anti-Dühring, Engels[5] (Engels, 1954:67) identifies his assertion with Hegel's example of the boiling or freezing of water at specific temperatures, qualitative (discontinuous) leaps arising from quantitative (continuous) changes. [6] (Hegel, G.W.F. 1842: 217) Engels's "dialectics" of quality and quantity presented in his Anti-Duhring, became the founding text of dialectical materialism and orthodox for the Marxism. Until the present time there are political consequences related to the problem of Engels's appropriation of Hegel's Science of Logic. For both Marx and Engels (1848), the "law" of transformation of quantity into quality was the central key to the change from one mode of production to another. Dialectical materialism approach sees history as unfolding in qualitatively distinct stages such as ancient slavery, feudalism, and capitalism. (Barkley Rosser, J.,Jr., 1998/2000:5) Engels (1940, pp. 18-19)[7] confronted the contradiction between the apparently simultaneous acceptance of discontinuity arising from the idea of qualitative leaps and of continuity arising from the 'fuzziness' implied by the interpenetration of opposites in the dialectical approach.

Although Engels's discussion in *The Dialectics of Nature* was reasonably current with regard to science for the time of its writing (the 1870s and early 1880s), much of its content is seen to be scientifically inaccurate by today's standards, and many of its examples thus hopelessly muddled and wrongheaded. (Barkley Rosser, J.,Jr., 1998/2000:5)

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[5] Engels, F. 1954. Anti-Dühring: Herr Eugen Dühring's Revolution in Science, Moscow, Foreign Languages Publishing House

[6] Hegel, G.W.F. 1842. Enzyclopadie der Philosophischen Wissenschaften im Grundrisse, Part 1, Logik, Vol. VI, Berlin, Duncken und Humblot - 1842, p. 217

[7] Engels, F. 1940. The Dialectics of Nature, New York, International Publishers, 1940, p.18-19)

In contrast, Hegel's though and writing does not suffer the distance of time and appear scientifically accurate to the subject of contemporary topology as specific discipline of modern mathematics, although topology was just emerging in her protophenomenal form from Euler and Leibniz during Hegel's time.

Engels's law of transformation and the passage of quantitative changes into qualitative changes, as all of his three laws of dialectics become cliché in the mode of thinking of quality and quantity. These three principles established by Engels are not only oversimplified, but also misleading at best, establishing something quite self-evident, trivial and common. The notion of gradualness and gradual changes were criticized from the standing point of dialectical materialism as not leading to turning point and new quality, thus the new quality and qualitative change may appear in the new measure only through abrupt changes and qualitative leap. Due to this assumption, the notion of Qualitative quantity in Hegel's dialectics, remained inapparent. The attempt of dialectical materialism to separate the concept of quantity-quality transformations from the historicist system of Hegel, falls apart. (Coombs, 2013:29) and the attempts in classical Marxism to conceptualize a novelty-bearing event out of Hegelian dialectics necessarily reach an impasse. (Coombs, 2013:29).

Hegel's dialectics of qualitative and quantitative demonstrates two notions. The first is related with Hegel's determined quality, where discontinuous transformation implies and change occurs through a leap. The second notion is the topological notion of qualitative quantity, where gradual and continuous transformation implies and change is topological. The exhibit form of the qualitative quantity is topological continuous transformation without leap or abrupt displacements in the equilibrium.

These two notions of Hegel's qualitative and quantitative dialectics can be revealed through the catastrophe theory and chaos theory. The word catastrophe comes from Greek tragic drama and refers to the sudden twist of development in the plots. The catastrophe is related with the rhetorical figure of 'metalepsis'. Catastrophe theory is a method for describing the evolution of forms in nature and it is particularly applicable where gradually changing forces produce sudden effects. Catastrophe theory is interdisciplinary in character linking mathematics, biology, social sciences and philosophy. Catastrophe theory is represented by using topology since one of the central concerns of topology is to study the properties of spaces that do not change under a continuous transformation, that is, translation, rotation and stretching without tearing.

The catastrophe theory will be applied as moments of catastrophe by overlapping two opposing situations as a twist in the narrative, and the physical fold on the surface.

Linking mathematical definition of catastrophe models, such as the dynamic and static forms with the two notions of Hegel's dialectic of qualitative and quantitative, we could associate Hegel's dialectic of quanity and qualitative quantity with the structural stability, where a model is structurally stable if its qualitative behaviour is unchanged by small perturbations of the parameters, and with structural stability, where a model is structurally stable if its qualitative behaviour is unchanged by small perturbations of the parameters. Hegel's dialectic of quantity and determined quality can be associated with the catastrophe, where a sudden change in state is presented.

For René Thom this becomes the mathematical model of morphogenesis, of qualitative transformation from one thing into something else, following the analysis of D'Arcy Thompson (1917) of the emergence of organs and structures in the development of an organism. Furthermore, Thom explicitly links this to dialectics, albeit of an idealist sort:

"Catastrophe theory...favors a dialectical, Heraclitean view of the universe, of a world which is the continual theatre of the battle of between 'logoi,' between archetypes." [8] [9] (Thom, 1975A, 1975B:382). (Barkley Rosser, J.,Jr., 1998/2000:12)

More generally, Thom argues that catastrophe theory showed how qualitative changes could arise from quantitative changes as in Hegel's dialectical formulation.[10] (Barkley Rosser, J.,Jr., 1998/2000:12)

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[8] Thom, R. 1975A. Structural Stability and Morphogenesis: A General Outline of a Theory of Models, Reading, W.A. Benjamin; and Thom, R. 1975B. Catastrophe theory: its present state and future perspectives, in Dynamical Systems-Warwick 1974, Lecture Notes in Mathematics No. 468, Berlin, Springer-Verlag

[9] Thom, René, with response by E. Christopher Zeeman. 1975. "Catastrophe Theory: Its Present State and Future Perspectives," in *Dynamical systems-Warwick 1974*. Lecture Notes in Mathematics No. 468. Anthony Manning, ed. Berlin: Springer-Verlag, pp. 366-389.

[10] See Rosser (2000b) for further discussion.

Rene Thom’s work “From Catastrophes to Archetypes: Thought and Language” aimed to extend the techniques and assumption of catastrophe models of morphogenesis to human processes and societies. Thom used mathematical notations and language only to express vague correspondences among neurobiologycal states, thought and language. R. Thom established that “the sequence of our thought and our acts is a sequence of attractors, which succeed each other in catastrophes”. According to Thom language translates the mental attractors of our brain. When one wishes to formulate a sentence expressing idea, it was mathematically projected onto a space of admissible sentences, where several attractors prompeted. One was eventually chosen and the sentence was uttered.

In 1970, Thom presented sophisticated catastrophe theory model of language. He developed a visual representation of the verbs associated with spatio-temporal activity. This was, Thom would say 20 yars later, a “geometrization of thought and linguistic activity”.

Thom classified syntactical structures into 16 categories. He claimed that “the topological type of the interaction determines the syntactical structure of the sentence wich describes it.”

According to Thom, meaning and structure were no more independent. Rene Thom constructed a modeling practice which, roughly speaking, used topologically informed means of transformation, biologically inspired raw materials that he adapted to mathematical practice.

The idea of cobordism was deeply explored by René Thom, but the roots of cobordism is due to Henry Poincaré in his Analysis Situs and in its complementary papers. His idea of homology is very close to the modern framework elaborated by Thom. Homology classes were first defined rigorously by Henry Poincaré in his seminal paper "Analysis situs", *J. Ecole polytech.* (2) 1. 1–121 (1895).

Recalling beyond Heidegger's use of Aletheia, Gadamer establishes (Gadamer HG., 1976), [11] that language is an "element" within which we live in a very different sense than reflection. Language completely surrounds us like the voice of home which prior to our every thought of it breathes a familiarity from time out of mind. Heidegger refers to language as the "house of being". Gadamer concludes at the end of his essay, that ..But the language-ness of all thought continues to demand … Dialectic must retrieve itself in hermeneutics (Gadamer). (Gadamer HG., 1976)

Hegel's dialectic of quantity and determined quality can be associated with a catastrophe form that shows most of the phenomena occurring in catastrophe models is that of the three dimensional cusp catastrophe, Behaviour observable in such a dynamical system can include bimodality, inaccessibility, sudden jumps, hysteresis, and divergence. If what one wishes to do is to examine the structural stability of a particular pattern of bifurcation, or perhaps more specifically to compare the topological characteristics of two distinct patterns of discontinuities in economics, then proper catastrophe theory is clearly the most appropriate method to use for sufficiently low dimensional systems with gradient dynamics derived from a potential function. (Barkley Rosser, J.,Jr., 2004) The idea that huge, sudden, and revolutionary changes might happen had considerable widespread appeal, especially among more dissident intellectuals.

The applicability of Hegel's topological notion of qualitative quantity to the catastrophe theory or in reverse, of catastrophe theory to Hegel's logic as topology as logic of toposes of change, a logic which with it's categories and notions can provide both understanding and models for explanation of how small changes in certain parameters of a nonlinear system can cause equilibria to appear or disappear, or to change from attracting to repelling and vice versa, leading to large and sudden changes of the behaviour of the system or gradual and continuous change of system's behaviour.

[11] "Die Idee der Hegelschen Logik (1971) in Hans-George Gadamer, "Hegel's Dialectic: Five Hermeneutical Studies", translated into English by P. Christopher Smith and collected in (New Haven: Yale University Press, 1976). These five essays are "Hegel and Heidegger,"; "Hegel's Dialectic of Self-consciousness"; "Hegel and the Dialectic of the Ancient Philosophers,"; "Hegel's 'Inverted World,'; and "The Idea of Hegel's Logic," 75-99, "The Idea of Hegel's Logic" is one of the five essays by Hans-Georg Gadamer known in his "Hegel's dialectics: Five Hermeneutical studies" (1971).

The two notions of Hegel's dialectic of quanity with the determined quality and the qualitative quantity, can be associated with description of two types of dialectics as proposed by Martin Zwick (Zwick, M. 1978).

There are two distinguishable types of dialectics, one which results in victory of one of the opposing forces, and second which gives rise to a compromise or synthesis. Some dialectical phenomena are best modeled with the cusp, but others are more complex and more appropriately grasped with the butterfly, the butterfly of reconcillation, where the struggle of opposites within the cusp bifurcation set is itself negated. (Zwick, M. 1978).

The first notions of Hegel's dialectic of quanity with the determined quality can be associated as logic of the cusp catastrophe, and the second notion of Hegel's qualitative quantity can be associated as logic of the butterfly, the butterfly of reconcillation.

Perhaps, dialectic would retrieve itself through the language and hermeneitics in catastrophe theory as well. If the 'cusp' catastrophes could be well illustrated and explained through the well known "law" of dialectics of transformation of quantity into quality, thus through Hegel's notion of quantity and determined quality, the 'butterfly' catastrophe not only illustrates but embeds the logic and topology of Hegel's notion of qualitative quantity.

There is striking analogy between Hegel's fourfold of infinities [1/. the bad qualitative infinity; 2/. the good qualitative infinity; 3/. the bad quantitative infinity; 4/. the good quantitative infinity] or the fourfold cobordism of the categories of:

- quantitative quantity (in the domain of Chronochora – Abstract Space and Abstract Time);
- quantitative quality (in the domain of Chronotopos –Meaningful Place and Abstract Time);
- qualitative quantity (in the domain of Kairochora – Abstract Space and Meaningful Time);
- qualitative quality (in the domain of Kairotopos – Meaningful Place),

and **the four parameters of the butterfly catastrophe**, where the volatile dyad is changed into a precarious triad and then into a stable tetrad.

The topological notion of Hegel's logic contains the seed of topological hermeneutics with "genuine infinite, a circle closed on itself" …The infinite that wants to be unlimited, because as Hegel points out – "there are two worlds, one infinite and one finite, and in their relationship the infinite is only the limit of the finite and is thus only a determinate infinite, an infinite which is itself finite." (Miller,A.V. trans.,1990. Hegel's Science oif Logic)

Once visualized in topological space through the notion and model of Rene Thom's 'cobordism', Hegel's fourfold model of multiplicities (infinities) build on the logical interrelations and interdependence between the categories such as quality and quantity, the model that unfolds the 'qualitative quantity' notion of place within the time, could be exhibited also by the bifurcation diagrams.

The striking resemblance between the philosophical or dialectical hermeneutics of the categories of quality and quantity, seen through the model of 'cobordism" - the "pair of pants" – or - the cobordism of hermeneutical circle showing how dialectical hermeneutics (must) retrieve itself in topology, namely the Hegel's fourfold of infinities (Quantitative quantity; Quantitative quality; Qualitative quantity; Qualitative quantity), could be thought as **circle or four circles placed in a 3-manifold, and the pitchfork bifurcation**, presented in the figure bellow, where the **supercritical** and **subcritical** components in the diagram, draw the **two half planner cilinder** from our example of **cobordism**.

**Methodology, theoretical framework and structure**

In my methodological approach I follow the consideration that method and subject matter in Hegel are homologically equivalent. In the very last chapter of Hegel's Science of Logic, method and subject matter supposedly conjoin. Method is the one and only subject. We have the Understanding, its negation, Dialectical Reason, and the Negation of the Negation -Speculative Reason. [12] (Carlson 2005).

Fundamental method of this dissertation is detailed analysis of texts, both Hegel's own texts and text from various sources within the area of philosophy, philosophical topology (perhaps

[12] David Gray Carlson, 2005, Why Are There Four Hegelian Judgments?, p.114:125, in Hegel's theory of the subjects, David G. Carlson, ed. 2005, Palgrave Macmillan 2005

first coined by Jeff Malpas) and mathematics, in particular topology, whereby individual sentences and words are analysed step by step with an exanination of their internal and external contexts. From a pragmatic and constructivist perspective, contextual responsiveness is employed with the purpose to gain insight by juxtaposing methods conducted using clearly defined and diverse research paradigm.

Language is the universal medium of understanding and Hegel's language, syntax and semantic is approached here through interpretive methods of applied hermeneutic, in particular the method of topological hermeneutics that facilitate the topological reading of Hegel. Hegel's Logic and category of 'qualitative quantity' is discussed in relation with **Rene Thom's topological theory of language and topological syntax.** Angel Lopez Garcia, who introduced the "topological linguistics", gives a rather critical reading of Thom`s proposals related to verbal semantics and the structure of basic propositions. Garcia compares Thom's specific analysis of verbs with the tradition of structuralism (Hjelmslev, Jakobson, Halliday, Chomsky) and the models: "Liminar Grammar" and "Topological Linguistics" proposed by himself. [13](Garcia, 1990)

In relation with Hegel's Science of Logic, in particular his categories from the Objective Logic,[14] within all over than 80 categories, presented by Hegel, in the present study I follow Rene Thom's work "From Catastrophes to Archetypes: Thought and Language", and Thom's catastrophe theory model of language, his visual representation of the verbs associated with spatio-temporal activity, his classified syntactical structures into 16 categories, where he claimed that "the topological type of the interaction determines the syntactical structure of the sentence wich describes it."

The novel result of the study is the proposed model of Hegel's fourfold of infinity builds on the concept of topological cobordism, deeply explored by René Thom, after Henry Poincaré's Analysis Situs, Thom's idea of homology and the homology classes, first defined rigorously by Henry Poincaré in his seminal paper "Analysis situs", *J. Ecole polytech.* (2) 1. 1–121

[13] Garcia, Angel Lopez, (1990), Introduction to Topological Linguistics - Annexa-LynX. Valencia-Minnesota, 1990.

[14] Hegel's "Encyclopedia of Philosophical Sciences", Part One, referred to as The Lesser Logic, See "Hegel's Logic", translated by William Wallace, with Foreword by J N Findlay, Clarendon Press 1975. First published 1873

(1895), also presented by D'Arch Thompson's concept of homology and structural stability "On Growth and Form" (1917).

Structural analysis techniques are employed to achieve thesis development related objectives, to process the different sources of research data and to integrate their parts into the whole, to facilitate the fusion of horizons - the transition from one level of understanding to next, from Hegel's language, syntax and semantics, pictorial thinking and powerful use of metaphors such as metalepsis, from Hegel's mathematics to Hegel's topology.

The language and syntax of Hegel's Logic and the rhetorical power of expression determine the need topology to be regarded and discussed here not only as discipline of mathematics but also in the rhetorical mode – topological reading and metaphors, in particular Hegel's use of metalepsis and the theories related with the concept of metalepsis. Hegel's enterprise aims at 'creating a new philosophical glossary by exploiting existing ambiguities and connotations of ordinary language. In declaring this program [. . .], Hegel specifically offers to make systematic distinctions between terms that are usually considered to be synonyms, especially the set: Existenz, Dasein, Wirklichkeit, etc.' (Yovel 1981: 117). Foolowing Yovel, (Yovel 1981) and Berto, (Berto 2007:20), here I am investigating Hegel's Dialectics as a semantic theory.

Interpretation employed in the present thesis follow the transition from typological mode to the topological mode proposed by Jay L. Lemke as mixed-mode semiosis - Typological vs. Topological Semiosis, [15] [16] from "typological" meaning-making, meaning-by-kind /natural language/ to "topological" meaning-making, meaning-by-degree /visual language/, with continuous variation or "topological" meaning." If meaning-by-kind is qualitative, the meaning-by-degree is quantitative, and with this methodological approach only the

[15] Jay L. Lemke, "Topological Semiosis and the Evolution of Meaning" – http://www-personal.umich.edu/~jaylemke/webs/wess/index.htm

[16] Jay L. Lemke, Typological vs. Topological Semiosis – .http://www-personal.umich.edu/~jaylemke/webs/wess/tsld002.htmSee: Jay L. Lemke, "Mathematics in the middle: measure, picture, gesture, sign, and word, and Opening Up Closure: Semiotics Across Scales".

topological notion of Hegel's qualitative quantity could reveal Hegel's Dialectics as topological semantic theory.

Mathematical Topology (as mathematical and formal thinking) has a place in the structure of language and the structure of language can inform us about the structure of the world we think about. Logic ('the science of the Idea in and for itself' – §18), relates to the most fundamental (structural relationships between) categories in language, i.e. it consists of categories without which the world would certainly be unintelligible, distinctionless white noise (such as Being, Becoming, the One and its Other) without however considering the application of these to the world itself. Hegel's use of "die Mitte" is interpreted through the topological notion of 'betweenness', 'in-between' in the pure meaning of Plato's 'methaxis' and Aristotle's 'metalepsis' or exactly in accordance with Voegelin's concept of metalepsis as 'in-between-ness'. The mediator is between the 'outside' and 'inside', between the 'quality' and 'quantity', between the 'time' and 'place', between the 'image' (bild) and the 'concept' (begriff). One of the strongest presence of topological cognition in Hegel is his use of *Vorstellung (Picture-thinking, Figurative thinking).* The verb *vorstellen* literally means "place before" (*vor* = before; *stellen* = to place) and directs to the topos (place), having a topological characteristic. The first language of infinity is the image! The image is the form of recollection. Donald Philip Verene emphasizes the role of the image in philosophical text, stating that "Any philosophical text depends upon images; they are always present. The reader can look first not for arguments in the work but for these root images. Once found, the reader can look for the questions that can be drawn forth from the images. The reader will then see how the image is directing and providing support for the question, which carries the reasoning process of the text forward. What are the images? What are the questions embedded in them?" (Verene 2007:xiv-xv).

Interpretations presented in the thesis are build on results of the mixed method and analytical approach, produced through triangulation of sources, with examination of the consistency of different data sources from within the same method, comparing philosophers and researchers with different view points. Analyst triangulation is employed using multiple analyses to review findings with the goal to understand multiple ways of seeing the research issues and thesis.Through theories perspective triangulation, multiple theoretical perspectives are used to examine and interpret the research hypotesis and relevant data.

In relation to the language, Hegel's language and philosophy, and with the purpose to build relation between Hegel's categories and concept and the mathematical, topological models, some important mathematical, in particular algebraic topological models and concepts are employed in the discussion and results of the present thesis, such as manifold, cobordism, graph theory, topological data analysis, homology, via the new theory of **persistent homology,** simplicial complexes, Betti numbers, **barcodes**. The result of my demonstration shows how the logical structure of concepts and syntax in Hegel's logic can be presented as topological space – a series of simplicial complexes.

The theories I implement in relation to Hegel's logic are concerned with Lucien Tesnière **and Rene Thom topological theory of language and topological syntax** (Wildgen and Brandt 2010:57), Thom's concept of versal unfolding[17] [18] and his catastrophe theory model of language, a visual representation of the verbs associated with spatio-temporal activity, related with what Thom calls a "geometrization of thought and linguistic activity", due to my assertion that syntactical analysis of Hegel's language demonstrate strong dependence relations between superordinate and subordinate lexical entities. There is a hierarchy of connections in Hegel's language, not only between lexical entities in a sentence, but also between different terms, categories, concepts and notions. Something more, the structure and relations between the core terms in Hegel demonstrate not only hierarchy but heteronomy – horizontal relation between lexical and logical entities. This structure of syntax in Hegel's text revieals both vertical and horizontal direction organized in topological mode. There is a strong emphasis on relations between lexiacal entities in Hegel, where dependence and subordination is mutually oriented in simultaneous way, both in vertical (hierarchical) and horizontal (heterarchical) order. In Hegel, language is structured in terms of dependency relations between superordinate and subordinate parts (with the verb as the kernel and other word classes. The structural configurational order of Hegel's language, the topological space of narrativ, prevails over the linear, combinatorial order. The schematization of sentence structure in Hegel does not simply mirror or reproduce its linear order.

[17] Bruce, B. and D.N. Mond, eds. Singularity Theory, Cambridge, England: Cambridge U.Press,1999, page xi.

[18] Peter Tsatsanis, On Rene Thom Sinificance for Mathematics and Philosophy, Scripta Philosophicae Naturalis 2:213-229 (2012), p.223-224.

Rene Thom's catastrophe theory is discussed and applied to Hegel's dialectic, with the assertion that some dialectical phenomena are best modeled with the cusp catastrophe model (as the transition from quantity to quality and establishing of the new measure through abrupt changes), but others are more complex and more appropriately grasped with the butterfly catastrophe model of reconcillation (the gradual transition of qualitative quantity), where the struggle of opposites within the cusp bifurcation set is itself negated. (Zwick, M. 1978).

Core place within my methodological interpretation of Hegel's Logic is the concept of manifold and topological notion of 'cobordism' (Henry Poincaré, René Thom), also the concept of homology elaborated by Thom and first defined rigorously by Henry Poincaré in his seminal paper "Analysis situs", *J. Ecole polytech.* (2) 1. 1–121 (1895).

In addition my topological interpretation of Hegel's qualitative quantity employs the concept of homology related with the works of D'Arch Thompson, especially his "On Growth and Form" (1917). In the last chapter of "On Growth and Form", D'Arcy Thompson's illustrates his "cartesian transformations" of animal forms. Thompson's mappings are referred to as "rubber sheet" mappings. D'Arch Thompson suggested that one should study the change from one biological form to another by examining the unique mathematical object that maps between them in accord with biological homologies.

Within the discussion of Dialectics and Catastrophe, I suggest an model of Cobordism of Hegel's fourfold of multiplicities (infinities) implemented and exhibited in bifurcation diagrams, namely the Pitchfork Bifurcation Diagrams - Supercritical Pitchfork Bifurcation Diagrams ($b < 0$) and Subcritical Pitchfork Bifurcation Diagrams ($b > 0$).

Following William Lawvere's suggestion that a significant fraction of dialectical philosophy can be modeled mathematically through the use of "cylinders" (diagrams of shape Δ) in a category, wherein the two identical subobjects (united by the third map in the diagram) are "opposite" (Lawvere, 1996), [19] my proposition and interpretation of Hegel's Logic implements two topological constructs. The first based on cylinders is the cobordism and the second based on the shape Δ addresses the simplicial complex.

[19] F. William Lawvere, 1996, Unity and identity of opposites in calculus and physics, Applied Categorical Structures, June 1996, Volume 4, Issue 2, pp 167-174

With the series of research papers and projects between 2011-2014, published and not officially published but available for review under academia.edu, I have demonstrated how applied research and empirical research methods, based on implementation of Hegel's Logic, in particular topological notion of qualitative quantity, can be used for the purpose of prospective research in the area of social dynamics, in particular within the field of applied ethics, ontology and epistemology, economics, auditing and law, emphasizing on topological approaches to the said fields.[20]

As we know, both qualitative and quantitative research methods can be used to collect data and information through the field research. As a complimentary to the traditional qualitative and quantitative methods, the novel method of 'topological qualitative quantity' can be tested

[20] See:Borislav Dimitrov:

**2014**, Auditor Independence within An Auditing Analysis Situs: Topology of Places as factor for enhancing auditor independence, competence, and audit quality, between the global and local: Topological Approach to Audit Dynamics, Focused on Auditor Independence, Competence and Audit Quality through Qualitative quantity methodology and Topological Data Analysis, Philosophy of Science for Social Science, Lund University, Faculty of Social Science; **2014**, The Struggle of Cultural Identity between the Dichotomies of Society and Community, between Liberalism and Communitarianism: Dialogue or becoming Topological? Philosophical topology of intercultural (identity) relationships, Study presented at the International Conference 'The Individual and Society: Challenges of Social Change", April 5th, 2014, Sofia, Bulgaria (Bulgarian Academy of Science and Arts, Serbian Royal Academy of Science and Arts, European Center of Business, Education and Science, published in the Conference edition collection 'The Individual and Society: Challenges of Social Change' (ISBN 978-954-411-151-9), 2014, p. 266-296; 2013 "Topological Ontology and Logic of Qualitative quantity", https://www.academia.edu/3237237/Topological_Ontology_and_Logic_of_Qualitative_quantity; **2012**, A Topological Approach to 'The Hospital of the Future': Topological Model based on the qualitative quantity research method, Amazon; 2012 "The Topological Approach of Qualitative quantity Implemented in Autopoietic Law and Audit: The Cultural Phenomenology of Qualitative quantity and … The Cultural Phenomenology of Law and Auditing as Autopoiesis", „Ariadne – Topology and Cultural Dynamics – Institute for Cultural Phenomenology of Qualitative quantity", http://ariadnetopology.org/3.html; **2012** "Cultural Phenomenology of Law and Topological Approach to Law", A Series of papers presenting the essentials of Topological Approach to Law : Qualitative quantity - The Cultural Phenomenology of Literature and … The Cultural Phenomenology of Law; Law and Literature Movement; Cognitive Science and The Law - Topological Approach To Law; Phenomenology of Law; Law and Social Choice: Qualitative quantity, Topological Social Choice and Topological Approaches to Law; The proposition of Qualitative quantity mode of Inquiry in the Classic Debate – Qualitative vs Quantitative research; The Topological approach of Charles Sanders Peirce's qualitative-ness and The Topological Qualitative quantity; The philosophy of Émile Boutroux – a profound influence on Henri Poincaré and Charles Peirce. „Ariadne – Topology and Cultural Dynamics – Institute for Cultural Phenomenology of Qualitative quantity", http://ariadnetopology.org/3.html; **2011** "The Relevance of Topological Approach, based on Qualitative quantity research method, to Audit Dynamics and Auditing Research – Cultural Phenomenology of Audit and Auditing research", „Ariadne – Topology and Cultural Dynamics – Institute for Cultural Phenomenology of Qualitative quantity", http://ariadnetopology.org/Cultural_Phenomenology_of_Audit_Dynamics_and_Auditing_Research_web.pdf for the ability to capture and provide insights on the processes exhibiting gradualness and continuous and transformation (persistence), emphasizing on the relation and relationships in space seen as topological space (topological notions and concept such as ‘homeomorphism’, ‘homology’, ‘boundaries’, ‘closeness’, ‘part and whole’, ‘inclusion’ and exclusion’) .

Following Ivan Punchev’s inquiry addressing Hegel’s Logic and dialectical method and mathematics, and paraphrasing Punchev’s question that embeds the relationship between philosophy and mathematics, with my own emphasis on topology, indeed the question - How to build a theory of philosophical-topological or topological-philosophical understanding through categorial "interpretation" of topological "structures"? – methodlogically, with the present thesis I explore such hypothesis and suggest the conclusion that the language elements, categories and notions in Hegel’s syntax in Science of Logic can be perceived as the data collected that can be mapped and analyzed through Topological Data Analysis, a recent mathematical method for analyzing data that has had new, dramatic, and unexpected applications to statistics among other areas.

Topological data analysis uses a branch of mathematics called algebraic topology to capture the shape of a point-cloud data set that "persists" in a dynamical setting. For this purpose the data, themes or issues will be represented as shape of a point-cloud data set that "persists" in a dynamical setting.

The main problems subject of Topological Data analysis could be recognized in Hegel’s Science of Logic, and these are the following:

- how one infers high-dimensional structure from low-dimensional representations; and
- how one assembles discrete points into global structure.

Both of the problems are very much related with the role of the philosopher and applied philosophy - to assemble discrete point into global structure and to enhance or improve the low-dimensional representations into the high dimensional structure.

Topological Data Analysis focusses on continuous flow, recognizing that the human brain can easily extract global structure from representations in a strictly lower dimension, i.e. we infer a 3D environment from a 2D image from each eye. The inference of global structure also occurs when converting discrete data into continuous images, e.g. dot-matrix printers and televisions communicate images via arrays of discrete points.

The main method used by topological data analysis is:

1. Replace a set of data points with a family of simplicial complexes, indexed by a proximity parameter. This converts the data set into global topological objects.

2. Analyse these topological complexes via algebraic topology — specifically, via the new theory of **persistent homology**.

3. Encode the persistent homology of a data set in the form of a parameterized version of a Betti number which will be called a **barcode**.

In Topological Data Analysis, the primary mathematical tool considered is a homology theory for point-cloud data sets—**persistent homology**—and a novel representation of this algebraic characterization— **barcodes**. Topological Data Analysis considered the shape of data.

For the purpose of the present thesis I discuss the relation between Topological Data Analysis and Hegel's claim about gradualness, and "the attempt to explain coming-to-be or ceasing-to-be on the basis of gradualness of the alteration" (the passage of § 777 from Hegel's "The Science of Logic" /The Greater Logic/, where Hegel asserts that:

"In thinking about the gradualness of the coming-to-be of something, it is ordinarily assumed that what comes to be is already sensibly or *actually in existence*; **it is not yet perceptible only because of its smallness.** Similarly with the gradual disappearance of something, the *non-being* or other which takes its place is likewise assumed to be *really there*, **only not observable**, and *there*, too, not in the sense of being implicitly or ideally contained in the first something, but *really there*, **only not observable**." /§ 777/ - /*bold by me B.D*/.

Applicable to the Hegel's statement is the discussion provided by Magnus Bakke Botnan in his work "Three Approaches in Computational Geometry and Topology - Persistent

Homology, Discrete Differential Geometry and Discrete Morse Theory" [21] about the discrete presentation of data /information/ and our ability to perceive gradualness of information and the continuous nature of the shape of data. Botnan states that "One of the most remarkable properties of the human brain is the ability to infer the world as a three-dimensional space. We do not see three spatial dimensions directly, but from experience we know how to visualise three dimensions via sequences of paired planar projections. In other words, we know how to extract global structures by studying representations from a strictly lower dimension. Another skill developed is how to infer a continuum from discrete data. As an example, consider the painting The Seine at La Grande Jatte by the French artist Georges Seurat. This painting consists of discrete data points and is obviously noisy. Nonetheless, we have no problems perceiving the tree by the waterline, the person in the kayak or the sailboat. Rather than altering out noise qualitatively it is favourable to have a quantitative measure." [22]

The result of my discussion is based on the graph theory and demonstrates how the nodes from the structure of Hegel's Science of Logic can be represented as a network and graph. For the purpose of such demonstration and simplification in the following lines and illustration I use four elements, corresponding to the four syllogisms in Hegel. Graphs are among the most ubiquitous models of both natural and human-made structures. They can be used to model many types of relations and process dynamics in physical, biological and social systems. Many problems of practical interest, such as *data*, issues, themes and clusters, can be represented by graphs. Gpaphs are used to represent networks of communication, data organization, computational devices, the flow of computation, etc.

In addition, the result of my discussion demonstrates how the toposes from the structure of Hegel's Science of Logic[23] (in particular from II. Second Part - Magnitude (Quantity) with three chapters: Chapter 1 – Quantity - Chapter 2 – Quantum – Chapter 3 - Quantitative Infinity, and will take the following subsection with 9 themes from each of the three chapter: A. Pure Quantity - B. Continuous and Discrete Magnitude - C. Limitation of Quantity (from Chapter I Quantity); A. Number - B. Extensive and Intensive Quantum - C. Quantitative

[21] Magnus Bakke Botnan in his work "Three Approaches in Computational Geometry and Topology - Persistent Homology, Discrete Differential Geometry and Discrete Morse Theory", 2011
[22] Magnus Bakke Botnan in his work "Three Approaches in Computational Geometry and Topology - Persistent Homology, Discrete Differential Geometry and Discrete Morse Theory", 2011

[23]

Infinity (from Chapter 2 Quantum) and A. The Direct Ratio - B. Inverse Ratio - C. The Ratio of Powers (from Chapter 3 The Quantitative Relation or Quantitative Ratio) – can be represented through the concept of topological persistence from Topological Data Analysis. Such result demonstrate how the transition from typology to topology is possible. In this demonstration I am using Euler Characteristic of the shape, the simplicial complexes, Betti numbers, barcodes.

**The structure of the thesis** is mixture of classic structure with Introduction, Methods, Results, Discussion and thematic chapters. The discussion of methodology and theoretical framework are exposed in the introductory chapter yet the development of both method and theoretical framework flow in-progress through the discussion integrated into the themes of the individual chapters. The review of literature is embedded in the discussion, segmented into the series of thematic chapter on several topics.

An overview of the present thesis in part was submitted in 2014 with pending publication in Sophia Philosophical Review under the title Hegel's Analysis Situs: Topological Notions of Multiplicity in Hegel's Fourfold of Infinities.[24]

The section 5 and 6 [25] from the fourth chapter of this study was presented at conference Evolution of Hierarchical Systems, held in Sofia, 2014, and at the International Conference ‘The Individual and Society: Challenges of Social Change", in Sofia, 2014, before the Bulgarian Academy of Science and Arts, Serbian Royal Academy of Science and Arts, European Center of Business, Education and Science, and subsequently published in an expanded form in the conference edition: Evolution of Hierarchical Systems, Sofia, Faber

[24] Dimitrov, Borislav. (2014), Hegel's Analysis Situs: Topological Notions of Multiplicity in Hegel's Fourfold of Infinities, pending publication, Sophia Philosophical Review, 2014

[25] Part 5. Dialectics and Catastrophe: Cobordism of Hegel’s fourfold model of multiplicities (infinities) exhibited in bifurcation diagramsvPitchfork Bifurcation Diagrams - Supercritical Pitchfork Bifurcation Diagrams (b < 0) and Subcritical Pitchfork Bifurcation Diagrams (b > 0); and Part 6. Dialectics and Chaos: Rethinking of the evolution of hierarchical systems through Hegel’s fourfold model of multiplicities exhibited in Feigenbaum Diagram - The relations between the algebraic topology and evolution - The Role of Heterarchy and Heteronomy in Evolution.

Publishing House, September 2014 [26] and in the Conference edition collection 'The Individual and Society: Challenges of Social Change'. [27]

***Megiston topos: hapanta gar chorei***<br>
Μέγιστον τόπος· άπαντα γαρ χωρεί<br>
"Space is the greatest thing, as it contains all things"

-- Thales

[26] Dimitrov, B. (2014), Philosophical topology and Topological philosophy as the mode of thinking of Evolution of Hierarchical Systems: The Role of Heterarchy and Heteronomy in Evolution, Conference Edition: Evolution of Hierarchical Systems, Sofia, Faber Publishing House, September 2014, p.285-318

[27] Dimitrov, Borislav. (2014), The Struggle of Cultural Identity between the Dichotomies of Society and Community, between Liberalism and Communitarianism: Dialogue or becoming Topological? Philosophical topology of intercultural (identity) relationships, Study presented at the International Conference 'The Individual and Society: Challenges of Social Change", April 5th, 2014, Sofia, Bulgaria (Bulgarian Academy of Science and Arts, Serbian Royal Academy of Science and Arts, European Center of Business, Education and Science, published in the Conference edition collection 'The Individual and Society: Challenges of Social Change' (ISBN 978-954-411-151-9), 2014, p. 266-296.

## Chapter 1 Topology, Hegel, Categories and Philosophical Topology

### 1.1. Topology

Topology is a major area of mathematics concerned with properties that are preserved under continuous deformations of objects, such as deformations that involve stretching, but no tearing or gluing. In topology, any continuous change which can be continuously undone is allowed. So a circle is the same as a triangle or a square, because one can just `pull on' parts of the circle to make corners and then straighten the sides, to change a circle into a square. Then one can just `smooth it out' to turn it back into a circle. These two processes are continuous. During each of these two actions, nearby points at the start are still nearby at the end. In topology we can transform a spatial body such as a sheet of rubber in various ways which do not involve cutting or tearing. We can invert it, stretch or compress it, move it, bend it, twist it, or otherwise knead it out of shape. Certain properties of the body, the properties of the ‘qualitative quantity’ (Dimitrov B., 1989-2014), will in general be invariant under such transformations, which is to say under transformations which are neutral as to shape, size, motion and orientation. The ‘qualitative quantity transformations’ can be defined as being those which do not affect the possibility of our connecting two points on the surface or in the interior of the body by means of a continuous line.

Topology (Analysis Situs) is the analysis of situating and forms of location of specific objects in specific spaces. Incompletely and imprecisely speaking, length of line segment, size of angle or area of the plane play an important role in geometry. In general topology, however, those characteristics are not substantial. From a topological point of view, the triangle is identical to square, straight line to line segment without ends, and plane to sphere without one point. Thanks to its generality, topology can study abstract spaces, exceeding highly Euclidean/non-Euclidean paradigm, used often in philosophical argumentation. (Skowron, 2014)

Topology as an area of mathematics is concerned with the properties of space that are preserved under continuous deformations, such as stretching and bending, but not tearing or gluing. Topology is concerned with the relationships between different forms, shapes, properties of space, where connectedness is an important topological property. Although there is no absolute distinction between different areas of topology (algebraic topology, differential topology, geometric topology, here we regard to general topology. Within the list of general topology topics are such topics as . . . Absolutely closed – Accessible - Accumulation point – Limit point - Almost discrete (space) – Approach space (a generalization of metric space based on point-to-set distances, instead of point-to-point) – Boundary – Bounded – Closure – Completeness – Component – Connected – Connected component – Cpntinuous – Continuum – Cover - Covering – Dense – Density - Developable space – Development – Discrete – Distance – Exterior – Homeomorphism – Homogeneous – Homotopy – Limit – Limit point – Loop – Metric - Metric invariant – Metric Map – Metric space – Metrisable (A space is metrizable if it is homeomorphic to a metric space) – Neighborhood – Net – Open – Open set - Partition of unity – Path – Path connected – Point.

In philosophy, the scientific knowledge was associated with geometry since the antiquity to the beginning of 20th century. Topology is concerned with the conceptual analysis of spatial notions, such as “space in general”, “connectedness”, “neighborhood”, “approximation”, “convergence”, “continuity”, “mappings”, “transformations”, “boundedness”, and many others.

As Bartłomiej Skowron asserts in his paper *The Forms of Extension, Substantiality and Causality* – “Topological properties in the strict sense are the invariants under some transformations, under homeomorphisms, which are bicontinuous one-to-one functions.“ (Skowron, 2014)

These transformations are good tools to describe the objects’ natural change over time (but not only over time) while preserving the identity of these objects.

It is often thought that mathematics is the science of quantities (length, number, etc.). Contemporary research in mathematics, however, considers also quality. Topology is an example of quality thinking in mathematics. Angles and lengths are not very important, qualities such as connectedness or denseness are only important. (Skowron, 2014)

More recently Thomas Mormann in many of his papers (Mormann, 1995; 1996; 1997; 2000; 2013) develops ontological issues with topology. Mormann discusses topology in general in his *Topology as an Issue for History of Philosophy of Science* (2013)". In his *Topological Aspects of Combinatorial Possibility* (1997) Mormann considered combinatorial worlds as mappings from individuals to properties. He draws a line between possible and impossible combinations by imposing structural constraints on the relation between a set of individuals and a set of properties, namely he forced on that relation to be a function. He also proposed to treat the complex individuals and complex properties as open sets. On that basis he could define possible worlds as continuous functions. In the concluding remarks of the above mentioned paper, Mormann writes as follows: A world is, so to speak, a *topologically structured* totality of states of afairs, or, to express somewhat more generally, it is a structural gestalt.

As Arkady Plotnitsky asserts in *Badiou's later Experimenting with ontologies: sets, spaces, and topoi with Badiou and Grothendieck* (2012), "While both geometry and topology are concerned with space, yet they are distinguished by their different ways of studying space. Geometry (geo-metry) has to do with measurement; topology disregards measurement and scale and deals only with the structure of space qua space, for example, with the essential shapes of figures, seen as continuous spaces. Distances are generally irrelevant. It is only continuity (as connectivity) or conversely rupture of continuity that matters, which is why topology defines space via its so-called open subspaces, such as those (called neighborhoods) around each point. (Plotnitsky 2012:334)

In topology, "Insofar as one deforms a given figure continuously (ie, insofar as one does not separate points previously connected and, conversely, does not connect points previously separated), the resulting figure is considered the same. The proper mathematical term is `topological equivalence'. Thus, all spherical surfaces, of whatever size and however deformed, are topologically equivalent, although some of the resulting objects are no longer spherical, geometrically speaking.” (Plotnitsky 2012:335) Such figures are, however, topologically distinct from the surfaces of tori, since these two kinds of surfaces cannot be transformed into each other without disjoining their connected points or joining the disconnected ones: the holes in tori make this impossible. This is sometimes expressed by saying that, rather than measuring distances, as with geometry, topology `measures' (counts) the number of holes in a spatial object. (Plotnitsky 2012:335)

For Plotnitsky, “Topology is mathematical not by virtue of mathematizing spatiality by measurement, as geometry does, but by virtue of relating the architecture of spatial objects to algebraic or numerical properties of algebraic or arithmetical objects. The number of holes in a given object such as the surface of a sphere with no holes in it vis-a-vis that of a torus, which has one hole in it (or the surfaces of pretzel-like figures, each with several holes in it)öis the simplest example of this kind of relation.” (Plotnitsky 2012:335)

Plotnitsky discusses the field of topology known as `algebraic topology' which studies topological spaces by relating them to algebraic objects, particularly the so-called groups. Groups are defined by abstract elements and a multiplication-like operation upon them, resulting in the elements of the same kind. He asserts, “Thus (glossing over technical specifics), whole numbers form a group with respect to addition, but not multiplication, since an inverse of a whole number is a fraction. Rotations of the circle or of the two-dimensional surface of the three-dimensional sphere also form a group, with its operation defined as that of performing consecutive rotations. In the case of the surface of the sphere, the order of rotations may change the outcome, which is expressed mathematically by saying that this group is not commutative. Groups play a major role in algebraic topology, and in category and topos theories, especially in the so-called homotopy and cohomology theories, which deal with certain groups, defined by the topology of the corresponding spaces and essential for studying these spaces. In the case of two-dimensional surfaces these groups are linked to the numbers of holes in them, as described above, and these groups are, accordingly, different for spheres and tori, for example." (Plotnitsky 2012:335)

Through its Latin root (locus) and Greek root (topos), place already etymologically informs locational and topological analysis. Understood in terms of the Greek topos, place is itself directly tied to the idea of topological notions such as 'relation', 'between-ness', 'limit' or boundary' (see Aristotle, 1983: 28 [212a5]).

Our world is undergoing profound change due to the advanced research in topology and significant importance of topological thinking and topological meaning making. Topology itself is the science of the change. Topology is extremely applicable to the complex dynamics systems. The unique and modern treatment of topology today is employing a cross-disciplinary approach.

Topology has been transformed from a theoretical field that highlights mathematical theory to a subject that plays a growing role in nearly all fields of scientific investigation.

### 1.2. Why Topology? – Hegel's Logic as topological place of thinking: the topos of being/logos

Back in 1989, as a Research Associate at Institute for Philosophical Research at Bulgarian Academy of Science, I have established Hegel's category and notion of 'qualitative quantity' (Dimitrov, 1989a) as vocamen of my research interests in philosophy, in particular within the area that can be defined as philosophical topology.

In two papers "Quality of quantity" (Dimitrov, 1989) and "Quality and Time" (Dimitrov, 1990), I have established the resurgence of G.F.W. Hegel's category "qualitative quantity" historically left "inapparent" arguing that this category is the real breakthrough from the cliché of the known first law of dialectics – the the law of transformation of quantity to quality and the appearance of the new quality as qualitative leap. Due to the gradual and continuous notion of qualitative quantity, linking Henri Poincare's Analysis Situs with Hegel's qualitative quantity, I have established and supported the argument that the exhibit form and notion of the Qualitative quantity is Topology and topological homeomorphism. Based on the exploration of D'Arcy W. Thompson's "Growth and Form" (1917) findings and examples from Hermann Haken (Haken, 1983), illustrating the qualitative quantity notion in the structural stability, in "Quality of quantity", I have argued that topology is the field of qualitative quantity and topological homeomorphism is exhibit form of this category. The Qualitative quantity is applicable to the complex dynamic systems as complimentary method to the qualitative and quantitative approaches and methods in the paradigm shift from Typological to Topological thinking.

Since my first publications on the qualitative quantity (Dimitrov, 1989; 1990) until my recent publications (Dimitrov, 2012; 2014), for the past twenty five years, the significant developments of the science and philosophy of science related to the concept of qualitative quantity emerged, evidently enriching the grounds of my thesis. Topological thinking is now undergoing aresurgence, with some highly practical research and applications [28] and the inquiry within philosophical topology, as I call it outside of the Hegel's context, has become increasingly central to my work over the last twenty five years. I believe that the emergence of 'topological' inquiries presented in philosophy after Hegel . . . is indebted to Hegel's topological thinking.

The manner in which Hegel develops his concepts and categories in the Science of Logic can be described as extension. From etymological point of view, the extension is that, what spreads, *extends* (from Latin *ex*- "out" + *tendere* "to stretch") between one thing and another. The extending is what is "between" this and that, what from its nature must be between some ends. (Skowron, 2014) Bartłomiej Skowron emphasizes that extension is "being between" and "being between is being from – to" and "being between is being everywhere (or almost everywhere) between." (Skowron, 2014) All mentioned moments of extension, i.e. spatial dimension (immersing in space), divisibility, impenetrability, being everywhere "between" in specific sets, determine various kinds of extension but do not consist of its definitional perspective, for we do not know yet, what is the very extension. M. Rosiak described it as the possibility of co-existence of a multitude of objects, abundance of diferences between places. (Skowron, 2014)

[28] ATACD project /A Topological Approach to Cultural Dunamics/, funded by EU, a research network based on the mathematical theories of topology.

The new direction for discussion on extension are discussed by Bartłomiej Skowron in his paper *The Forms of Extension, Substantiality and Causality*, (Skowron, 2014)

**As** Bartłomiej Skowron asserts, "The appearing characteristics of extension are often concerned pure geometrical notions, such as space, distance, solid (and its shape). In a natural way, when we want to give an example of extending structure, we quote *continuum*, 3-dimensional Euclidean space or plane. It makes sense, because in philosophy space was conceptualized by geometrical notions. But, on the other hand, Bergson and Ingarden warn against geometrization of philosophical issues, and especially against geometrization of extension of time." (Skowron, 2014) Skowron emphasizes that "it seems (that) geometry lost its explanatory power in philosophy, and especially in metaphysics. And it is so, indeed. Nowadays, however, geometry, being after all ancilla philosophiae, is replaced by another mathematical science – topology. (Mormann, 2013).

I do not regard myself as the only person to make the claim of the emergence of new area of topological reading or philosophical topology. For the last twenty years Jeff Malpas, Steven Crowell, Joseph Fell and Reiner Schьrmann, from very different perspectives, advanced topological readings of Heidegger, Gadamer, Husserl. [29] (Jeff Malpas, Self, Other, Thing)

Jeff Malpas, who to my knowledge first introduces the issue of philosophical topology in series of papers, claims that "topology is present in Heidegger and, though less explicitly, in Hegel." (Jeff Malpas, Self, Other, Thing) Malpas's works are focused mainly on Heidegger, and just touches Hegel very slightly – "but Hegel will also have a role to play". (Jeff Malpas, Self, Other, Thing) For Malpas, the primary emphasis is "on gaining further insight into the idea of topology itself, along with the ideas it encompasses and to which it relates." (Jeff Malpas, Self, Other, Thing) My assertion is that the concepts and notions of Hegel's Logic and the manner Hegel develops these, exhibits topological properties in the strict sense the 'properties' that are invariants under some transformations, under homeomorphisms, which are bi-continuous one-to-one functions.

[29] Jeff Malpas, Self, Other, Thing, http://philevents.org/event/show/13584

According to Malpas, "Topology or topography is a mode of philosophical thinking that combines elements of transcendental and hermeneutic approaches. It is anti-reductionist and relationalist in its ontology, and draws heavily, if sometimes indirectly, on ideas of situation, locality, and place. Such a topology is present in Heidegger and, though less explicitly, in Hegel. Such a topology is also evident in many other recent and contemporary post-Kantian thinkers in addition to Kant himself." (Jeff Malpas, Self, Other, Thing)

Following Malpas's claim that "the concern with the thinking of place - place – topos, Ort, Ortschaft - as the place of thinking immediately brings topology and phenomenology close together." And "the question about the place in which thinking has its origin is the central question of philosophical topology or topography – topology is an attempt to think the place of thinking"[30] and "the place of thinking that is opened up through this transformation and re-orientation is the place that thinking never leaves: it is the topos of being, the topos of the hermeneuien, the topos that also belongs to logos"[31], I argue that Hegel's Logic, in particular his categories of qualitative and quantitative, his concepts of space, time and place, indeed present Logic as place of thinking as topos of being or topos of logos, present Hegel's Logic through topological notions.

Following Malpas's claim that "Place cannot be other than what is given in the multiplicity of places"[32] my assertions directs to Hegel's fourfold of multiplicity.

Malpas explores the elements of such a topology with particular reference to the understanding of the self, and especially in regard to the way such a topology is articulated through the mutual inter-relation between self, other, and thing. Malpas's topological approach is extended also to hermeneutics and topology of understanding. Malpas asks important questions such as "Might understanding itself be spatially or 'topologically' structured? Moreover, given the way language also intrudes here, one might ask what the role of space and place might be tin relation to language or to metaphor and image? From the

[30] Jeff Malpas, The Place of Topology: Responding to Crowell, de Beistegui, and Young Jeff Malpas https://www.academia.edu/18545068/The_Place_of_Topology_Responding_to_Crowell_Beistegui_and_Young
[31] Jeff Malpas, The Beckoning of Language: Heidegger's Hermeneutic Transformation of Thinking, p.25 http://jeffmalpas.com/downloadable-essays/
[32] Jeff Malpas, The Place of Topology: Responding to Crowell, de Beistegui, and Young Jeff Malpas https://www.academia.edu/18545068/The_Place_of_Topology_Responding_to_Crowell_Beistegui_and_Young

standpoint of contemporary hermeneutics, concerned as it is with both understanding and language, these ought to be viewed as significant questions, even though they are also questions upon which hermeneutics has, with some notable exceptions, tended not to reflect. In what follows, my aim is to explore some of the spatial, and especially the topological character of understanding, and so also to explore the connection between hermeneutics and what I have elsewhere referred to as philosophical topology or topography."[33]

Malpas's discussion and argument is directed at showing that "not only is understanding imbued with the spatial and the topological, but that hermeneutics is itself essentially topological in character."[34] [35]

Analyzing the place and hermeneutics, Malpas claims that: "The language of understanding is deeply imbued with ideas and images of place and space. To speak of 'understanding' is itself to draw upon a sense of 'standing in the midst of' or 'between' (from the Old English, understandan) – one might even say, then, that to understand is 'to draw near' or 'to be close to'. Heidegger points to the character of the German Verstдndnis as having "the original sense of 'standing before' [Vorstehen]: residing before, holding oneself at an equal height with what one finds before oneself, and being strong enough to hold out" – and here too there is surely also a sense of standing 'near to'. The French, comprendre, on the other hand, a term which also enters into English as 'comprehend', does not draw upon any idea of standing 'before' or 'near', but the idea on which it draws is no less spatial or topological, namely, of grasping or seizing – even of taking in or bringing together. "[36]

[33] In 'Place and Situation', in Routledge Companion to Philosophical Hermeneutics, edited by Jeff Malpas and Hans-Helmuth Gander (Abingdon: Routledge, 2015), pp.354-366, and 'The Beginning of Understanding: Event, Place, Truth', in Jeff Malpas and Santiago Zabala (eds), Consequences of Hermeneutics (Chicago: Northwestern University Press, 2010), pp.261-280 – see also 'Self, Other, Thing: Triangulation and Topography in Post-Kantian Philosophy', Philosophy Today, 59 (2015), pp.103–126. Although not always addressed in so direct or explicit a fashion, the connection between hermeneutics and topology is a theme that can be said to run throughout my work – it is already present, for instance, even if couched in slightly different terms, in Donald Davidson and the Mirror of Meaning (Cambridge: Cambridge University Press, 1992).

[34] Ibid.

[35] Jeff Malpas, 29. Place and Situation, From Jeff Malpas and Hans-Helmuth Gander (eds), The Routledge Companion to Philosophical Hermeneutics, in press, https://www.academia.edu/6962665/Place_and_Situation See: 4. Conclusion: Hermeneutics as Philosophical Topology: "…hermeneutics can also be seen as a form of 'philosophical topology' – where such a topology is itself seen as essentially hermeneutical in character. To understand place, then, is also to place understanding, and vice versa."

[36] Jeff Malpas, Place and Hermeneutics: Towards a Topology of Understanding, . . . . . https://www.academia.edu/12073967/Place_and_Hermeneutics_Towards_a_Topology_of_Understanding

For Maplas ‘understanding’ bring such spatial and 'topological' associations that may be seen as an example of the primacy of bodily metaphor in the manner in which we speak and think about understanding, also in relation with how our speaking and thinking is constructed in our 'inner space' of the mind and its activities. Here Malpas relates to the work of George Lakoff and Mark Johnson ‘Metaphors We Live By’. Although Malpas claims that “The question as to whether there is a fundamentally spatial and topological character to understanding that is indicated by the prevalence of spatial and topological imagery and idea in the structure of our speaking and thinking about understanding never really emerges as even an issue.”[37]

Malpas demonstrates “the ways in which place and topology are indeed present in Gadamer's work – the topological character of understanding is thus something that emerges in Gadamer no less than in Heidegger, and indeed, is present even in the very temporality of understanding.” [38]

Malpas argues that “if understanding is topological, then one would expect to find topological modes and figures at work in Gadamer just as they must also be at work in all thinking and all understanding – one of the tasks of a philosophical topological ought.” [39] For Malpas, the task is about “retrieving the topology that is inevitably present within the history of philosophy in general and so to make explicit the topological underpinnings that are present even in the work of the most seemingly atopic thinkers.” [40]

Malpas claims that “The topological is at work in Gadamer no less than Heidegger.” And “In Gadamer, however, the topological character of his thinking is not merely present as part of the general topology that governs all thinking, but instead appears, if sometimes implicitly, in the very articulation of the hermeneutical as such. One only needs to reflect on the topological character of notions such as horizon and situation to see how this is so. Yet if hermeneutics is itself essentially topological, then not only will the topological character of hermeneutics be evident in key hermeneutical notions, but the very thematization of the hermeneutical will itself bring a topological orientation with it even if the topological orientation is not itself thematized. This seems to be very much the case with Gadamer. It is significant, from a

[37] Ibid.
[38] Ibid.
[39] Ibid.
[40] Ibid

topological perspective, that, when talking about the formative influences on his thinking, especially in regard to Heidegger, Gadamer..”[41]

In our re-thinking of Hegel’s Logic through topology, one may ask: why topology? There are many areas of mathematics and mathematics discussed by Hegel.

Steven Crowell’s question - *Is Transcendental Topology Phenomenological?* (Crowell, 2011) – receives the immediate answer from Jeff Malpas, who claims that “the reason for this is that phenomenology is itself essentially topological.” [42]

Malpas argues that “One of the key philosophical questions, the very first question, in fact, concerns the origin of our thinking – where does thinking begin? Already to ask this question is to invoke a topological perspective, since it is specifically a question that asks after the ‘where’, the place, in which thinking has its origin.”[43]

### 1.3. Discussion and Chronicle of Philosophical Topology

As a discipline of mathematics, topology has existed for only seventy years, though antecedents go back centuries. (Blackwell, 2004:15)[44] The emergence of an entirely new discipline within mathematics is a rare event in the history of science. The creation of topology – the science of properties of spaces and figures that remain unchanged under continuous deformations – represents a phenomenon of this kind, but of a distinctly modern variety.

Topology is a generalization of geometry that studies spaces with the degree of generality appropriate to a specific problem. One central concern of topology is to study the properties of spaces that do not change under a continuous transformation, that is, translation, rotation,

[41] Ibid.
[42] Jeff Malpas, The Place of Topology: Responding to Crowell, de Beistegui, and Young Jeff Malpas https://www.academia.edu/18545068/The_Place_of_Topology_Responding_to_Crowell_Beistegui_and_Young
[43] Jeff Malpas, The Place of Topology: Responding to Crowell, de Beistegui, and Young Jeff Malpas https://www.academia.edu/18545068/The_Place_of_Topology_Responding_to_Crowell_Beistegui_and_Young
[44] Brent M. Blackwell, Brent M., 2004, Cultural Topology: an Introduction to Postmodern Mathematics http://reconstruction.eserver.org/044/blackwell.htm

and stretching without tearing. One such concept is expressed by the concept of dimension: a curve is one dimensional; a surface has two dimensions; ordinary space, three; and the space-time of general relativity, four. Mathematicians faced with the problem of characterizing a space locally to a Euclidean space use the notion of manifold. An n-dimensional manifold is a space M, such that a neighborhood V exists around each point p of M in one-to-one correspondence with a subset W of the n-dimensional Euclidean space R. The study of manifolds is called differential geometry, and the classification of all manifolds of a given dimension is an important problem of topology.[45] (Aubin, 2004:101)

Topology is the mathematics of continuity, where continuity is the study of smooth, gradual changes, the science of the unbroken, and discontinuities are sudden, dramatic, places where a tiny change in case produces an enormous change in effect. As Ian Steward asserts in his discussion on topology, continuity and discontinuity, “A potter, moulding a lump of clay in his hands is deforming it in a continuous fashion; but when he breaks a lumb of clay off, the deformation becomes discontinuous. Continuity is one of the most fundamental mathematical properties of them all, so natural a concept that its basic role only become clear a hundred years ago, so powerful a concept that is transforming mathematics and physics, so elusive, a concept that even the simplest questions took decades to answer. Topology is kind of geometry, but a geometry in which lengths, angles, areas, shapes are infinitely mutable. A square can be continuously deformed into a circle, a circle into triangle, a triangle into parallelogram. Topology studies only those properties of shapes that are unchanged under reversible continious transformations.” (Stewart, 1989).

Stewart asks – “What are the archetypal topological properties?” and asserts “To the untutored ear they sound nebulous, abstract, woolly. Connectedness just alluded to, is an example. One lump /of clay/ or two? …It requires new concepts, concepts not part of everyday experience, concepts for which no words exists.” (Stewart, 1989).

Topology bears comparison with the calculus, probability theory or number theory in that the first ideas about a new field called *Analysis Situs* or *Geometria Situs* were communicated among a handful of mathematically-minded intellectuals in the late seventeenth and early

[45] Aubin, David, (2004), Forms of explanation in the catastrophe of Rene Thom: topology, morphogenesis, structuralism, in *Growing Explanations: Historical Perspective on the Sciences of Complexity*, ed. M. N. Wise, Durham: Duke University Press, 2004, 95-130.

eighteenth centuries. (Epple, 1998:300) Two important and interrelated strands in the practice of the exact sciences in the 19th century will be considered in which topological ideas came to be relevant for natural philosophy. The first of these two strands was concerned with topological issues that arose in the context of a dynamical theory of physical phenomena, a theory advocated in particular by British natural philosophers during the last third of the 19th century. (Epple, 1998:299)

Only gradually over the course of the 19th century was a consensus reached about the nature of problems in topology. Nevertheless, after crossing the threshold to a scientific discipline in the full sense of the word in the first decades of this century, topology became one of the core research fields of mathematics, and topological arguments have come to play a role in virtually every other field in mathematics and the mathematical sciences. If one may reasonably speak of *genuinely modern* mathematical disciplines, then topology certainly belongs among them. (Epple, 1998:300)

The history of topology is relatively short, but the roots of 19th century topology are embedded in philosophical problems about the nature of extended substances and their boundaries which go back to Zeno and Aristotle. (Zimmerman, 1996:1)

As Zimmerman states, "…it seems that there have always been philosophers interested in these matters, questions about the boundaries of three-dimensional objects were closest to center stage during the later medieval and modern periods. Are the boundaries of an object actually existing, less-than-three-dimensional parts of the object — that is, are solids bounded by two-dimensional surfaces, surfaces by one-dimensional "edges" or "physical lines", edges by dimensionless "simples"? If not, how does a perfectly spherical object manage to touch a perfectly flat object — what part of the sphere is in immediate contact with the plane, if the sphere has no unextended parts? But if such parts be admitted, are we not then saddled with "actual infinities" of simples, lines, and surfaces spread throughout each continuous object — the boundaries of all the object's internal parts? Does it help any to say that these internal boundaries exist only "potentially"? These questions were still in the air as mathematicians and natural philosophers developed the notions which were to become the basis for topology." (Zimmerman, 1996:1) Zimmerman notes that Bolzano and Cantor, "situated at the very headwaters of modern topology, were familiar with the scholastic debates in which these questions arose.” (Zimmerman, 1996:1), (Steele, 1950)

Leonard Euler discovered the first topological property in 1750, but the second was not discovered for another century. The first real work in topology did not appear until the turn of the twentieth century when Brouwer, considered the father of modern topology, unified some of Poincaré’s work on differential equations and published the first fixed point theorem in 1909. Little comfort to the non-mathematician, topologists themselves have a hard time defining what it is they do. Though it started as a kind of geometry (colloquially referred to as a rubber-sheet geometry), today topologists are an eclectic lot spanning the gamut of mathematical pursuit from analysis (calculus), to algebra, and geometry. In *Experiments in Topology*, Steven Barr seems to have his hand on the pulse of the early process of disciplinization when he claims that topology is “curiously hard to define”. Recognizing an overt process at work, he recognizes an inherent “postmodern” tendency when he claims that topology is “its own goal” (though, of course, he does not use this term). (Blackwell, 2004:15)

For Brent Blackwell, Topology is a postmodern science. (Blackwell, 2004:15) The post-war work in topology that has spread to nearly every corner of mathematics, moves the discipline in this direction as well: towards a redefining of its own scope….Topology changed the conditions of the game by re-defining what it means to construct, suggesting that perhaps extrinsic properties were no longer sufficient to describe a figure. Since topology analyzes the qualitative properties of figures, it becomes the first branch of mathematics to be in a position to make meaningful, qualitative statements about the “culture” of their realities as well. (Blackwell, 2004:41)

In 1750, Leonard Euler discovered the first topological property, called the Euler number. For every closed, unbound, and finite polyhedron with $v$ vertices, $e$ edges, and $f$ faces, the equation $v - e + f = 2$. What he discovered is that all polyhedra share a common property.

Thirty four years before the birth of Hegel (1770), Leonhard Euler published his paper on the Seven Bridges of Konigsberg (1736), regarded as one of the first academic treatises in modern topology. Forty years after the appearance of Hegel’s Phenomenology of Spirit (1807), the term "Topologie" was introduced in German in 1847 by Johann Benedict Listing in

Vorstudien zur Topologie. Listing's essay Vorstudien zur Topologie tries to convince scientists of the importanceof topology. The first real work in topology did not appear until the turn of the twentieth century when Brouwer, considered the father of modern topology, unified some of Poincaré's work on differential equations and published the first fixed point theorem in 1909. (Blackwell, 2004:15)

Little comfort to the non-mathematician, topologists themselves have a hard time defining what it is they do. Though it started as a kind of geometry (colloquially referred to as a rubber-sheet geometry), today topologists are an eclectic lot spanning the gamut of mathematical pursuit from analysis (calculus), to algebra, and geometry.

In *Experiments in Topology*, Steven Barr seems to have his hand on the pulse of the early process of disciplinization when he claims that topology is "curiously hard to define". (Blackwell, 2004:15) And while it is a relatively simple task to characterize topology as the study of continuity to non-mathematicians or as the formal study of the features of geometrical figures that remain invariant under spatial transformations to mathematicians, neither fully captures the "sense" of topology. (Blackwell, 2004:15)

In 1865, the German mathematician August Möbius, began work on a strange one-sided, two-dimensional figure that now bears his name (though Johann Listing actually published his findings on the "Möbius" band four years earlier). What Möbius discovered is that when he tried to align triangles along the surface of the band, the orientation of the triangles had changed by the time he finished covering the surface. He described this "property" of one sidedness as "non-orientability". What this means is that when an orientable figure (one with bilateral symmetry like our bodies, for instance) traverses the length of the one-dimensional figure, the right and left sides of it become inverted as if viewed in a mirror. The Möbius band is a perpetual mirror world in which it is impossible to tell which image is real and which is a reflection. If our friend with the eye-patch over his/her right eye traveled along a Möbius band, eventually the patch would switch to the left eye. With Möbius' discovery of the second topological property in 1865, a disciplinary narrative of topology develops, as Blackwell asserts –"one that reveals the latent postmodern tendencies in certain kinds of mathematical thinking." (Blackwell, 2004:18)

Topological properties like orientability are thus the most fundamental characteristics of figures and spaces as they are invariant across drastic (albeit allowable) kinds of spatial transformations. In a sense, these properties form the core nature of all topologically similar figures, whereas geometrical properties vary even among similar figures. For example, a triangle and a circle are the same kind of topological figure because their topological properties are the same, even though their geometrical qualities are drastically different. By following the rules of topological transformation, a triangle can be "rounded" into a circle, which does not change its intrinsic qualities. Only the extrinsic quantities (distances, angles, degrees, volumes, dimensions, etc.) change during topological transformations. (Blackwell, 2004:18)

For Blackwell "… standard classifications (such as the distinction between topology and geometry) are necessary, not only to provide a familiar correlative for the non-mathematician, but more importantly, to provide a context, as topology can also be described as the mathematics of context." (Blackwell, 2004:15)

For Blackwell, "Topology is like geometry: it is concerned with the shape of space and how we understand (visualize) figures within different kinds of spaces. However, whereas geometry is concerned with quantitative properties of geometrical structures such as lengths, degrees, and areas, topology is concerned with their qualitative properties. But despite this difference, both disciplines share a concern for the kinds of transformations that are allowed. Euclidean geometry -- the familiar, planar world high school students explore with varying degrees of success -- is the most restrictive of geometries. The only acceptable transformations are turning and flipping. (Blackwell, 2004:22)

Two objects are considered "equal" in Euclidean space if they can be exactly mapped one on top of the other (every point on one figure corresponds with every point on the other). Even though two triangles may have the same interior angles (they are "similar"), if the lengths of their sides are different, then they are not "congruent" triangles. In this sense, topology is a much less restrictive kind of geometry. For example, the concept of distance is crucial for topology, but actual distances (numerical values) are irrelevant. More than turning and flipping, topology allows for all continuous transformations such as bending and stretching, but discontinuous transformations such as cutting and puncturing are not allowed. A two-dimensional disk and a three-dimensional pyramid are topologically the same kinds of figures since continuous transformations can change one into the other. (Blackwell, 2004:22)

Conceptually, topology is the study of continuity. Rather than measuring angles and distances, topology looks for holes and sutures. Ironically, while Euclidean geometry focuses on minute and precise ratios, it is most efficient when dealing with the most simple of figures. In order for Euclidean geometry to determine the surface area of a complex figure such as a coffee mug, it must first break down the complexity into simple elements (essentially a cylinder and a torus -- the donut shape of the handle). On the other hand, topology is able to analyze very complex structures in their totality, without disassociation. In fact, as we shall see, topology is in a position to make qualitative statements about figures only *because* objects are understood in their complex totality. (Blackwell, 2004:23)

While both geometry and topology study figures and objects (real, impossible, or imaginary), each has its own set of rules and operations that are allowed. And it is this axiomatic that distinguishes geometry from topology. As I suggested, the fundamental relationship in geometry is congruence through an exact mapping of points. To be sure, geometry has developed many theorems that allow one to avoid such an exact mapping, but the mapping itself is essential. If this map cannot be produced through deduction or induction, theorems or postulates, then there is no congruence, only difference. In this way, geometry is concerned only with manipulating the extrinsic properties of a figure, that is, with its variable properties. (Blackwell, 2004:24)

Topology, on the other hand, is interested only in the intrinsic or permanent properties of figures. The concept of distance, while crucial in topology is only important in so far as it remains consistent *as a concept*. For geometry, only the value of distance is important. The nature of that value is irrelevant. For example, geometry is not concerned with the nature of roundness, only the value or “kind” of that roundness (elliptical, hyperbolic, circular, etc.). In topology, extrinsic determinations of distance or degree do not fundamentally change the nature of a topological object or its reliance on a consistent definition of distance. As a kind of rigorous, qualitative geometry, early pioneers in the field of topology began by distinguishing among types of spaces (again, open and unbound figures), rather than among different perceptions of those spaces (such as projective geometry). (Blackwell, 2004:25)

We discover in topology that changing the concept of distance changes the topology of a figure. For example, while the definition of distance is consistent among congruent topological objects, it is not consistent across all objects. The concept of topological distance is consistent between a circle and a triangle and between a donut and a coffee cup respectively, but between these two classes of topological objects, the concept of distance is different. Many early topologists were trained as geometers, and much of their work evolved out of a dissatisfaction with the kind of space Euclid envisioned. As my first example demonstrated, Möbius discovered that the shape of space determines what kind of orientation is possible. In the 1850's, Riemann discovered that while our reality seemed to be Euclidean, it was not. Riemann was among a growing number of mathematicians who became dissatisfied with some of Euclid's assumptions about the nature of space, particularly his parallel postulate. Keeping the rest of Euclid's system in tact, figures like Bolyai, Lobachevsky, and Riemann were able to create consistent geometries free of contradictions in which the parallel postulate did not hold. Early non-Euclidean geometries thus discovered that the shape of space determines the kind of geometry that is possible within it. (Blackwell, 2004:26)

In thinking about the order backwards (space determining the kinds of figures that are possible within it), non-Euclidean geometers were able to challenge the way mathematics thought about its objects of study as well. Unlike geometry, which can only compare quantitative differences (since sameness is so narrowly defined), topology actually compares the qualitative similarities between objects. In geometry, complexity is only understood as an instrument and extension of simplicity. Geometry distinguishes between circular and elliptical to arrive at quantitative statements about their *differences*. Topology understands complexity in itself, in order to develop a detailed and formal method for dealing with the qualitative *similarities* among objects. In this way, topology is the basis for geometry since the nature of roundness (in a formal and rigorous way) forms the basis for concepts such as circular and elliptical. As a kind of "postmodern" geometry, topology analyzes the nature of the ground upon which its own self-construction lies. Even when dealing with the same "object," geometry and topology approach that object in a radically different way. A common illustration to distinguish the two pursuits is the example of the London Underground.

Geometry can determine the distance from Piccadilly Square to King's Cross, while topology is more interested in how many different trains the trip will take. (Blackwell, 2004: 27)

Around 1679, Leibniz himself particularly in De Analysis Situs, considers similarity, the 'qualitative' aspect of a figure as opposed to its quantitative aspects: "Beside quantity, figure in general includes also quality of forms. And as those figures are equal whose magnitude is the same, so those are similar whos form is the same." (Leibniz, Philosophical papers and letters, 391 in Giovanelli, 2011:143). As Marco Giovanelli asserts, Hegel's conception of infinitesimal calculus and his conception of infinity seems to effectively recuperate an important aspect of Leibniz's philosophy. (Giovanelli, 2011:143).

In 1736 Leonhard Euler published his paper on the *Seven Bridges of Konigsberg* (1736), regarded as one of the first academic treatises in modern topology. In 1750 Euler discovered the first topological property, called the Euler number. For every closed, unbound, and finite polyhedron with $v$ vertices, $e$ edges, and $f$ faces, the equation $v - e + f = 2$. What he discovered is that all polyhedra share a common property.

Hegel's discussion on Euler's view of infinitesimals and Leibniz's of fixed infinitesimals provided in the *Science of Logic* (1807) demonstrate his attention to the subject that will become known later as topology or Analysis Situs. [46] [47]

[46] *The idea of infinitely small quantities (latent also in increment and decrement) is far inferior to the mode of conception [just] indicated. The idea supposes them to be of such a nature that they may be neglected in relation to finite magnitudes; and not only that, but also their higher orders relative to the lower order, and the products of several relative to one.—With Leibniz this demand to neglect (which previous inventors of methods referring to this kind of magnitude also bring into play) becomes more strikingly prominent. It is this chiefly which gives an appearance of inexactitude and express incorrectness, the price of convenience, to this calculus in the course of its operation*. (Hegel, G. (1961). Science of Logic. Tr. Johnston & Struthers. Allen & Unwin., p.274)

[47] *In this regard Euler's idea especially must be cited. On the basis of Newton's general definition, he insists that the differential calculus considers the ratios of incrementa of a magnitude, while the infinitesimal difference as such is to be regarded wholly as nil.—It will be clear from the above how this is to be understood: the infinitesimal difference is nil only quantitatively, it is not a qualitative nil, but, as nil of quantum, it is pure moment of a ratio only. There is no magnitudinal difference; but for that reason it is, in a manner, wrong to express as incrementa or decrementa as as differences those moments which are called infinitely small magnitudes. ... the difficulty is self-evident when it is said that for themselves the incrementa are each nil, and that only their ratios are being considered; for a nil is altogether without determinateness. Thus this image, although it reaches the negative aspect of Quantum and expressly asserts it, yet does not simultaneously seize this negative in its positive meaning of qualitative determinations of quantity, which, if torn away from the ratio and treated as Quanta, would each be but a nil.* (Hegel, G. (1961). Science of Logic. Tr. Johnston & Struthers. Allen & Unwin., p.275-276)

Between 1794 – 1795 Johann Gottlieb Fichte made his first major exposition of the *Wissenschaftlehre*. (Foundation of the Entire Science of Knowledge).

There are strong correspondences between Hegel and Fichte concerning the notion of Qualitative and Quantitative, in the particular the notion of Qualitative quantity and Quantitative quality. Friedrich Hardenber, known as Novalis concerned himself with the scientific doctrine of Fichte in the period of 1795–1796. Fichte's philosophy influenced greatly Novalis's world view. Novalis not only read Fichte's philosophies but also developed Fichte's concepts further, transforming Fichte's *Nicht-Ich* ("not I") to a *Du* ("you"), an equal subject to the *Ich* ("I"). The first complete translation in English of Novalis's "Fichte Studies", is the book edited and translated by Jane Kneller, published in 2003 by the Cambridge University Press ( Kneller, 2003:177). In part of the book entitled "Group VI: 569-667, summer to fall, 1796", we can see the following table of Fichte's categories given by Novalis:

> 599. Quantity
>
> Unity: Modal Quantity
> Plurality: Modal - Qualitative Quantity
> Totality: Modal – Qualitative relative quantity
>
> Relation:
> 1. – modal relation
> 2. - modal – qualitative relation
> 3. – modal – quantitative qualitative relation
>
> Quantity:
> 1. Reality – modal quality
> 2. Negation – modal relative quality
> 3. Limitation – modal relative quantitative quality
>
> Modality:
> 1. Qualitative modality
> 2. Qualitative relative modality
> 3. Qualitative – relative quantitative modality …
>
> ( Kneller, 2003:177)

From the similarities between Hegel's Qualitative quantity and Fichte's Qualitative quantity, we can derive the comparable elements between Hegel and Fichte about their notions of Infinity. One of the most significant propositions of Fichte's *Wissenschaftslehre*, is the movement of knowledge in the process of the Sublime, the Sublime defined by Kant's Third *Critique* as an attempt to "present the infinite". In Fichte's *Wissenschaftslehre* the sublime is the dynamics of the mind, Mathema of the mind. Fichte promotes the concept of the Infinity

(Sublime) to the level of a gnoseological process following from the critique of objectifying knowledge to the definition of knowledge beyond representation. Perhaps Fichte's notion of infinity based on his notion of Qualitative quantity and Quantitative quality is reflected in Hegel's categorical fourfold of infinities.

In 1809 Louis Poinsot, during the first lecture that he gave at the Paris Science Institute, declared his public affiliation to Leibniz and Carnot, and his rejection of Euler, with respect to his work in constructive geometry. The singularity of Poinsot in this war of ideas is of the utmost importance because he is the only mathematician in history who explicitly understood ***analysis situs*** as an epistemological instrument of warfare.

The year 1807 marked Hegel's *Science of Logic (1807)* and *Phenomenology of Spirit (1807).* Hegel's Science of Logic (1812, 1816) was revised in 1831. In 1816 Hegel published his Encyclopedia of the Philosophical Science, a summary of his entire philosophical system, revised in 1827 and 1830. Year later in 1831 Hegel died.

In 1827 Hermann Grasmann begins his studies of theology at the University of Berlin.

In 1837 Bolzano published his *"Theory of Science" (Wissenschaftslehre)*, providing logical foundations for all sciences, building on abstractions like part-relation, abstract objects, attributes, sentence-shapes, ideas and propositions in themselves, sums and sets. The logical theory of Bolzano developed in his work *"Theory of Science"* has come to be acknowledged as ground-breaking. Bolzano was mainly concerned with three realms: (1) The realm of language, consisting in words and sentences; (2) The realm of thought, consisting in subjective ideas and judgements; (3) The realm of logic, consisting in objective ideas (or ideas in themselves) and propositions in themselves.

In 1862 Franz Brentano's first writings, the dissertation On the Several Senses of Being in Aristotle appeared, followed by his habilitation thesis, The Psychology of Aristotle (1867) and Philosophical Investigations on Space, Time and the Continuum. Brentano's dissertation played a decisive role in the young Heidegger's thought on being, and thus in his development of a new type of phenomenology, distinct from the Husserlian one.

The distinction between parts and wholes play a prominent role in Bolzano's system. Bolzano's work "The Paradoxes of the Infinite" was greatly admired by Charles Sanders Peirce, Georg Cantor and Richard Dedekind. Bolzano's works was rediscovered by Edmund Husserl[48] and the Polish philosopher and logician Kazimierz Twardowski [49], both students of Franz Brentano. Through Husserl and Twardowski , Bolzano became a formative influence on both phenomenology and analytic philosophy. The influence of Bolzano on Heidegger is witnessed by Heidegger himself. [50] In Bernard Bolzano's account of continuity, for example, we have "the first attempt at a mathematical formulation of the topological notion of connected." [51] (Wilder, 1978:721)

Franz Brentano's work on continuity resembles Bolzano's in blending mathematical and physical concerns. Brentano assumes that definitions of continuity like those found in Cantor, Dedekind, and Poincaré should be judged by their adequacy to our concepts of real continua, such as space, time, and extended bodies. (Cf. Brentano, Philosophical Investigations on Space, Time and the Continuum,trans. by Barry Smith (London: Croom Helm, 1988), pp. 39-44 and 138-149 ("Addendum to the treatise on what is continuous" and "Nativistic, Empiricist and Anoetistic Theories of our Presentation of Space"). But Brentano's work, unlike Bolzano's, could hardly have had an impact upon the development of topology (cf. note 14 below).

In 1847 Johann Benedict Listing introduced in German the term "Topologie" with his essay *Vorstudien zur Topologie* an attempt to convince scientists of the importance of topology. In Listing's work, the phenomena of orientation and knotting play a central role.

[48] Hermes Scholz, Concise History of Logic (1931), English translation: New York: Philosophical Library, 1961, p. 47. : "With such illogicality did things happen in the history of logic which we are pursuing here that this great, born logician fell prey to a fate which beats the fate of Joachim Jungius. For the latter at least was read, and read by a Leibniz; but that cannot even be said of Bolzano. Hence we cannot even maintain in his case that he was forgotten. All the greater is the merit of Edmund Husserl who discovered Bolzano."

[49] the founder of the Lvov-Warsaw School of logic, together with Alfred Tarski and Jan Lukasiewicz, Kazimierz Twardowski formed the *troika* which made the University of Warsaw, during that period, the most important research center in the world for formal logic.

[50] Martin Heidegger, Preface to: William Richardson, Heidegger. Through Phenomenology to Thought, The Hague: Martinus Nijhoff, 1963. p. X.: "The first philosophical text through which I worked my way, again and again from 1907 on, was Franz Brentano's dissertation: On the Manifold Sense of Being in Aristotle."

[51] R. L. Wilder, "Evolution of the Topological Concept of 'Connected'", American Mathematical Monthly, 85 (1978), pp. 720-26; quotation from p. 721.

In 1851 Riemann discovered that while our reality seemed to be Euclidean, it was not, between 1851 - 1857 Riemann presented his papers on complex analysis and his ideas on connectivity. Eight years later Helmholtz resumed his studies on vortex motion of perfect fluids and mentions connectivity numbers of spatial regions and published his seminal paper on vortex motion with the reception of Listing's work in the 1880's.

Between 1884 and 1887 Husserl attended the lectures of Franz Brentano. Under the mentorship of Brentano, Husserl came to view philosophy as complementary to science. Husserl became concerned with linking mathematics and philosophy. In 1887 Husserl completed his habilitation work "*On the concept of Number"* under Carl Stumpf's supervision. In 1891 Husserl published his first text, "*The Philosophy of Arithmetic"*, with a dedication to Brentano. In 1936 Husserl wrote *"The origin of Geometry" (1936).*

In "The origin of Geometry" (1936), Husserl argues that geometrical formations can serve as the model for any object whatsoever, due to their communicability as ideal objects. Geometrical formations or geometrical idealities are universal and non-perspectival. They are free from the contingencies of spatio-temporal existence. Geometrical formulation due to their ideality, must in principle be accessible to every rational human being, capable of being objects for all conscious subjects. One of the core concern of Husserl in this book is to explain how a geometrical formations which is initially confined to the solitary psychological life of the first or proto-geometer can become intersubjective, an object for the whole human community. According to Husserl, the medium that allows the proto-geometer to share his discovery with others in the same community is the speech. It is speech which brings ideality into the public realm. In order to transmite the discovery from generation to generations, the proto-geometres needs the writing. The writing gives history to the geometrical formations.Through inscription the geometrical formations are passed down to others, who will add their contribution formulating further theorems and axioms. The path of scientific progress and cultural development need writing. But there is something important for phenomenological reduction that Husserl notices. There is a price to be paid for the preserving of the discovery in history and this price is the loss of the conscious intentional states of the protogeometer's mind that led to the discovery.

Essential topological character of Husserl’s work is discussed by Jeff Malpas, who asserts that “the way in which Husserl understands the structure of meaningful experience is in terms of a set of notions that are themselves essentially topological in character, so that the structure of phenomenological presentation is identical with the structure of place.”(Malpas, 2011)

Within few decades following Hegel’s death (1831), during the last third of the 19th century topological ideas appears in the British natural philosophy.

Between 1855 and 1867, the Irish mathematical physicist William Thomson (1824 – 1907) and the Scottish mathematical physicists Peter Guthrie Tait (1831 – 1901) elaborated topological ideas. William Thomson did important work in the mathematical analysis of electricity and formulation of the first and second laws of thermodynamics, and formulated his vortex theory of the atom. Over the period 1855 to 1867, Thomson collaborated with Tait on a text book that founded the study of mechanics first on the mathematics of kinematics, the description of motion without regard to force. The text developed dynamics in various areas but with constant attention to energy as a unifying principle. Between 1870 and 1890 a theory purporting that an atom was a vortex in the ether was immensely popular among British physicists and mathematicians. About 60 scientific papers were written by around 25 scientists. Following the lead of Thomson and Tait, *the branch of topology called knot theory* was developed.

In 1860 Peter Guthrie Tait reads Helmholtz and begins thinking about quaternion analysis. One year later, in 1861 Listing publishes his Census der r¨aumlichen Complexe, and six years later, in 1867 Gauss’s *fragments on electromagnetism* are published, including the linking integral. Meanwhile in 1840 Hermann Grasmann submited his examination thesis to the commission in Berlin and Grasmann’s Extension Theory is born. In the year 1840 Hermann and Robert Grasmann study *Schleiermacher’s Dialectic.* Three years later Grasmann finishes work on the first volume of *Extension Theory* (A1).

In 1844 *Grasmann’s concepts of intensive and extensive quantity are highlighted in Ausdehnungslehre (1844).* In modern physics and especially in continuum mechanics and thermodynamics, a physical quantity associated with a physical system extended in space is called (1) intensive if it is a function on (the physical system extended in) space; (2) extensive if it is a density or linear distribution on (the physical system extended in) space. For instance for a solid body its temperature is intensive, but its mass is extensive: there is a temperature assigned to every point of the body (in the idealization of classical continuum mechanics anyway) but a mass is assigned only to every little "extended" piece of the body, not to a single point. This terminology in physics appears vaguely in Hegel's section Extensives und Intensives Quantum, in Science of Logic (1812) and Hegel's part II "Philosophy of Nature", second section "Physics", B, §298b in Encyclopedia of the Philosophical Sciences (1817), also more precisely in *Ausdehnungslehre (1844)* and in its fully modern form is maybe due to Richard Tolman in 1917.

Hermann Grassmann's *Die Wissenschaft der extensive Grössen oder die Ausdehnungslehre Erster Teil, die lineale Ausdehnungslehre*, introduced for the first time basic concepts of what today is known as linear algebra (including affine spaces as torsors over vector spaces) and introduced in addition an exterior product (§37, §55) on vectors, forming what today is known as exterior algebra or Grassmann algebra, hence in fact superalgebra. Grassmann advertizes his work as being the theory of extensive quantity. The modern way of speaking about this is that the elements of the exterior algebra he considered are differential forms on Euclidean space. (Lawvere 1995), (Lawvere 1996). [52]

Hermann Grassmann's *Ausdehnungslehre*, the 'theory of extension' went virtually unrecognised, even when he presented essentially the same results in an entirely different way in his revised version, from 1862. It took almost another decade before the relevance of Grassmann's discoveries to mathematics at large began to be recognized. Grassmann intended this to be a 'new branch of mathematics, explained through applications to the other branches of mathematics as well as to statics, mechanics, the theory of magnetism, and crystallography', as the subtitle proclaimed. Grassmann was the first mathematician to explicitly make a distinction between geometry, as the science of our physical space, and a purely mathematical treatment of abstract objects (which he termed 'extensive magnitudes')

[52] See also: William Lawvere, Grassmann's Dialectics and Category Theory, in Hermann Günther Graßmann (1809–1877): Visionary Mathematician, Scientist and Neohumanist Scholar, Boston Studies in the Philosophy of Science Volume 187, 1996, pp 255-264 (publisher) - and the similar text - William Lawvere, A new branch of mathematics, "The Ausdehnungslehre of 1844," and other works. Open Court (1995), Translated by Lloyd C. Kannenberg, with foreword by Albert C. Lewis, Historia Mathematica Volume 32, Issue 1, February 2005, Pages 99–106 (publisher)

that would have geometry as one of its applications but which would not be limited to three dimensions.

Nowadays, for the last few decades, in the series of works, William Lawvere established proposals for formalization of Hegel's objective logic in categorical logic (Lawvere 1991), (Lawvere 1992), (Lawvere 1994), (Lawvere 1995), (Lawvere 1997), claiming that proposals for formalizing some of Hegel's thoughts in terms of algebra may be identified in Hermann Grassmann, Ausdehnungslehre (1844).

In 1846 Grasmann was awarded a prize honouring Leibniz for his Geometrical Analysis. Ideas from Extension Theory receive public recognition for the first time.

In 1869 Klein discovers Grasmann by reading Hankel.

Back in 1853 Johann Carl Friedrich Gauss asked his student Riemann to prepare a *Habitationsschrift on the foundations of geometry*. Over many months, Riemann developed his theory of higher dimensions and delivered his lecture at Göttingen in 1854 entitled *Über die Hypothesen welche der Geometrie zu Grunde liegen* ("On the hypotheses which underlie geometry"). When it was finally published in 1868, two years after his death, the mathematical public received it with enthusiasm and it is now recognized as one of the most important works in geometry.

Between 1851 and 1857, Riemann's *papers on complex analysis* present his ideas on connectivity. Bernhard Riemann (1826–1866), who like Grasmann introduced the concept of n-dimensional manifolds to geometry, had teachers as prominent as Moritz Stern (1807–1894), Johann Benedict Listing (1808–1882), Carl Friedrich Gaus (1777–1855), Peter Gustav Dirichlet (1805–1859), Carl Gustav Jakob Jacobi (1804–1851) and Gotthold Eisenstein (1823–1852). Gaus "provoked" Riemann to choose a groundbreaking topic for the lecture he gave for his habilitation.

In 1865 Möbius, began work on a strange one-sided, two-dimensional figure that now bears his name (though Johann Listing actually published his findings on the "Möbius" band four years earlier). Two years later, in 1867 Gauss's fragments on electromagnetism were published, including the linking integral.

In 1871 at Göttingen, Christian Felix Klein made major discoveries in geometry. He published two papers *On the So-called Non-Euclidean Geometry* showing that Euclidean and non-Euclidean geometries could be considered special cases of a projective surface with a specific conic section adjoined. This had the remarkable corollary that non-Euclidean geometries was consistent if and only if Euclidean geometries was, putting Euclidean and non-Euclidean geometries on the same footing, and ending all controversy surrounding non-Euclidean geometry. Next year, in 1872 Klein's synthesis of geometry as the study of the properties of a space that is invariant under a given group of transformations, known as *the Erlangen Program (1872),* profoundly influenced the evolution of mathematics. This program was set out in Klein's inaugural lecture as professor at Erlangen, although it was not the actual speech he gave on the occasion. The *Program* proposed a unified approach to geometry that became (and remains) the accepted view. Klein showed how the essential properties of a given geometry could be represented by the group of transformations that preserve those properties. Thus the *Program*'s definition of geometry encompassed both Euclidean and non-Euclidean geometry.

Ernst Cassirer considered *Klein's Erlangen Programme* as a guide line for the epistemology of his "Critical Idealism" characterizing the task of epistemology as finding the ultimate invariants of scientifi c knowledge. In *Substanzbegriffund Funk tions begriff* and much later in *The Philosophy of SymbolicForms* he dedicated cen tral chapters to concept formation in geometry which he considered as a paradigmatic case for concept formation in science *überhaupt*. Cassirer emphasized in his philosophy of science the importance of geometry for philosophy of science, but offered only some general, passing remarks on the role of topology.

About the time Cassirer's Substance and Function was first published (1910), Kurt Lewin, the author of "Principles of Topological Psychology" (1936). was a graduate psychology student at the University of Berlin. He began attending Cassirer's lectures on the philosophy of science which left an indelible impression on him and strongly influenced his subsequent work. After being wounded in the war, Lewin completed his Ph.D. under Stumpf, and, like Cassirer, left Germany in 1933. Unlike Cassirer, however, Lewin went almost immediately to the United States where he became a famous, iconoclastic leader in the field of American social psychology. Lewin says “That correct qualitative analysis is a prerequisite for adequate quantitative treatment is well recognized in psychological statistics. What seems less clear is that the qualitative differences themselves can and should be approached mathematically” (p:31). Again, Lewin references Cassirer as one who “points out again and again that mathematization is not identical with quantification. Mathematics handles quantity and quality” (p:30-31).

In 1870 William Thomson tries to prove the dynamical stability of simple vortex configurations and to determine their fundamental vibrations. Three years later, in 1873 Maxwell publishes his Treatise, including Listing’s topological ideas, Thomson’s results on flows in multiply connected regions and Gauss’s linking integral.

In 1880 Cantor began arguing for an actual infinity of actual infinities and five years later in 1885, Henri Poincare first introduced the term ‘bifurcation’ [53] The same year Poincare published *Analysis Situs*, introducing the concept of homotopy and homology, now part of algebraic topology. Between 1899 and 1904 Poincare published five supplements to his paper.

In 1897 Bertran Russell started his philosophical career with the dissertation *The Foundations of Geometry.* (Bertrand Russell, *The Foundations of Geometry*. Cambridge: Cambridge University Press 1897). In the year 1903, Russell’s *The Principles of Mathematics* apears.

Between 1900 and 1901, Husserl published his “Logical Investigations”, a formal theory of part, whole and dependence that is used by Husserl to provide a framework for the analysis of mind and language of just the sort that is presupposed in the idea of a topological foundation for cognitive science. [54]

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[53] Henri Poincaré, L'Équilibre d'une masse fluide animée d'un mouvement de rotation, Acta Mathematica, t.7, pp. 259-380, sept 1885.

[54] Barry Smith, Topological Foundations of Cognitive Science, a revised version of the introductory essay in C. Eschenbach, C. Habel and B. Smith (eds.), *Topological Foundations of Cognitive Science,* Hamburg: Graduiertenkolleg Kognitionswissenschaft, 1994, the text of a talk delivered at the First International Summer Institute in Cognitive Science in Buffalo in July 1994.: http://ontology.buffalo.edu/smith/articles/topo.html
And Barry Smith, (ed.) 1982 Parts and Moments. Studies in Logic and Formal Ontology, Munich: Philosophia.)

In 1927 with *The Analysis of Matter*, Russell was engaged in *using topological methods for the "logical analysis" of space and time.* Russell's topological project was by far as the most sustained and detailed one. Russell developed his topological ideas with various degrees of precision and explicitness in several contributions, beginning with *Our Knowledge of the External World*, later in a more detailed way in *The Analysis of Matter*, and finally in *On Order in Time.* (Bertrand Russell, *Our Knowledge of the External Worlds as a Field for Scientifi c Method in Philosophy*. London: Routledge and Kegan Paul 1914; *The Analysis of Matter*, *op. cit*.; "On Order of Time", in: Russell, *Logic and Knowledge*. London: Routledge 1956, pp. 347-363 (orig. 1936).

Russell's talent for dealing with the conceptual tools of topology but his project did not find followers. Worse, no philosopher realized that Russell's sketch of a topological logical analysis had long been superseded by the ongoing evolution of topology.

Russell's attempt to introduce topological methods in philosophy of science for the logical analysis of philosophical and scientifi c notions remained unsuccessful. He wanted to show that the basic mathematical structures of physical space-time – usually conceived of as structured sets of spatial and temporal points (instants) – could be logically reconstructed from 'crude sense data', later to be characterized as 'events'. He credited Whitehead with the basic ideas of this approach.

In 1909, Brouwer, who is considered as the father of modern topology, unified some of Poincaré's work on differential equations and published the first fixed point theorem in 1909.

In the year 1917, D'Arcy Wentworth Thompson, who was Professor of Biology at University College Dundee 1885-1917 and Professor of Natural History at the University of St Andrews 1917-1948, published *On Growth & Form.* The father of D'Arcy Wentworth Thompson (1860 – 1948) was closest friend with Peter Guthrie Tait during their years at Cambridge. Structuralism and topology are indisputably linked with D'Arcy W. Thompson, who advocated structuralism as an alternative to survival of the fittnes in governing the form of species. In his classic "On Growth and Form", D'Arcy Thompson analyzed biological form in terms of physical forces. With his famous set of diagrams, Thompson showed family resemblances between species of fish by deforming grids through smooth coordinate transformation, suggesting that topology is basic to the overall plan of an organism. In the last chapter of “On Growth and Form”, D'Arcy Thompson's illustrates his "cartesian transformations" of animal forms. Thompson's mappings are referred to as "rubber sheet" mappings. D'Arch Thompson suggested that one should study the change from one biological form to another by examining the unique mathematical object that maps between them in accord with biological homologies.

D’Arcy W. Thompson’s structuralism influenced thinkers as the anthropologists such as Claude Levi-Strauss, Edmund Leach, and Edwin W. Ardener and the topological philosophy of Jacques Lacan, Jacques Derrida, Gilles Deleuze.

In 1920 Peter Guthrie Tait, who actively pursued topology in the decade 1876–1885’s become widely recognized and now-famous for his Tait’s knot tables.

In 1922 Rudolf Carnap’s first philosophical publication appeared - *Der Raum. Ein Beitrag zur Wis sen schafts lehre.* (Rudolf Carnap, *Der Raum. Ein Beitrag zur Wis sen schafts lehre Kantstudien Ergänzunghefte, 56*, 1922.) With this work Carnap sought to establish the topological structure of space as a modernized version of a Kantian *synthetic a priori*. In **1930** Carnap claimed that “philosophy of science just is logic of science”.

Between 1900 and 1923 Freud with his two "topographies" (the first dating from 1900 and the second from 1923), resorted to schemas to represent the various parts of the psychic apparatus and their interrelations. These schemas implicitly posited anequivalence between psychic and Euclidean space.

In 1927 Martin Heidegger’s *Being and Time* appeared.

Alfred North Whitehead articulated in several books and articles published between 1916 and 1929 - Mereotopology, a branch of metaphysics, and ontological computer science, a first-order theory, embodying mereological and topological concepts, of the relations among wholes, parts, parts of parts, and the boundaries between parts. In 1929 Whitehead published *“Process and Reality”*, the work that established the so called process philosophy. The book is a revision of the Gifford Lectures he gave in 1927-28. Whitehead put forward something like a topological philosophy, but it was not more than a sketch and had no infl uence on mainstream analytic philosophy of science.

In 1934 Lucien Tesnière published the article ”*Comment construire une syntaxe*” which preceded his monumental work on structural syntax, posthumously published in 1959 with the title *Eléments d'une syntaxe structurale (*Elements of Structural Syntax).[55] Tesnière's *Elements of Structural Syntax* proposes a sophisticated formalization of syntactic structures.He developed the concept of valency in detail, and the primary distinction between arguments (actants) and adjuncts (*circumstants*, French *circonstants*), which most if not all theories of syntax now acknowledge and build on. Tesnière argued that syntax is autonomous from morphology and semantics. Some of the central concepts in Tesnière's approach to syntax are 1) connections, 2) autonomous syntax, 3) verb centrality, 4) stemmas, 5) centripetal (head-initial) and centrifugal (head-final) languages, 6) valency, 7) actants and circonstants, and 8) transfer.

Wildgen and Brandt assert that in the developing topological theory of language, Thom is standing on the shoulders of the founding father of modern syntax, Lucien Tesnière (1893-1954).

Lucien Tesnière begins the presentation of his theory of syntax with the *'connection'*, the central concept for him. Connections are present between words of sentences. They group the words together, creating units that can be assigned meaning. Tesnière writes:

"Every word in a sentence is not isolated as it is in the dictionary. The mind perceives connections between a word and its neighbors. The totality of these connections forms the scaffold of the sentence. These connections are not indicated by anything, but it is absolutely crucial that they be perceived by the mind; without them the sentence would not be intelligible. ..., a sentence of the type *Alfred spoke* is not composed of just the two elements *Alfred* and *spoke*, but rather of three elements, the first being *Alfred*, the second *spoke*, and the third the connection that unites them – without which there would be no sentence. To say that a sentence of the type *Alfred spoke* consists of only two elements is to analyze it in a

[55] Wildgen, W., & Brandt, P. A. (2010). *Semiosis and catastrophes: René Thom's semiotic heritage*. Bern: Peter Lang., p.57

superficial manner, purely morphologically, while neglecting the essential aspect that is the syntactic link."[56]

Between 1935 and 1966, *"Elements of Mathematics series (book III – Topology)* Notations was introduced by Bourbaki include the symbol ∅for the empty set and a dangerous bend symbol, and the terms injective, surjective, and bijective.

In 1947 Emmanuel Levinas published *De l'Existence à l'Existent (Existence and Existents).*

In 1948 Ernst Robert Curtius (1886 – 1956), German literary scholar, philologist, and Romance language literary critic, published his study *Europäische Literatur und Lateinisches Mittelalter*, translated in English as *European Literature and the Latin Middle Ages*. With his study Curtius introduced the concept of 'literary topos" as scholarly and critical discussion of literary commonplaces, claiming that much of Renaissance and later European literature cannot be fully understood without knowledge of that literature's relation to Medieval Latin rhetoric in the use of commonplaces, metaphors, turns of phrase, or, to employ the term Curtius prefers, *topoi*". (Lind, L.R. (1951). "Rev. of Curtius, Europäische Literatur und lateinisches Mittelalter". The Classical Weekly 44 (14): 220–21.)

Klaus Ostheeren in his study on Curtius (Ostheeren, K., 1998), establishes that "Topological studies in various branches of knowledge, such as law, go back to this original meaning…but literary topology is firmly based on Curtius's metonymic use of "topos", wich some scholars traced back to Aristotle. Inaugurating modern topology as a method for historical, cultural and literary research, Curtius transformed the inherited conception of from technique of finding arguments in rhetorical persuasion into the patterns of thought and expression originally found by applying this technique, but now established as inherited and aquitened constituents of literary competence – the indispensable cognitive units and literary production, reception, and interpretation." (Ostheeren, K., 199, p.373).

Curtius's rhetorical modes of thought and expression crystalize into patterns or models wich he called topoi, their study being "topological research" or "topology" (toposforshung).

[56] The passage cited here is taken from the first page of the Éléments (1959[1969]) in Wildgen, W., & Brandt, P. A. (2010). *Semiosis and catastrophes: René Thom's semiotic heritage*. Bern: Peter Lang., p.57

Curtius practiced this branch of topology, synchronic or syntagmatic topology, with brilliant results in studies of Divine Comedy.

The links between the Curtius's 'literary topology' and mathematical concepts, similar to the modern topology as mathematical discipline, are not disputable. Mark A. Paterson in his book *Galileo's Muse: Renaissance Mathematics and the Arts*, asserts that "the branch of mathematics that deals with spaces like this, spaces that are different from space that we visualize most easily, is called topology. In visualizing a new three-dimensional space, finite but having No edge, Dante has invented a new topological space, the 3-sphere. If this imaginative feat had been recognized in his own time, and if the idea had been pursued and developed, Dante would today be considered one of the inventors of topology, and one of the great creative mathematicians of all time. As it is, he is not even a footnote to topology, which was only invented officially in the eighteenth century, and didn't really take off until the twentieth." (Peterson, M., 2011)

The presence of 'topology' in philology and literature is strong in the historical tradition and our currents, first derived from rhetorical notion of 'topos' and later directly linked with topology as mathematical discipline.

The importance of Curtius's rediscovery of topoi and topics extended beyond the medieval studies. In philosophy, sociology, political science, and jurisprudence topology has come to be regarded as a bridge over the historical gap that opened in the eighteenth century Europe, when – for not yet fully understood – topic gave way to logical thinking. (Gelley, A. 1974).[57] [58] [59]

[57] Gelley, Alexander. (1974), Ernst Robert Curtius: Topology and Critical Method. In *Velocities of Change.* Ed. Richard Mackey. Baltimore: The Johns Hopkins University Press (Gelley A, 1974)

[58] The spatial turn and 'topological turn' in Literary Theory and Textual Analysis (Ernest W. B. Hess-Lüttich, W.B. E. *2012)* is associated with the development of linguistics and structuralism. Topological thinking is presented in the studies of the Russian structuralist Vladimir Propp, in his analysis of Russian folk tales, where Propp identifies 31 constitutive elements. Levi-Strauss, who has reintroduced the concept of transformation independently and developed it further, refers to Propp in his essay Structure and Form: Reflections on a Work by Vladimir Propp, (1983). Topology is presented in semiotics of Jurij M. Lotman, in study of the symbolic space in literature as a result of culturally determined sign utilisations.

Ernest W. B. Hess-Lüttich, *(2012)* Spatial turn: On the Concept of Space in Cultural Geography and Literary Theory, (Vol. 5; 2012) Journal for Theoret*ical Cartography (*Ernest W. B. Hess-Lüttich, *2012)*

[59] In contemporary research of the so called 'digital humanities', **topology** is used as a means of modeling linguistic patterns to understand the spatial connectivity of literary texts. One of the best examples in this approach is Andrew Piper's 'theory of Topological Reading' (Piper, A. 2013), and his 'literary topologies'.

In 1948 Emmanuel Levinas's *Le Temps et l'Autre (Time and Other).*

In 1950 Martin Heidegger appeared with a lecture and formulation of the famous saying *Language speaks*, later published in the 1959 essays collection *Unterwegs zur Sprache.*

The topological philosophy is strongly presented in Martin Heidegger. The contribution in the development of an understanding of Heidegger's topology belongs to Jeff Malpas, who is

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See: Andrew Piper, "Reading's Refrain: From Bibliography to Topology." *ELH*. Special Issue on Reading. Ed. Joseph Slaughter (Summer 2013): 373-399. http://piperlab.mcgill.ca/pdfs/Piper_ReadingsRefrain.pdf (Piper A, 2013); Andrew Piper and Mark Algee-Hewitt, "The Werther Effect I: Goethe Topologically." *Distant Readings: Topologies of German Culture in the Long Nineteenth Century*. Ed. Matt Erlin and Lynn Tatlock (Rochester, NY: Camden House, 2014) 155-184. (Piper A & Algee-Hewitt M, 2014); Andrew Piper, (2013) A Theory of Topological Reading : http://txtlab.org/?p=188 (Piper A, 2013)

The effect of topology and metalepsis!
"..reading topologically alters our visual and cognitive relationship to the text, it also enables us to reconsider the place of conversion within reading as one of reading's most historically prominent emotional and affective ideals (as well conversion's secular correlate, the history of transgressive reading). In privileging a sense of restraint, a refraining *from*, topology moves us beyond our long held convictions of the palpable, the transformational, and the excessive when it comes to reading—the way reading moves us deeply, profoundly, and immeasurably—and toward the likely, the proximate, and the scalar. It moves us from a state of revolution to one of resolution, where reading's affections and attachments are reinscribed within a perspectival, literative system. Conversion (or transgression) no longer serves in an electronic milieu as reading's primary spiritual outcome, but instead as a theoretical initiation. Translation, a change of state, becomes the condition of topological reading rather than its end. (Piper. A., 2013, p.337)

Andrew Piper is director of a digital humanities laboratory at McGill University (.txtLAB). The lab 'explore the use of computational and quantitative approaches towards understanding literary and cultural phemonena in both the past and present' with the aim 'to engage in critical and creative uses of the tools of network science, machine learning, or image processing to think about language, literature, and culture at both the large and small scale.' Remarkable is the statement made by Piper in his *Reading's Refrain: From Bibliography to Topology* (Piper, A. 2013), emphasizing on the notion of manifold, and what I will discuss later as Hegelian 'Menge' and 'Mitte' and Hegel's topological notion of multiplicity. Piper asserts that "in its manifestation of the knowledge of the numerous and the multiple, which in the nineteenth century came to be known as *Mengenlehre* (the science of the manifold), topology is founded on a primordial act of translation." (Piper. A., 2013, p.380)

**The idea of Literary Evolution presented by Franco Moretti**, director of Stanford Literary Lab. Within the discipline of 'digital humanities', Moretti created the "quantitative history of literature"**.** In his book *Graphs, Maps, Trees: Abstract Models for a Literary History* (2005), Morettti used a quantitative approach to the study of literature that includes historical and comparative contexts and charts a cultural geography for literary genres. Moretti's other books include *Signs Taken for Wonders* (1983), *Modern Epic* (1995) and *Atlas of the European Novel 1800-1900* (1998). Moretti is at work on a five-volume collaborative study of the novel throughout all history and in all forms. In his triptych published in New Left Review —'Graphs', 'Maps' and 'Trees', with subtitle 'Abstract Models for Literary History', published later as book, Moretti offers intriguing and innovative approach based on the quantitative history, geography and evolutionary biology. In his essay „On Literary Evolution", Moretti uses evolution as a metaphor, linking evolutionary model with this of Darvin. Moretti ended his essay **„**Maps" with the quotation from D'Arcy Wentworth Thompson's *On Growth and Form*: "We rise from a conception of form to an understanding of the forces which gave rise to it [. . .] and in the comparison of kindred forms [. . .] we discern the magnitude and the direction of the forces which we have sufficed to convert the one form into the other". (Thompson, D'Arcy Wentworth. *On Growth and Form*. Macmillan, 1943.) In his last part from the triptych 'Trees', Moretti discusses the morphological tree of evolution. He claims that the evolutionary bibliography could be understood as prototype evolutionary science author of the books - "Heidegger and the Thinking of Place: Explorations in the Topology of Being"[60]and "Heidegger's Topology: Being, Place, World".[61]

The idea of philosophical topology is increasingly central to the work of Jeff Malpas, as he himself admits, concluding his work over the last twenty years. For Malpas "the question about the place in which thinking has its origin is the central question of philosophical topology or topography – topology is an attempt to think the place of thinking…the concern with the thinking of place the place of thinking immediately brings topology and phenomenology close together."[62]

The idea of place "topos", runs through Heidegger's thinking almost from the very start. It can be seen not only in his attachment to the famous hut in Todtnauberg, but in Heidegger's constant deployment of topological terms and images and in the situated, "placed" character of his thought and of its major themes and motifs.

According to Malpas, Heidegger's own work cannot adequately be understood except as topological in character, and so as centrally concerned with place - *topos*, *Ort*, *Ortschaft*". It was in fact Heidegger himself, in the 1969 Thor seminar, who stressed the topological dimension of his thinking, using the expression "topology of be-ing" [*Topologie des Seyns*] to replace the earlier expressions "meaning of being" and "truth of being.

Malpas argues that Heidegger's concepts of being and place are inextricably bound together. The human being, for Heidegger is always and already human being situated in place. Malpas demonstrates how this 'emplacement' became, for Heidegger, the central answer to the question of how anything, including human being, can exist and be the thing it is.

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and to think biology in the terms of bibliorgraphy. For Moretti the philogenesis is the base of bibliogenesis.

Moretti F, Signs Taken for Wonders (1983), Modern Epic (1995) and Atlas of the European Novel 1800-1900 (1998).

[60] Jeff Malps, "Heidegger and the Thinking of Place: Explorations in the Topology of Being", MIT Press, 2012
[61] Jeff Malpas, Heidegger and the Thinking of Place: Explorations in the Topology of Being, MIT Press, 2007
[62] Jeff Malpas, "The Place of Topology: Responding to Crowell, Beistegui, and Young", University of Tasmania and La Trobe University, Australia, 2011

In the 1980s, Joseph Fell and Reiner Schürmann, from very different to Malpas perspectives, advanced topological readings of Heidegger.[63][64]

If the place (of philosophy) for Heidegger is in the very topos of thinking, the place of (ethical thinking) in Levinas is in the very topos of otherness, it is in "the very quality of difference". Heidegger's concept of thinking "the place of place" explains temporality and historicity.

In 1945 Warren McCulloch published *"A Heterarchy of Values Determined by the Topology of Nervous Nets"*.[65] McCulloch's work is associated also with the works of Gotthard Gunter's [66] doctoral thesis on Hegel [67].

In 1960 Gotthard Günther met Warren McCulloch and worked with him and Heinz von Foerster and Humberto Maturana. Günther's work was based upon Hegel, Heidegger and Spengler. From 1976 to 1980 Günther completed three studies "Contributions to the Foundation of an Operational Dialectic /"Beiträge zur Grundlegung einer operationsfähigen Dialektik/. Günther aim was to make dialectics operationable and contributed with his work and influence to the fields of cybernetics. The qualitative notion of numbers or the qualitative quantity in Gotthard Günther's thinking is unfolded in his "Number and Logos" [68], devoted to his frend and one of the fathers of cybernetics Warren McCulloch, bearing Günther's note: "Unforgettable Hours with Warren St. McCulloch". Warren McCulloch was the one who influenced Gregory Bateson's topological thinking [69]. Bateson took McCulloch's idea of heterarchy and adapted this to Bertrand Russsell's hierarchy of logical types in a rather peculiar way: logical types could systematically unravel circularity of information and how it

[63]Joseph P. Fell, Heidegger and Sartre: An Essay on Being and Place (New York: Columbia University Press, 1983)

[64]Reiner Schürmann, Heidegger on Being and Acting: From Principles to Anarchy, trans. Christine-Marie Gros (Bloomington: Indiana University Press, 1987).

[65] Warren McCulloch, "A Heterarchy of Values Determined by the Topology of Nervous Nets" /In: Bulletin of Mathematical Biophysics, 7, 1945, 89–93.

[66] Gotthard Günther, "Cybernetics and the Dialectic Materialism of Marx and Lenin", published at http://www.thinkartlab.com as an enlarged representation of a lecture Gotthard Günther did deliver at the University of Cologne (Köln, Germany) July 17, 1964. The paper was prepared under the Sponsorship of the Air Force Office of Scientific Research, Directorate of Information Sciences, Grant AF - AFOSR - 8 - 63 and 480-64.

[67] 1933, Fundamentals of a Theory of Thought in Hegel's logic, ("Grundzüge einer neuen Theorie des Denkens in Hegels Logik")

[68] Gotthard Günther, "Number and Logos", published at http://www.thinkartlab.com

[69] A special issue of the S.E.E.D. Journal (Semiotics, Evolution, Energy, and Development) in 2004 focused on Essays on Recursion, Difference, Dialectics, Maps and Territories in Celebration of Gregory Bateson's centennial.

becomes entangled in a multi-level universe, but logical typing cannot itself demonstrate appropriate communication.

In 1958 Alexander Grothendieck was already at the IHÉS Institut des Hautes Études Scientifiques (IHÉS).

In 1959 Jean Piaget with his Genetic Epistemology introduces his *"topological primacy thesis"*. Piaget and Inhelder claimed that the young child's intristic geometry was first of all topological and then, later, projective Euclidean.

In 1961 Emmanuel Levinas's *Totalité et Infini: essai sur l'extériorité*. (Totality and Infinity) appeared, and few years later in 1974 Levinas published *Autrement qu'être ou au-delà de l'essence* (Otherwise than Being or Beyond Essence).

There are two contemporary philosophers deploying a topological approach to the works of Levinas – J.Aaron Simmons, "Vision without image": A Levinasian Topology" and Silvano Petrosino, "La topologia di Levinas". Levinas's ethics can be originally presented in topological terms, where topology is used not in the pure mathematical mode, but as philosophical topology and topological thinking.[70]

Emphasizing the notion of qualitative quantity, in conclusion of these lines about the topological qualitative quantity in Fichte, Hegel and Heidegger, I would like just to quote the philosopher who is also one of the strong influences of Emmanuel Levinas's notion of time – Henri Bergson. In the work "Time and the free will", Bergson asserted that "..it is through the quality of quantity that we form the idea of quantity without quality'.[71]

On this ground and the above discussion on topological notion of qualitative quantity seen as cobordism, I would like to propose some application of Rene Thom's topological cobordism to the Ethics of Levinas.

[70] Borislav G. Dimitrov, 2013, "The Topological Alterity in Levinass chronotope As Ethical Analysis Situs", ttp://www.academia.edu/3237203/The_Topological_Alterity_in_Levinass_chronotope_As_Ethical_Analysis_Situs

[71] Bergson, Henri (1910): Time and Free Will, New York, MacMillan Co., p.123. (French original, entitled 'Essai sur les données immediates de la conscience', Paris 1889)

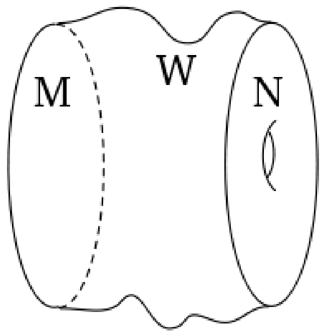
The radical contribution Rene Thom made for the linguistics and the devotion of Levinas on the language represents language as the natural philosophical ground on which we may approach Levinas's ethics from the standpoint of Thom's cobordism.

Lévinas's philosophy redefines language. Language for Lévinas is the "structure" of both the retraction and the inscription of infinity (*l'infini*), and this "structure" is what Lévinas calls *ethics.* It is possible to affirm that in Lévinas there is no ethical turn without a certain linguistic turn. Lévinas's linguistic turn is turned inside out: for him language contains the other's call and thus harbors a form of exteriority within itself. This inversion or fusion of interiority and exteriority recall the topological notion.

A topology of signification or a "morphogenesis of meaning" in the sense of Rene Thom is complete only if the dialogical aspect is adequately accounted for. Dialogical aspects and dialogics figure prominently in the ethical philosophy of Emmanuel Levinas. In topological terms, the dynamics (of category of Qualitative quantity) implemented in Levinas face-to-face relationship could be represented as a cobordism between a single circle (at the top), representing Levinasian *the third* and a pair of disjoint circles (at the bottom), representing *the self* and *the other*.

Cobordisms form a category whose objects are closed manifolds and whose morphisms are cobordisms. The main subject(s)object in Levinas, that could be seen as closed manifolds, are "the self" and the "other", and the "third", and these manifolds morph and bords in the notion of Levinas's 'Aborder Autrui'.

Cobordism had its roots in the attempt by Henri Poincare in 1895 to define homology purely in terms of manifolds. Poincaré simultaneously defined both homology and cobordism, which are not the same, in general. In mathematics, cobordism is a fundamental equivalence relation on the class of compact manifolds of the same dimension, set up using the concept of the boundary of a manifold. Two manifolds are *cobordant* if their disjoint union is the *boundary* of a manifold one dimension higher. The name comes from the French word bord for boundary.




In Levinasian ethics the manifolds of “Self” (M) and “Other” (N) are *cobordant* (aborder) if their disjoint union is the *boundary* of a manifold one ethical dimension higher. The notion of cobordism is simple - two manifolds M and N are said to be cobordant if their disjoint union is the boundary of some other manifold (W) or the “Third”. This manifold [of] recollection represent an opportunity for us to build a Ethical Analysis Situs[72] (topological ethics - applied ethics as conceptual tool for situational analysis and situation design), which is quite different from the discourse ethics model of dialogue and can be applied in reality. Topological Vorstellung could be applicable to the notions such as identity, dignity and well-being.

Between 1961 and 1962 Lacan introduces his topological ideas. Lacan’s seminar "Identification", unveiled a collection of topological objectsuch as the torus, the Möbius strip, and the cross-cap hat served pedagogical aims, but already he saw them as more than just models. In 1973, Lacan introduced the Borromean knot, claiming that topological objects are a real presentation of the subject and not just a representation.

Between 1952 and 1968 Claude Levi-Strauss’s “Social Structure” (1952) and Structural Antropology (1968), appeared embedding his topological thinking.

Between 1956 and 1960, Merleau-Ponty introduced the allusion to “topological space” in The Visible and the Invisible. Merleau-Ponty’s most explicit treatment of modern physics appears in La Nature (1956–60/2003), the course notes from his Collège de France lectures on the concept of Nature. The material in question is found in Part 2 of the First Course (1956–57), titled “Modern Science and Nature.” After introducing the subject by bringing out the

[72] Borislav G. Dimitrov, 2013, “The Topological Alterity in Levinass chronotope As Ethical Analysis Situs”, ttp://www.academia.edu/3237203/The_Topological_Alterity_in_Levinass_chronotope_As_Ethical_Analysis_Situs

contribution modern science can make to the ontological clarification of nature, Merleau-Ponty proceeds to focus on quantum mechanics. Using Laplacean ontology as his foil, he summarizes interpretively such quantum mechanical themes as complementarity, nonclassical logic, and the inherently probabilistic nature of microphysics. Then he broadens his scope to explore the philosophical significance of quantum mechanics. In raising the question of what would constitute a philosophy adequate to the phenomena of the microworld, Merleau-Ponty rejects both nominalism and idealism. “If a philosophy can correspond to quantum mechanics, it will be both a more realistic philosophy, of which the truth will not be defined in transcendental terms, and more subjectivist. The situated and incarnated aspect of the physicist must succeed the universal ‘I think’ of transcendental philosophy” (97).

In 1961 Edmund Leach published “Rethinking Antropology”, where he proposed topological implementation in Antropology.

In 1962 Derrida’s *“Edmund Husserl’s origin of Geometry, an Introduction” appeared.* This work refers to the book which Husserl wrote shortly before his death and published only posthumously - *“The origin of Geometry” (1936).*

The year 1971 mark Hans-George Gadamer’s *“Hegel's Dialectic: Five Hermeneutical Studies”, "Die Idee der Hegelschen Logik (1971)*

Between 1972 and 1980, Gilles Deleuze’s most popular works – the two volumes of *Capitalism and Schizophrenia:Anti-Oedipus (1972)* and *A Thousand Plateaus (1980),* both co-written with the psychoanalyst Felix Guattari, appeared. Deleuze’s metaphysical treatise *Difference and Repetition (1968)* is considered by many scholars to be his magnum opus. The book *A Thousand Plateaus* was translated into English by Brian Massumi. [73]

[73] Deleuze’s view on the interrelation of qualitative and quantitative is demonstrated in his “Nietzsche and philosophy”. /Originaly published in France in 1962/, In the chapter “3. Quality and Quantity”, Deleuze discussed qualitative quantity as the “difference in quantity” on the interpretation in Nietzsche: “Forces have quantity, but they also have the quality which corresponds to their difference in quantity: the qualities of force are called "active7' and "reactive".

The term “Phenomeno-topology” first appear in Alain Badiou’s “Deleuze: the Clamor of Being” /translated by Louise Burchill, 2000, The University of Mianesota Press/. Writing about Deleuze’s “topology of the outside”, Alain Badiou claims that: “Deleuze devotes innumerable pages to this stage of his ontological identification, multiplaying the cases and refining the investigations – to such a degree that some have believed him to do noting other than replace phenomenology by a phenomeno-topology.” The aspects of the qualitative quantity in Deleuze’s “Renewed Concept of the One” /title of the chapter from Alain Badiou’s “Deleuze: the Clamor of Being”/, could be found in the Badiou’s assertion that “Deleuze is indeed he who announces that the distribution

In 1975 Mitchell Jay Feigenbaum discovered the so called *Feigenbaum constants* and created the known after him *Feigenbaum diagram*. The same year Rene Thom published *Structural stability and Morphogenesis* - An Outline of a general theory of models" (1975).

In 1970, Thom presented sophisticated catastrophe theory model of language. He developed a visual representation of the verbs associated with spatio-temporal activity. This was, Thom would say 20 yars later, a "geometrization of thought and linguistic activity". Thom classified syntactical structures into 16 categories and claimed that "the topological type of the interaction determines the syntactical structure of the sentence wich describes it." According to Thom, meaning and structure were no more independent. Thom constructed a modeling practice which, roughly speaking, used topologically informed means of transformation, biologically inspired raw materials that he adapted to mathematical practice.

Thom contributed to the idea of versal unfolding. (Bruce, B. and D.N. Mond .1999) The term 'versal' is the intersection of 'universal' and 'transversal', and one of the Thom's insights was that the singularities of members of families of functions of mappings are versally unfolded if the corresponding family of jet extension maps is transverse to their orbit (equivalence classes) in jet space." (Peter Tsatsanis, P. 2012: 223-224) This insight of Thom led him to the Catastrophe theory with identified by him seven orbits of function singularities which can be met transversally in families of fewer parameters – the seven elementary catastrophes, which meant to underlie all abrupt changes (bifurcation) in generic four parametrers families of given dynamical systems.

These seven are: fold, cusp, swallowtail, butterfly, hyperbolic umbilic, elliptic umbilic, and parabolic umbelic. Rene Thom used transversality as the main tool to prove the existence of universal unfolding. Thom created a mathematically rigorous theory that showed "the true

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of Being according the One and Multiple must be renounced." Alain Badiou claims that "…as always Deleuze, going beyond a static /quantitative/ opposition always turns out to involve the qualitative raising up of the one its terms."

complementary nature of the seemingly irreconcilable notions of versality and stability, that is, preserving identity in spite of development.

If the 'cusp' catastrophes could be well illustrated and explained through the well known law of dialectics of transformation of quantity into quality, the 'butterfly' catastrophe illustrates the topological notion of qualitative quantity.

There is striking analogy between Hegel's fourfold of infinities [1/. the bad qualitative infinity; 2/. the good qualitative infinity; 3/. the bad quantitative infinity; 4/. the good quantitative infinity] or the fourfold cobordism of the categories [of - quantitative quantity - /in the domain of Chronochora – Abstract Space and Abstract Time/; - quantitative quality - /in the domain of Chronotopos –Meaningful Place and Abstract Time/; - qualitative quantity - /in the domain of Kairochora – Abstract Space and Meaningful Time/; - qualitative quality - /in the domain of Kairotopos – Meaningful Place] and the four parameters of the butterfly catastrophe, where the volatile dyad is changed into a precarious triad and then into a stable tetrad.

Martin Zwick, in his paper "Dialectics and Catastrophe" (Zwick, M. 1978), assert, that there are two distinguishable types of dialectics, one which esults in victory of one of the opposing forces, and second which gives rise to a compromise or synthesis. Some dialectical phenomena are best modeled with the cusp, but others are more complex and more appropriately grasped with the butterfly, the butterfly of reconcillation, where the struggle of opposites within the cusp bifurcation set is itself negated.

In the year 1995, the term "topology of meaning" emerged at the proceedings of the "Einstein meets Magritte" Conference, Brussels, Belgium (1995). The "topology of meaning" was introduced by R. Ian Flett and Donald H. McNeil in their paper "What's Wrong with this Picture? Towards a Systemological Philosophy of Science with Practice." [74]

In 1982 Lin Chen formulated and published his *"Topological Structure In Visual Perception". (1982)*

[74] Donald H. McNeil, "What's going on with the topology of recursion?", Science and Art: The Red Book of `Einstein Meets Magritte': The Red Book Vol 2 (Einstein Meets Magritte: An Interdisciplinary Reflection on Science, Nature, Art, Human Action and Society), Kluwer, 1999 - Flett, R. Ian, and Donald H. McNeil. 1995).

In 1988 Alain Badiou first publication of "Being and Event" appeared, translated in English only in 2005. In "Mediation Fifteen on Hegel", Alain Badiou recognizes the "qualitative quantity" as the core of the domain of "quantitative infinity", claiming that "Quantitative infinity is quantity qua quantity, the prliferator of proliferation, which is to say, quite simply, *the quality of quantity*, the quantitative such as discerned qualitatively from any other determination." [75]

Alain Badiou has introduces his philosophical topology through two works – Logic of Words (), as an attempt to rephrase his material dialectic philosophical project in terms of topos theory, and Theory of the Subject (2009).

In *Theory of the Subject* (1982/2009), Alain Badiou states that "In the opening onto a new metaphor, we will say that there is the algebraic disposition and the topological disposition. (Badiou, 1982/2009:208)

For Badiou, "Any subject effectuated the operations of a topological algebras, (Badiou, 1982/2009:209). But "What does the mathematics call "algebra"?" (Badiou, 1982/2009:201) asks Badiou, and elaborates 'topological', stating that "Topology steems – via the arguments of analysis – from the need for a mathematical guarantee in order to grasp movements. It lies at the origin of primitively vague notions such as location, approximation, continuum and differential. It is not aimed (as algebra) at what happens when two distinct and homogeneous event end up being combined under certain constraints but at what happens when one investigates the site of a terms, its surroundings, that which is more or less "near" to it, that which is separated from it in continuous variations, its degree of isolation or adherence. . . If the master concept of algebra is that of the law (of computation), topology is based on the motion of neighbourhood. . . . for the dialectical interpretation, its clear that an open set is one that serves as neighbourhoods for each of its points." (Badiou, 1982/2009:201)

For Badiou, "topology works perforce on the parts of a set, considered as families of neighbourhood of an element (of a point, one will say, thereby making that this time it is the

[75] Alain Badiou, Being and Event, Oliver Feltham (tr.), Continuum, 2006, see p. 168-169, The Arcana of Quantity

location that is crucial). It does not associate to each elements an other element, but rather imposes upon it the multiform configuration of its environment. .. The algebraic legislation produces difference based on the other (element) as the same. The topological disposition makes identity of the same according to the multiple – other of its neighbourhood. …All in all, the element by itself has no interest for topology. It is a discipline of the heterogeneous in that it finds to determinate the point by families of points, the included by what lies around it. Its aim is to make a rule out of approximation. Much more so than the being of a term, it seeks to establish its system of proximate differences." (Badiou, 1982/2009:211)

Badiou concludes that if "the algebraic alterity is combinatory; the topological identity is differential." He discusses the transition of quantitative to qualitative in Hegel's Logic as follow:

"In Hegel's logic there is an astonishing passage in which the becoming – real of the One guarantees the transition from quantity to quality. This is nothing less than the speculative birth of number. What is quantity? Hegel assumes: the unity of continuus and the discrete (SL 119). That is the dialectic of adherence of the neighbourhood (continuity) and elementary becoming (discreteness). "This act of distinguishing or differentiation is an uninterrupted continuity (SL 188). Algebra is tied to topology (p.212) It is therefore the category of matematics that the opposition between Spatial dimensions and the Point is only partially resolved in the line (Hegel Enciclopedia paragraph 256) (Badiou, 1982/2009:212)

And Badiou continues "Just like the Point, The Line has neither area not body, it is not spatial. Does not allow us to make any true qualitative distinctions in Space. The opposition between Spatial Dimensions and the Point is further resolved in the Plane. . . Hegel sets about engendering starting from the pure concept of One. That is to say, on the basic of the placed elements as such. This artifice has great interest for us. Why? (Badiou, 1982/2009:212)

Badiou assrets that "Hegel Introduces two operatory mediations in order to accomplish according to the One, the engederment of algebraic discreteness and topological continuity: repulsion, by which One posits itself as distinct from the multiple-of-One; and attraction, by which One amalgamates the multiple with itself. . . . repulsion draws its theme from division as the essence of the One. It is "the self-determinating of the oneq at first into many, and then, because of their immediateness, in others (SL 173). . . . . the passage of algebra, the repulsive One, the first One to rise up in revolt – or in disjunction – is readable only on account of its contradictory virtue of attraction. It immediate aspect is the dissidence within the homogeneus multiple – this multiple of ones about which Hegel brilliantly affirms that in the place sketched out by it, the One "becomes only one" (ibid), its act consists in polarizing the entire field by an attractive unification: just as a localized, popular apprising, if its carries the proposition of a new unity, disturb the algebraic homogeneity in the topological direction of a regenerated consistency. (Badiou, 1982/2009:213)

For Badiou, "Hegel's One One is topological approximation – Das Eine Eins – the One One is that One which, from having emerged as subject under the law of repulsion – attraction establishes itself at the crossing point between an algebraic concern – which makes it One One – and topological, attractive, coagulating consistency – which makes it One One…" (Badiou, 1982/2009:214)

Any material subject One for the One One and One One according to the Ones, articulates the algebra of its placement and the topology of its novelty. (Badiou, 1982/2009:214) Badiou concludes that "Algebra is the logic of belonging, topology, a logic of adherence." (Badiou, 1982/2009:216)

As Clayton Crockett asserts, "For Badiou, the subject emerges at the intersection of algebra and topology, or space and force. A materialist subject "effectuates the operations of a topological algebra (Theory of the Subject 209). Badiou needs both algebra and topology to think the subject. The subject comes to be out of the gap between topology and algebra, or the exceess of topological force over algebraic composition. (Crockett.2013:126)

Crockett states that "Badiou takes inspiration from Lacan's topology, but argues that Lacan never truly escapes algebraic structural-symbolic thinking." (Crockett.2013:127) Badiou radicalizes Lacanian topology by means of his understanding of set theory from Cantor to Cohen in a way that prefigurates the grandiose achievement of *Being and Event*. Topology comes "from the need for mathematical guarantee in order to grasp movement" (Theory of the Subject 210). Whereas algebra concerns "what happens when two distinct and homogeneous events end up being combined under certain constraints," topology aims at "what happens when one investigates the site of a term, its surroundings, that which is more of less 'near' to it," or its neighborhood (Theory of the Subject 211). Algebra "produces difference based on the other (elements) as the same. The topological disposition makes identity of the same according to the multiple-other of its neighborhoods (Theory of the Subject 211). So algebra refers to the elements that belong to a set, whereas topology deals with the parts of the set that are included even as this inclusion exceeds belonging, to put it in term of *Being and Event.* (Crockett.2013:127)

Algebra names the calculable in materialism, while topology "takes things by the pack." It metaphorically translates the functional in materialism. (Theory of the Subject 215). Algebra "is a logic of belongings; topology, a logic of adherence" (Theory of the Subject 216). What is interesting here is that topology names what in Being and Event is the exceess of inclusion or parts over belonging, and this is what gives an evental site its composition (Crockett.2013:127)

Topology, or what Badiou calls a topological materialism, deals with the intersection and inclusion of neighborhoods. Badiou explains that "given a neighbourhood a point, there exists a superneighbourhood of this point such that the first neighbourhood (the 'bigger' one) is the neighbourhood of each of the points of the second (the 'smaller' one) (Theory of the Subject 223). –Here include D'Arcy transformation . . . prim Topological multiplication of neighbourhoods amplifies inclusion, producing an asymptotic approximation of common terms, regions, and names. In this way we can see how topology can produce a kind of consistency of neighbourhooda, or regions in what Badiou calls a subjective process, in distinction from subjectivisation. Algebra applies to subjectivisation; if follows the causality that produces a subject by destroying its previous structural place, whereas topology supplies a way to formulate the consistency of subjective process that recomposes another order of the real beyond (Crockett.2013:128)

Badiou divides concepts, modalities, and temporalities in two, producing and replicating division. He explains that "the subject materializes the division of materialism," because it consists in "the topological upheaval of an algebra" (Theory of the Subject 257-58). This topological upheaval of an algebra is supported by Badiou's interpretation of set theory, specifically Cantor's theorem that says that "the cardinality of the set of parts of [a set] E is always superior to the cardinality of E itself (Theory of the Subject 261). There is something excessive and inexistent about the superiority of parts over sets themselves, which gives Badiou the opening to his theory of the subject. Cantor's theorem implies that "conceived topologically, bu inclusion of its parts, E destroys the totalizing law of the maximum of multiplicity that it is supposed to be (Theory of the Subject 266). The problem is that mathematicians, including Cantor, want to limit the implications of this radical theorem to algebra, and supply an order to legislate it. Such mathematicians "neglect the real of the neighbourhoods; they restrict the alterity of the Same to its algebraic filiations" (Theory of the Subject 266). The name given to this legislation is the continuum hypothesis, which asserts a smoot and ordered continuum of being (Crockett.2013:129)

For Badiou, "The point is what stitches together the subject and the objective multiplicities that appear in a world. A point is a topological operator of decision, a yes or no whereby a body is identical and takes shape in a world according to subjective force and truth. The subject becomes a subject by making a radical choice to take place in the world as body, which is iltimately not just an object but a body of truth. The points of the world compose a topological space for a subject/body to emerge. As Badiou asserts "points space out the world" - in the Logic of the Worlds (416). Points, for Badiou constitute space for existence by concentrating "the degree of existence, the intensities measured by the transcendental, into only two possibilities" (LW 416).

Badiou's topological approach to class struggle between proletariat and bourgeoisie, with references to Hegel, is discussed by Alberto Toscano, who commented that "In the seminar dated '14 February 1977', Badiou approaches the question of the proletariat/bourgeoisie relation from a topological angle. If we follow an economistic tradition, which sunders Marx's *Capital* from the concrete (strategic) analysis of concrete (political) situations, bourgeoisie and proletariat appear topologically exterior to one another—the first defined in terms of its ownership of the means of production, the second in terms of its separation (alienation) from them. The result of this purely external topology, is paradoxically to render the proletariat functionally interior or immanent to the bourgeoisie."[76] (Toscano, 2006). As Toscano asserts "Insisting with the topological vocabulary, Badiou writes that 'the politics of the proletariat is

[76] Alberto Toscano, The Bourgeois and the Islamist, or, The Ot her Subjects of Politics, Cosmos and History: The Journal of Natural and Social Philosophy, vol. 2, no. 1-2, 2006, source www.cosmosandhistory.org

in a situation of internal exclusion with regard to bourgeois politics, that is, with regard to its object'. The proletariat is thus both within and against the bourgeoisie, constantly 'purging' its intimate bourgeois determination. Its 'topology of destruction' means that it is enduringly engaged in an effort to dislocate and ultimately destroy the site of its existence (without this destruction, it might just be a mask or ruse of the bourgeoisie, as Badiou deems to be the case for the USSR ); but it can only do so, because of its originary impurity, in an immanent, dialectical combat with the bourgeoisie that internally excludes it. This topological vision transforms the standing of the bourgeoisie within Badiou's theory of the subject. . ." (Toscano, 2006).

In 1974 Derrida published *"Of Gramatology"* entitled *"Algebra: Arcanum and Transparence" (1974).* The genealogy of Derrida's philosophical algebra[77], especially his algebra of undecidables, could be traced trough Godel back in its roots in Leibniz. Derrida devoted to Leibniz an important section of *"Of Gramatology"* entitled *"Algebra: Arcanum and Transparence" (1974).* This work of Derrida deals with the logical algebra …of writing. The strange term is typical for Derrida sense. Leibniz's ideas concerning the possibility of making topology into a rigorous mathematical discipline were among his great contribution to mathematics. In the 19 century Leibnitz's topology was developed into modern topology in the works of Karl Friedrich Gauss, Bernhard Riemann, Henri Poincare and others. In his "The Double Session", devoted to Deleuze's topology, Derrida offers us philosophically geometrical topological perspective on or approach to fold. This perspective is Derida's philosophical algebra, which entails, as Arkady Plotnitsky asserts a certain topology or spatiality.

In 1954 Rene Thom, in his remarkable, paper "*Quelques proprietes globales des varietes differentiables"*, formulated the concept of 'cobordism', and gave the full solution to this problem for unoriented manifolds, as well as many powerful insights into the methods for solving it in the cases of manifolds with additional structure. It was largely for this work that Thom was awarded the Fields medal in 1958. Cobordism had its roots in the attempt by Henri Poincare in 1895 to define homology purely in terms of manifolds. Poincaré simultaneously defined both homology and cobordism, which are not the same, in general.

[77] Arkady Plotnitsky, "Algebras, Geometries and Topologies of the fold: Deleuze, Derrida and Quasi-Mathematical thinking (with Leibniz and Mallarme"

### 1.4. The current topological turn and topological approaches.

**Philosophical Topology and Epistemology: The new frontier of Topology as Epistemology**

There is a research shift in progress for the last decades bringing the fusion between these two disciplines and giving the grounds to approach the relationship between topology and epistemology in different way of seeing them. **Oliver Schulte and Cory Juhl** regarded topology as epistemology, in their two papers from 1996 - **"Topology as Epistemology"**[78] and "**Epistemology, Reliable Inquiry and Topology"**[79]**.** Schulte and Juhl recommends the second paper as a friendly introduction to the connection between topology and epistemology, discussing some topological interpretations of Carl Popper's falsifiability criterion. Subject of Schulte and Juhl work is an interpretation of point-set topology as the theory of inquiry for logically omniscient agents with no limitations on memory capacity, a proposal that transferred topology into a powerful tool for epistemology. One of the distinguished researchers in the field of merger between topology and epistemology is professor **Kevin Kelly** (Carnegie Mellon) **with his remarkable work "Topological Epistemology".**[80] Kevin Kelly authored a tutorial on Topological Epistemology. According to Kelly, traditionally, discussions of inductive inference and empirical justification are framed within logic and probability theory, and he argued at length that the right framework is topology. The main initiators of the research in topological epistemology are **Rohit Parikh, Lawrence S. Moss, Chris Steinsvold (Topology and epistemic logic) exploring** applications of topological ideas in modal logic, especially in epistemic logic, applications of topological ideas in epistemic logic and offering a topological semantics and completeness proof for the logic of belief**.**[81] The mathematician, logician, and philosopher **Rohit Jivanlal Parikh** is one of the authors of "Topological Reasoning and The Logic of Knowledge" (with Dabrowski and

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78 Oliver Schulte, Cory Juhl, Topology as Epistemology, (1996). The Monist vol. 79:1, 141-147. http://www.jstor.org/discover/10.2307/27903468?uid=3737608&uid=2129&uid=2&uid=70&uid=4&sid=21102214017207

79 Oliver Schulte, Cory Juhl, Epistemology, Reliable Inquiry and Topology, Department of Philosophy, Carnegie Mellon University, Pittsburgh, PA, Department of Philosophy, University of Texas at Austin, 1996: http://www.cs.sfu.ca/~oschulte/files/pubs/monist.pdf

80 Kevin Kelly, Topological Epistemology - http://www.illc.uva.nl/lgc/seminar/?p=1241and http://www.andrew.cmu.edu/user/kk3n/homepage/kelly.html

81 Rohit Parikh, Lawrence S. Moss, Chris Steinsvold, Topology and epistemic logic - applications of topological ideas in modal logic, especially in epistemic logic; applications of topological ideas in epistemic logic; a topological semantics and completeness proof for the logic of belief KD45: http://www.indiana.edu/~iulg/moss/TEL.pdf

Moss) [82] An interesting approach, that illustrates the implementation of topology in epistemology, to the known Gettier problem, is offered by Chris Steinsvold in his work **"Topological Models of Belief Logics"**. Steinsvold suggests that "the topological semantic may be an interesting way to consider Gettier problem in epistemology." Discussing the theme of knowledge operator, Steinsvold claim that "there is a topological operator which acts like justified belief". (P.26 /1.11. "A note on knowledge and Gettier") According to Steinsvold - "The topological semantics seems to support the thesis that knowledge is true justified belief." As Steinvold establishes, "Up until 1963 this would not have been controversial. Since then Gettier's celebrated article has totally undermined this simple thesis." The arguments given by Steinsvold are that the modal logic S4 has a topological semantics and S4 is widely used as a basis of knowledge. In addition, Steinsvold establishes that there is an interior operator which obeys the S4 axioms. This interior operator acts knowledge and that helps to interpret the topological semantic on intuitive level. Steinsvold proposed the question – Is there a topological operator which obeys a logic of belief? The answer he gives is positive - Topological semantics seems to support the thesis that knowledge is true justified belief. Modal logic S4 has a topological semantics S4 is widely used as a logic of knowledge. There is a topological operator wich obeys a logic of belief and this topological operator acts as belief and helps to interpret the topological semantic on intuitive level. The interior operator acts like knowledge. In other words, S4 is topologically complete.

Topology of justification in new epistemological sense is subject of research works of Sergei Artemov and Elena Nogina.[83] Artemov and Nogina proposed topological implementation in epistemology based on the justification Logic as a family of epistemic logical systems obtained from modal logics of knowledge by adding a new type of formula constructed topologically, implementing the principal epistemic modal logic S4 which includes Alfred Tarski's well-known topological interpretation, according to which the modality $2X$ is read

[82] Parikh, Dabrowski, Moss, "Topological Reasoning and The Logic of Knowledge", Annals of Pure and Applied Logic 78 (1996) 73-110.

[83] Artemov, S., and E. Nogina, Topological Models for Justification Logic, TANCL 2007, Oxford University; Sergei Artemov and Elena Nogina, THE TOPOLOGY OF JUSTIFICATION: http://www.logika.umk.pl/llp/1712/21-1712zw.pdf; Artemov, S., J. Davoren, and A. Nerode, "Modal logics and topological semantics for hybrid systems", Technical Report MSI 97-05, Cornell University, 1997.; Artemov, S., and E. Nogina, "Logic of knowledge with justifications from the provability perspective", Technical Report TR-2004011, CUNY Ph.D. Program in Computer Science, 2004.; Artemov, S., and E. Nogina, "Introducing justification into epistemic logic, *Journal of Logic and Computation* 15, 6 (2005), 1059–1073.; Artemov, S., and E. Nogina, "On epistemic logic with justification", pages 279–294 in R. van der Meyden (ed.), *Theoretical Aspects of Rationality and Knowledge. Proceedings of the Tenth Conference (TARK 2005), June 10–12,2005*, Singapore, National University of Singapore, 2005.; Artemov, S., and E. Nogina, "On topological semantics of justification logic",*Algebraic and Topological Methods in Non-Classical Logics III (TANCL'07)Oxford, England, August 2007*.

*the Interior of X in a topological space* (the topological equivalent of the '*knowable part of X*'). Artemov and Nogina extends Tarski's topological interpretation from S4 to Justification Logic systems with both modality and justification assertions. The topological semantics interprets *t:X* as a reachable subset of *X* (the topological equivalent of '*test t confirms X*'). The authors etablished a number of soundness and completeness results with respect to Kripke topology and the real topology for S4-based systems of Justification Logic.

Jean-Claude Pont [84]and Pierre Beaudry [85] emphasizes on the existence of two opposite lines of battle over which mathematicians fought over 214 years, from 1679 to 1893, and the epistemological difference between these two – algebraic topology versus analysis situs.

Although Pont did not reference this opposition explicitly among the correctly identified mathematicians, he knew that the ***analysis situs genealogy*** that he appended in the conclusion portion of his book revealed the two cited distinctly opposite forces that I will now identify clearly for you. Pierre Beaudry elaborated Pont's the ***analysis situs genealogy,*** highlighting with color these two different and opposed flows. The first flow (orange) represents the Platonic-Leibnizian epistemological group flowing through Vandermonde, Gauss, Riemann, Betti, including Poinsot; and the other flow (green) represents the Aristotelian sense certainty group, flowing through Euler, Cauchy, Listing, Dyck, and to Poincaré who is the founder of modern topology and of chaos theory.

Presently, The Topology project is developed at Tate Modern in collaboration with NTNU Trondheim (Norwegian University of Science and Technology), Goldsmiths, University of London, Ohio State University, and the Centre for Freudian Analysis and Research, London.[86]

From the announcement of the project we learn that :"Mathematicians in the first half of the twentieth century constructed Topology as a general theory of space. It initially emerged as an

[84] Jean-Claude Pont, Topologie Algébrique, des origines àPoincaré**,** (Algebraic Topology, from the origins to Poincaré), Presses Universitaires de France, 1974, p. 173

[85] Pierre Beaudry, 2012, Analysis Situs and the principle of reciprocity,

[86] The Topology project is developed at Tate Modern/ http://www.tate.org.uk/whats-on/tate-modern/eventseries/topology

understanding of space in terms of properties of connectedness and invariance under transformation. Within a few years of its inception, psychologists, psychoanalysts, architects, artists, scientists and philosophers had started to use the conceptual language of relationships, intensities and transformations of this new theory outside its original field of mathematics. Limit, boundary, interior, exterior, neighborhood, disconnection and cut were central notions that became ways of describing the fields of forces experienced by individuals. Static ideas of space as a container were replaced by understandings of movement-space, of multiplicity, differentiation and exclusive inclusion that in turn have led to new ideas of power, subjectivity, and creativity."

Presently, The Topologies of Social Change is an area of research within the ESRC Centre for Research on Socio-Cultural Change at The University of Manchester, UK, where the theme is introduced as "Topological approaches (that) seek to address the fluidity and elasticity of social life. We approach notions of stability and change through an appreciation of the spatial and material qualities of relations, looking specifically at the more affective, messy, and hybrid aspects of transformative practice."[87]

The International Center for Formal Ontology (ICFO), affiliated with the Warsaw University of Technology, Poland, announced the 5th International Ontological Workshop - Topological Philosophy (February 8 – 9, 2016).[88] [89]

In 2014, The Hydra Dialogues, a strategic research initiative at the Royal Danish Academy of Fine Arts, School of Architecture, Architecture, with University of Copenhagen, and Roskilde University, under the thema of Space and Form, addresses issues of morphology, **topology**

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[87] ESRC Centre for Research on Socio-Cultural Change at The University of Manchester, UK http://www.archive.cresc.ac.uk/research/cresc2/index.html

[88] the 5th International Ontological Workshop - Topological Philosophy (February 8 – 9, 2016). http://www.icfo.ans.pw.edu.pl/en/?page_id=15 https://sites.google.com/site/ontologicalworkshops/vth-international-ontological-workshop-2016-topological-philosophy

[89] There is information on the web pages of the ICFO about the Graduate School on Topological Philosophy and the 2-day graduate-level course on topological philosophy held for students and doctoral students. The theme of the Workshop is "Topological Philosophy" with focus on the use of topological concepts and topological tools in philosophy, especially in the ontology (but not exclusively).

The tutorial on Topological Philosophy is provided by Thomas Mormann, Professor of Philosophy at the University of the Basque Country in Donostia-San Sebastian, Spain. Prof. Mormann obtained his PhD in Mathematics from the University of Dortmund (1978) and Habilitation from the University of Munich. Prof. Mormann is the author 'Topology As An Issue for History of Philosophy of Science', available at https://www.academia.edu/1585214/Topology_as_an_Issue_for_History_of_Philosophy_of_Science and artifice in today's built environment. Since 2004 Study Department 6, Royal Danish Academy of Fine Arts, School of Architecture under direction of Cort Ross Dinesen has researched issues of cartography, topology and architectural drawing in seminars and reoccurring summer schools on the Greek island of Hydra. The focus has been to consider contemporary topography developed as a topology for architecture in the contemporary built environment. Based on this ongoing research a new step has been taken in the form of The Hydra Dialogues: five dialogues at the School of Architecture, 2013-2014, culminating in May 2014 in an international conference with renowned scholars as keynotes. (Murphy P, 2014)

A Topological Approach to Cultural Dynamics (ATACD) a project funded by EU Framework program (2007-2010), lead by Prof Celia Lury, Warwick, research network based on the mathematical theories of topology, involved 19 university as partners from fields as diverse as semiology, crtificial intelligence, sociology, philosophy and mathematical economics. (Lury C., Parisi L. and Terranova T., 2012)

Topological reading of Qualitative quantity within Hegel's fourfold (manifold) of multiplicity is subject that lack attention at all from Hegel's commentators. This conceptual gap, ontological, epistemological, phenomenological is widening within the 'linguistic turn', 'topological turn' and emerging topological approaches to various fields of social science and their need for deep conceptualization based on the philosophical categories, notions and concepts.

As John WP Phillips, who in his paper On Topology (2013)[90], critically examines the recent arguments asserting a topological turn in culture, the range of topologically informed interventions in social and cultural theory, remarkes that such contemporary fashionable notions of 'topological approaches' and 'becoming topological of culture' "demands a greater critical reflection than the notion of a 'topological turn' suggests."

[90] Phillips, John WP. (2013), On Topology, Theory, Culture and Society, 9/2013; 30(5):122-152: http://www.researchgate.net/profile/John_Phillips20/publications [accessed Mar 21, 2015].

I believe that such demand of critical reflection shall be based on Hegel's categories of logic, notions and concept, and following my assertion that 'topological' in intuitively presented by Hegel in his logic, dialectic and method, my intend with the present thesis is to argue how Hegel's categories of qualitative and quantitative, the concept of space, time and place can provide contemporary researcher with the power and methodology of such a greater critical reflection, thus re-address in the new and enhanced mode the variety of these toplogical approaches. The methodology of an applied philosophical topology shall be based on the topological re-reading of Hegel and topological hermeneutics.

### 1.5. How to build a theory of philosophical-topological or topological-philosophical understanding through categorial "interpretation" of topological "structures"

The question - how to build a theory of philosophical-topological or topological-philosophical understanding through categorial "interpretation" of topological "structures" is formulated as an interpretation of the similar question first proposed in Bulgaria by Ivan Punchev in regard to mathematical in Hegel.

The present thesis is an attempt to answer this question through topological reading of Hegel and assertion about the presence of topological in Hegel's Logic, first and second through proposing an model - the model of topological notion of cobordism.

**Ivan Punchev** (Пунчев, И. 2006, 2011) proposed the thesis that the mathematics is subject to dialectical-logical systematization, as well as the dialectical logic is subject to mathematical formalization.

Punchev's works demonstrates an original philosophical interpretation of mathematical notions and concepts implemented in the dialectical logic of Hegel (in Punchev: dialectical theory of logical-mathematical Intelligence). According to Punchev, in the development of philosophy and mathematics (mathematical and philosophical Mind) there are "problematic situations" that have a common "categorial" structure by which they are to be studies and the "method" of the study can not be other than "mathematical modeling" or "explication" of philosophical "categories" and together with it – categorial "interpretation" of the mathematical "structures".

Punchev illustrates his assertion using perhaps one of the clearest examples of mathematicians affiliated with "philosophical language" and of philosophers who articulate the language of mathematics. He recalls that in Georg Cantor an entire foundation of mathematical theory as set theory is a mathematical "explication" (or "model") of the basic "categories" of philosophy and logic from antiquity to modern times, and argues that G.W.F. Hegel in his "Science of Logic" has given a categorical interpretation not only of the basic concepts of the modern mathematics of his time, but also of the future mathematics of 2hte 0th century - "multiple theoretical" mathematics and "mathematical logic". (Пунчев, И. 2006, 2011).

According to Punchev, philosophical-mathematical and mathematical-philosophical Mind have one and the same "subject," to whom study is applied with the "polar-opposites" of its "form" methods, or studying the same "subject" in the two polar-opposite "forms" through "method", which is inherently "the same." The single "essence" of the subject and "double form" of the method – or the double "form" of the subject and the single "essence" of the method. .(Пунчев, И. 2006, 2011). Punchev provides an answers to the question imposed by himself—how to build a theory of philosophical-mathematical or mathematical-philosophical understanding? And indeed he does so through one "parallel" history of "mathematics" and "philosophy," the history that would be able to present their common "philogenesis" and in addition, a "parallel" history that would be able to present the overall "ontogenesis"[91] of mathematics and philosophy, namely the genesis of "mathematical" and "logical" structures in the process of individual development of "intelligence." Here, Punchev recalls the genetic "epistemology" of Jacques Piaget built as "experimental" and along with that as "mathematical" science, which according to Punchev now gives us such a theory for the ontogenesis of Reason (Mind). The genetic epistemology of Piaget, mentioned by Punchev in relation to "mathematical" science should be understood as topology. The genetic epistemology of Piaget that "today gives us such a theory of ontogenesis of Reason" (Пунчев, И. 2006, 2011) is actually topology.

[91] I would like to note here, that it was **Gilbert Simondon**, the French philosopher who highly influenced the philosophical topology of Delauze, who proposed with his philosophy of individuation, the thesis of topology and ontogenesis.

Punchev's direction (to Piaget), inevitably refers to the 'topological primacy thesis' [9293] of Jacques Piaget and Bärbel Inhelder (Piaget, J., and Inhelder, B., 1956),[94] (Ian Darke, 1982), the thesis on the priority development of topological spatial perception in infancy. According to Piaget and Inhelder, the first geometric idea in children is a topological and later they develop projective geometry idea known as Euclidean geometry.

In relation to Piaget and Inhelder's thesis, it shall be recalled that, in 1982, the Chinese scholar Lin Chen published his research "Topological structure of visual perception" (Chen, L. 1982), justifying the idea that topological properties are actually primitive and innate qualities of perception of objects. According to Chen, our visual system is sensitive to global topological properties and extraction of these topological properties is a major factor in the organization of human perception. The relationships between topology and psychology, respectively psychoanalysis can be traced from Freud to Lacan, but perhaps the first entry implicitly topology in psychology took place in 1936 when the German Gestalt psychologist Kurt Lewin published his famous work "*Principles of Topological Psychology*".( Lewin, K. 1936)

Hegel, highlights Punchev, formulated this law on the basis of the discovery of the social "evolution" and the individual "development" of Spirit, before Darwin's discovery of the biological "evolution" of the "types" in nature, and at a time when to biology was known only the development of the "individual" organism. (Пунчев, И. 2011)

Hegel is undoubtedly one of the great representatives of the tradition of philosophical mathematics and Cantor is one of the great founders of mathematical philosophy and logic.

[92] Piaget, J., and Inhelder, B., 1956. The Child's Conception of Space. London: Routledge & Kegan Paul.
The **topological** nature of the genetic "epistemology" of Jacques Piaget, built as "experimental" and together with it as "mathematical" science, which represents an unique theory of **ontogenesis** of the Mind, corresponds to the theory, or rather with the theories of evolutionary development. The problematic situation of evolution (hierarchy and heterarchy of evolutionary systems), in which mathematical (topological) models and philosophical categories interact and interrelate, directs us to to Hegel, where in the words of Punchev, is implemented "one of the greatest discoveries that summarized the outcome of the development of the "problem situation " in German classical philosophy", namely Hegel's Phenomenology of Spirit, the work that "gives us" a method of genesis of Mind (Spirit) as "parallel" **Embryology** and **Paleontology** of the human Spirit. The fundamental law, which connects the two forms of the **genesis** (Punchev named "sotsiogenetical law of Mind"), states that the stages through which passes out the "individual" Mind are reduced (generalization "summary") repetition of the stages through, which passes the "social" Mind in its philogenesis."

[93]

[94] See also Ian Darke, "A Review of Research Related to the Topological Primacy Thesis," *Educational Studies in Mathematics,* Vol. 13, No. 2 (May, 1982), pp. 119-142.

Perhaps the strongest link between philosophy and mathematics is expressed by both Hegel and Cantor, in their concepts and notions of the unity and interaction of philosophical categories of quality and quantity. In Cantor's theory, the categories of quality and quantity are represented in mathematical notions of set theory and multiplicity as the quality of quantity.

My assertion is that 'quality of quantity' is one of the main issues in Hegel's concept of multiplicity. Hegel's dialectical logic of quantity and quality is presented in a topological model, namely in Hegel's fourfold of infinities—bad qualitative infinity; good qualitative infinity; bad quantitative infinity; good quantitative infinity. These four dimensions of infinity and the categories of quality and quantity do not form a single measure, but four measures, or: quantitative quantity; quantitative quality; qualitative quantity; qualitative quality.

Ivan Punchev insightfully shares a particularly significant finding, which unfortunately has remained almost unnoticed by his contemporaries, Bulgarian philosophers, namely that in the sections on "being for itself" in the sphere of Quality and Quantity, we may discover strikingly deep dialectics of the contemporary notions and concepts of modern mathematics as "multiple"—the dialectic of the One and the Many , "full" and "empty" multiplicity—anticipating by a whole half century Cantor's concept of "multiple". He especially anticipates the notion of the 'current'—the actual infinite "multiple." Cantor himself admits that, reading Hegel, he understood by Hegel's concepts almost nothing, but the fact remains that Hegel is a precursor to Cantor's "set theory " and before Cantor, Hegel implemented in his logic and dialectics the Actual infinity as own deep foundation.) (Пунчев, И. 2011)

The question proposed by Punchev — *how to build a theory of philosophical-mathematical and mathematical-philosophical Mind* (Пунчев, И. 2011) — can be set in particular with a specific type of mathematics, topology. As such, the answers to this question could be sought and interpreted in the *philosophical approach to topology* or the *topological approach to philosophy*, namely philosophical topology or topological philosophy. The proposed question could be paraphrased in these terms: *how to build a theory of philosophical topology or topological philosophy.*

Following and paraphrasing Punchev's reference to mathematical and philosophical and philosophical mind, I may assert, that the development of Topological Philosophy and Philosophical Topology, shall be based on the dimensions of the concept of "philosophical-mathematical (FM) and mathematical-philosophical (MF) Reason - ("knowledge of the general common" or "general common knowledge").(Пунчев, И. 2011)

In his work, "The doctrine of Hegel's Logical forms: concept, notions, statement", Punchev states that "contemporary experimental and mathematical study of the Nature and the structure of knowledge acquired this species, which has it in "Objective logic" of Hegel. "Being" of material objects is seen as Qualitative and Quantitative determined "events" in the structure of space and time. (Пунчев, И. 2011)

The comments made and the conclusions drawn by Punchev are extremely valuable to the approach and methods of scientific research in the debate between two paradigms— those centred on quantitative or quality approach. The triad of statements presented by Hegel in the three forms of Idea and the Sciences related to them ("Science of Logic", "Natural philosophy" and "philosophy of the Spirit"), together with the "rule" of relationships forming an "absolute triple deduction," constitute grounds to consider the quality of quantity as a logical form and method outside a formal-logical treatment. Moreover, the dialectics of Hegel's logical form is a quantity-quality "system of relations"—method, description and design.

Punchev states that Hegel created his 'dialectical logic' that is as universal 'systems theory' or 'universal system theory' - a credo, which he announced in the 'Preface' to the 'Phenomenology of Spirit'": the true form of truth is its logic system, and the form of 'consistency' is a concept. (Hegel). (Пунчев, И. 2011)

The relationship between philosophy and topology, or specifically between topology and the Hegelian system of logic and the logic of systems, has its roots in the universal trend towards algebraization of mathematics, and thus in logic. As Punchev states, "Following the general trend of algebraization in mathematics and hence in logic, in the late 20th century any 'logical system' has been seen as algebraic, and the mathematical theory of "categories" and "funktors" became the most universal algebraic method not only of modern mathematics, but of logic.

For explanation will only say that the mathematical category (as the Set of all sets in a universal and all morphism among them who fulfill certain axioms, so that a philosophical category can be represented as a "concept" for the mathematical category." (Пунчев, И. 2011)

The Topological nature of "Qualitative quantity" (as far as the exhibit form of this category is topological homeomorphism) illustrates how a philosophical (dialectical) category can be represented as a 'notion' of mathematical categories or in inversely, but truly, how the categories and methods of algebraic topology can illustrate the mathematical-dialectical logic and dialectics of qualitative quantity as an topological dialectical logic.

### 1.6. Categories

For Gadamer, "as a matter of fact, Plato's dialectic is not a method at all. The real construct of Hegel's dialectical method are the categories."[95]

The category is the basis of the idea of the new science of logic which Hegel expressly opposes to the traditional form of logic. For Hegel, after Kant, logic could no longer remain formal logic limiting itself to the formal relationship of concept, judgment, and syllogism.

For Kant the understanding contains a mere 12 categories corresponding to the 12 forms of judgment: unity, plurality, totality, reality, negation, limitation, possibility, actuality, necessity, substance, causality, and reciprocity (Kant 2003: 113). [96] By contrast, Hegel's *Science of Logic* can be understood as a solid reworking and expansion of Kant's list of categories. While Kant elaborates only 12 categories, Hegel expounds over 80, including such central and fundamental terms as being, essence, existence, actuality, necessity and universality (Hegel 1969: 15-22).[97]

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[95] "Die Idee der Hegelschen Logik (1971) in Hans-George Gadamer, "Hegel's Dialectic: Five Hermeneutical Studies", translated into English by P. Christopher Smith and collected in (New Haven: Yale University Press, 1976). These five essays are "Hegel and Heidegger,"; "Hegel's Dialectic of Self-consciousness"; "Hegel and the Dialectic of the Ancient Philosophers,"; "Hegel's 'Inverted World,'; and "The Idea of Hegel's Logic," 75-99

[96] Kant, I. (2003) Critique of Pure Reason, trans. by Norman Kemp Smith, Hampshire: Palgrave Macmillan.

[97] Hegel, G.W.F. (1969) *Science of Logic*, trans. by A.V. Miller, New York: Humanity Books.

Hegel famously observed that Kant failed to take note of the *content* of the *categories themselves,* in Kant the categories are presupposed, taken over from classical logic as concepts self-evidently valid and comprehended (Hegel 1892: 50-52, 83; Hegel 1969:33, 594, 789).[98] [99] Hegel's *Science of Logic* represents an exposition of the dialectical relations that constitute the immanent content of the categories. While Kant posited that the categories taken on their own are empty, thoroughly devoid of content, Hegel claimed that, "to assert that the categories taken by themselves are empty can scarcely be right, seeing that they have content, at all events, in the special stamp and significance which they possess. Of course, the content of the categories is not perceptible to the senses, nor is it time and space: but that is rather a merit than a defect" (Hegel 1892: 91).[100]

Kant maintains that our thoughts can only have a determinate content when they are bound to an intuition which is delivered to us by the senses; whereas for Hegel, the content of a concept is first and foremost that which it must be *opposed to* in order for it to emerge in its clarity and distinctiveness, in order for it to be more than an empty name (Hegel 1892: 152; Kant 2003: 93). [101] [102] The key to understanding Hegel's idea that the categories each have their own specific content lies in understanding **what counts as a "content" for Hegel**. For example, Hegel will posit that the "content" of the concept of infinitude (*Unendlichkeit*) is first and foremost everything contained in the concept of finitude (*Endlichkeit*), of which it is the negation (Hegel 1969: 143). [103]

Hegel seeks to give logic a new scientific character by developing the universal system of the concepts of the understanding into a "whole" of science. Hegel's starting point is Kant's traditional theory. In contrast with Kant, for Hegel categories are not simply formal determinations of statements or thinking.

On this new understanding of the categories, Hegel build on his logic as the ideal of a science of logic, as an syntesis of the doctrine of Being and the doctrine of Essence in the doctrine of Concept.

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[98] Hegel, G.W.F. (1892) *The Logic of Hegel*, trans. by William Wallace, Oxford: Clarendon Press.
[99] Hegel, G.W.F. (1969) *Science of Logic*, trans. by A.V. Miller, New York: Humanity Books.
[100] Hegel, G.W.F. (1969) Science of Logic, trans. by A.V. Miller, New York: Humanity Books.
[101] Hegel, G.W.F. (1892) *The Logic of Hegel*, trans. by William Wallace, Oxford: Clarendon Press.
[102] Kant, I. (2003) *Critique of Pure Reason*, trans. by Norman Kemp Smith, Hampshire: Palgrave Macmillan.
[103] Hegel, G.W.F. (1969) *Science of Logic*, trans. by A.V. Miller, New York: Humanity Books.

Hegel’s doctrine of Being follows Kant’s table of categories including the categories of quality and quantity. The doctrine of Essence and the doctrine of the Concept, on the other hand, explicate the categories of relation and modality.

According to Kant's Theory of Knowledge, the mind has its own pure concepts by which it organizes the flux of sensory impressions into substances, qualities, and quantities, and into causes and effects. The the mind is furnished with twelve pure concepts or categories. The Pure Concepts of the Understanding are:

Quantity: Unity - Plurality - Totality

Quality: Affirmation - Negation - Limitation

Relation: Substance - Accident- Cause-effect - Causal-reciprocity

Modality: Possibility - Actuality - Necessity

Kant maintains that the mind actively interprets the world rather than passively receiving and recording in memory.It is the categories of our own minds that organize the sensory flux and give it meaning. Substances with qualities, quantities, related as causes and effects or in reciprocal causation.

The transcendental deduction of the pure a priori concepts of understanding resulted in the well-known table of categories (quantity, quality, relation and modality), corresponding to the logical functions in all possible judgments. In respect of the a priori side of all knowledge, at least four conditions must be satisfied. What must first be given is the manifold of pure intuition. The second factor involved is the synthesis of this manifold by means of the imagination, although this does not yet yield knowledge. The *manifold* of intuition can only be united in the concepts which give unity to this pure synthesis. In all knowledge of an object there is unity of the concept, which may be entitled qualitative unity. This qualitative unity is not the category of unity, since we must look yet higher for it, namely in that which itself contains the ground of the unity of diverse concepts in judgment. Bringing the synthesis of imagination to concepts is a function which belongs to the understanding. An object is that, in the concept of which, the manifold of a given intuition is united. All all unification of representations demands unity of consciousness in the synthesis of them. Consequently, it is the unity of consciousness that alone constitutes the relation of representations to an object, and therefore their objective validity and the fact that they are modes of knowledge; and upon it therefore rests the very possibility of the understanding'. This unity of consciousness /compare the equivalent expressions: 'transcendental unity of self-consciousness'; 'transcendental unity of apperception'; and 'cogito', i.e. the 'I think'/, contains no manifold. In 'the representations "I am", nothing manifold is given'. To act as unifying instance, the cogito cannot contain any manifold. This reasoning also explains Kant's earlier remark, referring to the understanding as 'an absolute unity'.

In Kant's Critique of Pure Reason, the categories are pure concepts which give unity to the pure synthesis. The system of categories parallels the system of forms of judgment:

- Quantity (unity, plurality, allness)
- Quality (reality, negation, limitation)
- Relation (inherence/subsistence, cause/effect, community)
- Modality (possibility/impossibility, existence, nonexistence, necessity/contingency)

Every judgment combines presentations in four ways:

- Quantity (universal, particular, singular)
- Quality (affirmative, negative, infinite)
- Relation (categorical, hypothetical, disjunctive)
- Modality (problematic, assertoric, apodeictic)

Hegel himself acknowledges that his own logic is a first attempt which lacks ultimate perfection. It means that by pursuing multiple paths of derivation, one could work out, as he himself did in his teaching, the fine distinctions of what had only been given in outline form in the *Logic.*

The methodological necessity in the interconnection of concepts as they unfold according to their specific dialectic, is not necessity in the absolute sense. Hegel’s point is not only that in his *Logic* he did not complete the enormous task before him, but beyond that, in an absolute sense, that it cannot be completed.

The categories of ‘quality’ and ‘quantity’ address words that are derived from members of a family of Latin adverbs beginning with the prefix "qu-". The both word address an interrogative use with the verb *quaero*, to seek, to ask, to inquire. The word ‘qualitative’ derives from ‘So *qualis*?’ (I ask: what sort?), or in Hegel’s terms ‘quality’ direct to the ‘one’ and what ‘*is*’, to ‘something’ that *is* different to ‘other’. The word *quantus* direct to ‘many’ and means ‘I ask: how much?. The prefix "qu-" is a kind of question mark that translate *qualis* and *quantus* respectively as "(?)sort" and "(?)degree." In Latin, these two words addresses with questions the ‘time’ and ‘space’ - *quando* (time); *quamdiu* (elapsed time); *quo* (place), reason and distance: *quia* or *quare* (reason); distance, *quoad* (distance); *quot* (number) and *quoties* (frequency); *quotus* (number in series), and so on.

It was Kant who derived the categories from the logical functions of judgement, yet stating that we cannot explain ‘why we have precisely these and no other functions for judgement” (Critique of Practical Reason). Before Hegel, the reason why we employ precisely these and no other categories remains inexplicable. Kant never explains why we should think in terms of these categories and he never explains why the logical functions from which the categories are derived are necessary. Following Fichte, Hegel explains the categories ‘in their necessity”. For Hegel, the categories must be derived, not from presupposed forms of judgement, or from our presuppositions, but from what Hegel calls the sheer ‘simplicity of thinking’.For Hegel, the categories must be derived from the indeterminate being of thought, from the indeterminate thought of being. Hegel’s speculative logic exhibits a presuppositionless derivation of the categories. Kant puts quantity at the head of his table of categories and sees no entristic logical connection between quantity and quality. Hegel first derives the categories of quality and then shows how quality itself leads logically to quantity. For Hegel there is a clear difference between quality and quantity and thinks that quantity is made necessary by quality, yet quality and quantity are not just two distinct categories, but they are related logically, the quality makes the quantity necessary. For Hegel quantity itself is a distictive kind of quality.

For Hegel quantity is characterized by continuity, it represent the unity of the moments of continuity and discreteness, quantity divides itself into the distinct forms of continuos and discrete magnitude.Continuity and discreteness are moments of one another and quantity clearly anticipates the logical structure of essence, quantity remains with quality and measure a moment of being. (Houlgate, 2014)

There are strong correspondences between Hegel and Fichte concerning the notion of Qualitative and Quantitative, in the particular the notion of Qualitative quantity and Quantitative quality. Friedrich Hardenber, known as Novalis concerned himself with the scientific doctrine of Fichte in the period of 1795–1796. Fichte's philosophy influenced greatly Novalis's world view. Novalis not only read Fichte's philosophies but also developed Fichte's concepts further, transforming Fichte's *Nicht-Ich* ("not I") to a *Du* ("you"), an equal subject to the *Ich* ("I"). The first complete translation in English of Novalis's "Fichte Studies", is the book edited and translated by Jane Kneller, published in 2003 by the Cambridge University Press (Kneller, J. ed., trans. 2003). In part of the book entitled "Group VI: 569-667, summer to fall, 1796", we can see the following table of Fichte's categories given by Novalis:

> 599. Quantity
>
> Unity: Modal Quantity
> Plurality: Modal - Qualitative Quantity
> Totality: Modal – Qualitative relative quantity
>
> Relation:
> 4. – modal relation
> 5. - modal – qualitative relation
> 6. – modal – quantitative qualitative relation
>
> Quantity:
> 4. Reality – modal quality
> 5. Negation – modal relative quality
> 6. Limitation – modal relative quantitative quality
>
> Modality:
> 4. Qualitative modality
> 5. Qualitative relative modality
> 6. Qualitative – relative quantitative modality …(Kneller, J. ed., trans.2003::177)

From the similarities between Hegel's Qualitative quantity and Fichte's Qualitative quantity, we can derive the comparable elements between Hegel and Fichte about their notions of Infinity. One of the most significant propositions of Fichte's *Wissenschaftslehre*, is the movement of knowledge in the process of the Sublime, the Sublime defined by Kant's Third

*Critique* as an attempt to "present the infinite". In Fichte's *Wissenschaftslehre* the sublime is the dynamics of the mind, Mathema of the mind. Fichte promotes the concept of the Infinity (Sublime) to the level of a gnoseological process following from the critique of objectifying knowledge to the definition of knowledge beyond representation. Perhaps Fichte's notion of infinity based on his notion of Qualitative quantity and Quantitative quality is reflected in Hegel's categorical fourfold of infinities.

The sharp distinctions between the these two philosophies of Fichte and Hegel are well known, but the topoi between Hegel and Fichte is evident in one of the most important component of the philosophical thinking – the notion of quality and quantity, the notion that could bridge the "disagreements" between these two philosophers, bridging the two cores – Hegel's Self as Absolute and Fichte's Other.

Following Fichte's claim about "Self As Other", topologically we can see the Absolute present in both – in the Self and in the Other. The topological approach here is possible to be elevated on the ground of the thesis of "mathematical" Fichte, in particular the thesis of Mathesis in Fichte, thus the transformation of Self (I) and Other (Non-I) in Fichte's "Self as Other" could be seen as topological transformation and perhaps in the Fichte's notion of quality and quantity the true grounds of the Topological Hegel or The Topological Notion of Qualitative quantity can be re-elevated.

The mathematical concept of maniford could be traced to Leibniz, but it was Reimann who elaborated the concept. As Arkady Plotnitsky asserts in his influencial work "Algebras, Geometries and Topologies of the fold: Deleuze, Derrida and Quasi-Mathematical thinking (with Leibniz and Mallarme" – "*the noun "maniforld" marked the end of dialectics and the beginning of topology*".

For Hegel, the logic is the most abstract of the object totalities and this is the reason for him to begins this section with the Being as universal principle of everything. It will take ten (out of a total of 14) subsections for him to elaborate dialectically mathematically important concepts such as (determine Number), Numbers and arithmetical operations, from the universal principle – Being. Here Hegel regards some oppositions and their resolution as self-evident, such as the opposition between Being and Nothing and its resolution in Becoming and

Presence. From Presence until the start of section B, Quantity, Hegel uses these questions explicitly as a tool to drive his exhibition onwards (Hegel $1830_3$, $1817_1$: §§89-98). After that, the quantitative and its moments (including Measure) are again not explicitly discussed this way (Hegel $1830_3$, $1817_1$: §§99-107).

Hegel develops important mathematical categories systematically out of other more abstract categories and this reflects on the meaning of these categories and how in turn Hegel's account reflects on the mathematics in which the categories are utilized.

Mathematical categories presuppose abstract categories in common language. So, contrary to popular belief, the mathematical mindset is founded on languages like English, French, German, Dutch and the like. It therefore is not a language in its own right.

The totality of interrelated categories brings the inner nature of the object totality to light.[31] [32]

This means that categories are only fully defined when the exhibition is complete. Before that happens, the categories have to remain flexible as we need to view them from different erspectives with each step that is taken towards concretization. One of the consequences of this is that rigorous definitions of categories by means of assumptions are essentially ruled out.

Roland Williamson relates Hegel's dialectical method with Saussure's structuralist linguistics, asserting that the essence of Hegel's "dialectical method" is a *linguistic* method and this enabled Hegel to determine the *linguistic* relations underpinning the categories - a method which is moreover in deep continuity with Saussure's structuralist linguistics (Hegel 1969: 441, 831-36). (Roland Williamson, 2009:4). For Saussure, as he puts it in his Course in General Linguistics, concepts are to be "defined not positively, in terms of their [meaning], but negatively by contrast with other items in the same system" (Saussure 1983: 115).

In his *Science of Logic*, Hegel does not provide exact definition for each of the categories, the definition that one would expect to find in a dictionary. Instead he elaborates the category's immanent linguistic relations with other categories (Hegel 1969: 795-800, 834-36). [104] Hegel

[104] Hegel, G.W.F. (1969) *Science of Logic*, trans. by A.V. Miller, New York: Humanity Books.

exhibits the self-contradictory nature of any of his categories, the transitions they must undertake over into one another. Hegel demonstrated how each category is to be linguistically dependent upon others (necessity upon contingency, identity upon difference, being upon nothing etc.).

For Hegel, the categories are presented in our minds. In this regard Hegel asserts the following:

"The forms of thought are, in the first instance, displayed and stored in human language. Nowadays we cannot be too often reminded that it is *thinking* which distinguishes man from the beasts. Into all that becomes something inward for men, an image or conception as such, into all that he makes his own, language has penetrated, and everything that he has transformed into language and expresses in it *contains a category* – concealed, mixed with other forms or clearly determined as such, so much is logic his natural element, indeed his own peculiar nature. (Hegel 1969: 31-33)[105]

The dialectical method is the means to the cognition of the linguistic relations underpinning the categories, and the categories are the means to the cognition of empirical reality. It follows that the dialectical method must be applicable to at least some aspect of empirical reality, namely, the dialectical aspect. (Roland Williamson, 2009:6/7).[106]

Hegel discerns that this dialectical aspect can be isolated in Nature by examining not so much the secure findings of the individual sciences *as the transition between the sciences*: the transition from physics to biology, from biology to anthropology etc. (Hegel 1970: 20-21, 24-27, 443-45)[107], (Roland Williamson, 2009:6/7). As Williamson asserts, "Hegel's own account of such dialectical transitions within his *Philosophy of Nature* is severely outdated, but I contend that this is less a result of any flaw in the dialectical method as it is a result of the severe limitations of the science of his day. Indeed the very fact that these sciences continue to develop today in general isolation from one another (e.g. one can be a neuroscientist without being a physicist) demonstrates that the categories being employed in each science differ

[105] Hegel, G.W.F. (1969) *Science of Logic*, trans. by A.V. Miller, New York: Humanity Books.
[106] Roland Williamson, 2009, Hegel Among the Quantum Physicists, International Journal of Žižek Studies, 3 (1) (2009).
[107] Hegel, G.W.F. (1970) Philosophy of Nature, trans. by A.V. Miller, Oxford: Oxford University Press.

radically. To use the simplest example, the category of "life" is inapplicable to the domain of objects examined in physics and chemistry, and can only be legitimately applied to the objects studied in biology (Hegel 1969: 761-66; Hegel 1970: 18-19, 270-75).[108] [109] The dialectical method can thus be productively employed in examining the *transition* from e.g. chemistry to biology, that is, in examining the metaphysical emergence of the category of life in physical reality. However, while dialectical transitions are ubiquitous and transparent in the abstract domain of the logical categories (since the dialectical method is, after all, a *linguistic* method), such transitions are counter-intuitive and uncanny when they take place in physical reality. (Roland Williamson, 2009:7).[110]

In this regard, as Williamson concludes, that "Žižek's work on quantum mechanics represents a transparent reworking of Hegel's *Philosophy of Nature*: he is reapplying Hegel's dialectical method to what one might call the zero-level dialectical transition in physical reality, namely, the quantum mechanical phenomenon of the wave function collapse." And "Žižek's work on quantum mechanics thus demonstrates the immediate relevance and applicability of Hegel's dialectical method to contemporary philosophical and scientific problems: the "paradoxes" which had traditionally plagued quantum physicists simply dissolved away once the dialectical method was mobilized for their apprehension, resulting in the current physical theory of quantum decoherence. (Roland Williamson, 2009:7).[111]

Hegel's logic and philosophy, especially his philosophy of nature is mediated coincidence of logic and the world to which logic applies. (Paterson 2006:2) For Hegel, the science and spirit (ethics, art, religion, philosophy itself) is structured by the logical categories, and 'logic' is understood in terms of rational structure. (Paterson 2006:2) As Paterson argues "the assertion as ``p ∨ ∼p'', the law of the excluded middle, treated as formal logic, is only `the tip of the iceberg'' as far as its rational content is concerned. . . A common structure of thought underlies all of human knowledge and experience, not in any ``subjective'' sense but as expressing the reality that is forcing itself through human knowledge and experience. In this

[108] Hegel, G.W.F. (1969) *Science of Logic*, trans. by A.V. Miller, New York: Humanity Books.
[109] Hegel, G.W.F. (1970) Philosophy of Nature, trans. by A.V. Miller, Oxford: Oxford University Press.
[110] Roland Williamson, 2009, Hegel Among the Quantum Physicists, International Journal of Žižek Studies, 3 (1) (2009).

[111] Roland Williamson, 2009, Hegel Among the Quantum Physicists, International Journal of Žižek Studies, 3 (1) (2009).

way, Hegel answers the fundamental question of how knowledge and truth over *all* of human experience relates. (Paterson 2006:2) I agree with Paterson for whom "Hegel had the courage to try to understand the relation in terms of his profound logic – the logic of the world is the logic of thought, for we are thinking *beings*." (Paterson 2006:2)

## Chapter 2 From Hegel and Mathematics to Topological (in) Hegel

In relation to *Hegel's Logic and mathematics*, this study builds on and contributes to works of **Alan L. T. Paterson** (Paterson 1994, 1997a, 1997b, 1999, 2000, 2002, 2004, 2005), **David G. Carlson** (Carlson 2000, 2002, 2003a, 2003b), **Dirk Damsma** (Damsma D. 2008; 2010; 2011; 2015), **Arkady Plotinsky** (Plotnitsky, 2009), **William Lawvere,** who established proposals for formalization of Hegel's objective logic in categorical logic (Lawvere 1991), (Lawvere 1992), (Lawvere 1994), (Lawvere 1995), (Lawvere 1997), claiming that proposals for formalizing some of Hegel's thoughts in terms of algebra may be identified in Hermann Grassmann, Ausdehnungslehre (1844), in particular Lawvere's suggestion that a significant fraction of dialectical philosophy can be modeled mathematically through the use of "cylinders" (diagrams of shape Δ) in a category, wherein the two identical subobjects (united by the third map in the diagram) are "opposite" (Lawvere, 1996).

In relation to *Hegel's Logic and topology*, the present study builds on and contributes to work of **Jeff Malpas,** who to my knowledge first introduces the issue of 'philosophical topology' in series of papers, claims that "topology is present in Heidegger and, though less explicitly, in Hegel."[112] For Malpas, the primary emphasis is "on gaining further insight into the idea of topology itself, along with the ideas it encompasses and to which it relates."[113]

Although Malpas's works are focused mainly on Heidegger, and just touches Hegel very slightly – "but Hegel will also have a role to play", [114] **my assertion is that the concepts and notions of Hegel's Logic and the manner Hegel develops these, exhibits topological properties in the strict sense the 'properties' that are invariants under some transformations, under homeomorphisms, which are bi-continuous one-to-one functions.**

[112] Jeff Malpas, Self, Other, Thing, http://philevents.org/event/show/13584
[113] Ibid

[114] Ibid

In regards of *topological (in) Hegel*, the present study builds on and contributes to work of Alain Badiou's "Being and Event" (1988), where he recognizes the "qualitative quantity" as the core of the domain of "quantitative infinity", claiming that "Quantitative infinity is quantity qua quantity, the prliferator of proliferation, which is to say, quite simply, *the quality of quantity*, the quantitative such as discerned qualitatively from any other determination." [115]

Although Alain Badiou in "Being and Event" (1988), states that "the good quantitative infinity is a properly Hegelian hallucination". (Badiou, A. 2006: 168-169), **my arguments versus Badiou's disagreement with Hegel, follow the position of Jonas Jervell Indregard, (Indregard, J.J. 2010)** who claims that "in fact, Badiou *does not object* to Hegel's deduction of the good *qualitative* infinite. In fact, Badiou recognizes that there is not, in Hegel, a simple dichotomy between qualitative and quantitative infinity, but rather a fourfold of infinities: the bad qualitative infinity, the good qualitative infinity, the bad quantitative infinity, and the good quantitative infinity." (Indregard, J.J. 2010).

***With an exception of Andrew Hass - Hegel and the Problem of Multiplicity* (2000).[116] (Haas, A. 2000), *there has not been an investigation on the thesis proposed here - Topological Notions of Multiplicity in Hegel's Fourfold of Infinities.***

Andrew Hass, proposed and answered a question with great importance, the question of understanding the core and essence of Hegel's logic and dialectics in terms of homology and topology – *What does it mean to think multiplicity as two-sided, as subject to the process or movement of double-edged, de-limitation, to the logic of the concept and its relational borders, frontiers, horizons, thresholds? . . . and What does it mean to think multiplicity as both quality and quantity?* (Haas, A. 2000: 115)

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[115] Alain Badiou, Being and Event, Oliver Feltham (tr.), Continuum, 2006, see p. 168-169, The Arcana of Quantity

[116] Haas, A. (2000), Hegel and the Problem of Multiplicity. SPEP Studies in Historical Philosophy. Evanston: Northwestern University Press (Haas A, 2000)

***In contrast with Haas, who does not provide explicit direction toward topology, my response to Haas's question is 'topology', and the 'topological notions of Hegel's multiplicity', the 'topological notion of Hegel's fourfold of infinities', and the 'topological cobordism of Qualitative quantity'.***

## 1. Hegel and Mathematics - Previous Literature on Hegel and Mathematics

Hegel's determination of the categorial foundations of mathematics. Hegel provided some tantalizing insights into the nature and essence of mathematics. (Damsma 2015).

The relationships between Hegel's Logic[117] and Mathematics, Hegel's dialectical transformations and important concepts in his Wissenschaft and Encyclopädie der philosophischen Wissenschaften are discussed, explained and amended with modern-day insights in the research work of **David G. Carlson** (Carlson 2000, 2002, 2003a, 2003 b), **Alan L. T. Paterson** (Paterson 1994, 1997a, 1997b, 1999, 2000, 2002, 2004, 2005)[118], **Dirk Damsma** (Damsma D. 2008).

David Gray Carlson's "A Commentary on Hegel's Science of Logic" is the first English language full commentary on The Science of Logic, and it is considered a major advancement in the study of Hegelian philosophy. Carlson has devised a system for diagramming every single logical transition that Hegel makes, many of which have never before been explored in English. The topological approach in Hegel's Science of Logic is evident in Carlson's diagrams for clarifying the argumentative structure of each move of the text in the fashion of complicated Venn diagram called a Borromean Knot /Lacan's famous Borromean knots/. Carlson presents Hegel's logic in the form of pictorial triads of overlapping concepts, in the quite topological mode of Lacanian knots (Carlson 2003a).

[117] The two Logics

[118] On Paterson's website (https://sites.google.com/site/apat1erson/) are available all of his papers on the Hegelian philosophy of mathematics. Three of these are about the philosophy of Number (1997b; n.d.; 2000), one is about the 'Hegelian Concept and set theory' (2007) and one (2002) is about the Hegelian philosophy of mathematics in general. In each of these, Hegelian philosophy is proposed as a solution to the problems 'which arise out of the existence in mathematics of self-referential, non-constructive concepts (such as *class*)' (Paterson 2002: 143).

The "inapparent" Hegel and topological notion of his philosophy is revealed by Carlson as we read in the publisher's review of the book: "The author has devised a system for diagramming every single logical transition that Hegel makes, many of which have never before been explored in English. This reveals a startling organizational subtlety in Hegel's work which heretofore has gone unnoticed.

In the course of charting Hegel's logical progress, the author provides a vigorous defence and thorough explication of unparalleled scale and scope." Carlson's diagrams are really evoking the topological qualitative quantity of the Lacan's famous Borromean knot, where no one of the rings is directly tied to the other, but if you cut *one* of the rings the other two slip away.

The qualitative quantity is the third ring linked with quality and quantity in the measure and if one cut off the qualitative quantity, the notion of the two – quality and quantity will remain uncomplete and non-topological.

Some attempts to formalize Hegel's dialectical logic using set theory are implemented in the works of **Reinhold Baer** (Baer, R. 1932), **Michael Kosok** (Kosok, M. 1972), **Graham Priest** (Priest G. 1989).

In set theoretical terminology one might say that Hegel sought to resolve Russell's paradox (Russel, B. 1903) long before it was formulated.

In the series of papers and his PhD thesis, **Dirk Damsma** investigates the set theory and geometry in Hegel (Damsma 2011)[119], Qualitative and quantitative analysis in systematic dialectics (Damsma 2010)[120], the articulation of systematic-dialectictical methodology (Damsma 2015)[121], the dialectical foundation of mathematics in Hegel (Damsma 2011)[122] -

[119] D. Damsma (2011). Set theory and geometry in Hegel. In A. Arndt, P. Cruysberghs & A. Przylebski (Eds.), *Geist? - Tl. 2 Vol. 2011. Hegel-Jahrbuch* (pp. 54-58). Berlin: Akademie Verlag.

[120] D. Damsma (2010). *Qualitative and quantitative analysis in systematic dialectics: Marx vs. Hegel and Arthur vs. Smith.* (Preprints). Amsterdam: University of Amsterdam, School of Economics.[go to publisher's site]

[121] D.F. Damsma (2015, January 09). *On the articulation of systematic-dialectical methodology and mathematics.* Universiteit van Amsterdam (viii, 201 pag.). Supervisor(s): prof.dr. J.B. Davis & dr. G.A.T.M. Reuten.

[122] D. Damsma (2011). *On the dialectical foundations of mathematics.* (Preprints). Amsterdam: University of Amsterdam.
Available from:

dialectical determination of mathematical concepts in Hegel's Encyclopädie der philosophischen Wissenschaften (1830, 1817), and investigates the insights that can be gained from such a perspective on the mathematical. Damsma offers discussions on the determination of Numbers and arithmetical operations from Being showing that the One and the successor function have a qualitative base and need not be presupposed. He demonstrates how for infinite Intensive Magnitudes (cardinals) there exists an Extensive Magnitude through which they gain meaning. Damsma concludes that the 'bad' in Hegel's 'bad infinity' is a trifle problematic. Damsma's suggestion is that if 'Dasein' is interpreted as the whole of perception in the present, Place can be viewed as the spatial Now, Motion as the passage from Place to Now and Matter as the actual (as opposed to observed) Presence of the natural realm.

Damsma asserts that Hegel provided some tantalizing insights into the nature and essence of mathematics. (Damsma 2011). Damsma states that Hegel's correct recognition of the subject of mathematics as an external reflection on many distinguishable but divisible elements has received praise (Baer 1932: 104; Fleischhacker 1982: 194); as well as Hegel's views on the nature of the mathematical infinite (Baer 1932: 112; Ellsworth de Slade 1994: 213; Lacroix 2000: 311-315).

Hegel's determination of the quantitative bears topological characteristics. Mathematical categories, like Discrete and Continuous Magnitude, Number, Spatial Dimensions, the Point and the Line, are developed and ordered, posited by Hegel in topological mode, along other categories within Hegel's philosophical framework.

The two types of infinities distinguished by Hegel, the bad or metaphysical and the true infinite, bears topological characteristics as well. According to Hegel the true infinite category is involved in the mathematical infinite (Lacroix 2000: 303).[123] The bad infinite is the

http://www.researchgate.net/publication/254914726_On_the_dialectical_foundations_of_mathematics [accessed Jul 21, 2015].

[123] Lacroix, Alain (2000). The Mathematical Infinite in Hegel. *The Philosophical Forum, 3-4, XXXI,* 298-327. DOI: 10.1111/0031-806X.00043

unreachable infinity of an endless progression and is best represented by a straight line (Inwood 1992: 141)[124], (Ellsworth de Slade 1994: 212-213)[125].

The reason to look for a topological in Hegel and mathematics – topological (in) Hegel (Hegel's Logic), lies first as the reason to look for a philosophy of mathematics in Hegel, and that is the rigor and precision of mathematics and definitions for mathematics. From this rigor and precision of mathematics springs the general topology with her various disciplines, springs the rigor and precision not only of topology as mathematics but language, syntax, semiotics, visual rhetorics.

Once a category or subject is rigorously defined, it is set apart from all possibilities that are not captured by the definition. When worked with, these rigorous definitions therefore eventually call up their own negation. That is, while the mathematical implications of these definitions become clearer and clearer, so do their shortcomings.61 In other words, to truly understand some definition and its implications also implies an understanding of its limits. Thus, the rigorous definitions of mathematics call up their own negations (Damsma 2011)[126] (Paterson 1997a: 14; Tóth 1972: 36-38[127]).

Hegel's Quantity is a Quantity beyond all Quantity and the development of this process is implicitly topological, in that Quantity is forever beyond the finite: whatever operations you perform using finite quantities; the result will always be finite again (Hegel 1812, 1813, 1816:

[124] Inwood, Michael (1992). *A Hegel Dictionary*. Oxford: Blackwell. John (unknown date). The Gospel according to Saint John. In Moses, Peter, Luke *et. al. The Holy Bible: American Standard Version.*

[125] Ellsworth de Slade, H. (1994). Das wahrhafte Unendliche und die Unendlichkeit der Mathematik; Eine Studie zu den Entsprechungen zwischen Hegels Bestimmung des wahrhaften Unendlichen in der "Wissenschaft der Logik" und seiner Auffassung der Infinitisimal-Mathematik. Heidelberg: Philosophisch-Historischen Fakultät der Ruprecht-Karls-Universität Heidelberg.

[126] D. Damsma (2011). *On the dialectical foundations of mathematics.* (Preprints). Amsterdam: University of Amsterdam. Available from: http://www.researchgate.net/publication/254914726_On_the_dialectical_foundations_of_mathematics [accessed Jul 21, 2015].

[127] Tóth, Imre. (1972). *Die nicht-euklidische Geometrie in der Phänomenologie des Geistes; Wissenschaftstheoretische Betrachtungen zur Entwicklungsgeschichte der Mathematik.* Frankfurt am Main: Horst Heiderhoff Verlag. -- In D. Damsma (2011). *On the dialectical foundations of mathematics.* (Preprints). Amsterdam: University of Amsterdam.; As Damsma asserts, Tóth illustrates this point in relation to the development of non-Euclidian geometry. Interestingly, many authors, like Aristotle in the third century B.C. and Saccheri and Lambert in the 18th century A.D., already knew that a non-Euclidian geometry was possible in principle, but except for Aristotle they all dismissed this type of geometry as untrue (Tóth 1972: 20-23). Thus, the Euclidian system clearly calls up its own negation, even though this negation was only accepted as a true possibility in the 19th century A. D.. Within both axiomatic geometrical systems, the other system can be shown to be false, so the two are truly oppositional. Yet this opposition lead to a more comprehensive dialectical understanding of the nature of geometry (Tóth 1972: 36-40).

282, 2.2Bc; Lacroix 2000: 314).[128] The bad infinite is only a potential infinity that cannot be reached by finite mathematicians. It is beyond our grasp by its very definition. For Hegel all things in the world are finite, but this fact itself is infinite. 'Finite entities develop, change, pass away and give rise to other entities' (Inwood 1992: 295)[129] ad infinitum. This passage itself is the basis for Hegel's conception of the true infinity (Hegel 1812, 1813, 1816: 163, 1.2Cc; Lacroix 2000: 315).[130]

Hegel's Logic and philosophy of mathematics can be used to make conceptual sense of the development of topology and the nature of topology in general. Same as with the development of the non-Euclidian geometry for which Hegel doesn't provide any account in his writing, is not to say that he gave any explicit account on the first seeds of topology development during his time. [131] (Paterson 2004/2005: 46).

The mathematician deals with 'finite objectivities that thought posits in its infinite self-development' (Lacroix 2000: 315). To Hegel the most important example of an application of the true infinite in the realm of Quantity is the differential calculus. Thus, while at this limit disappear as specified Quantities, their relation reappears as a qualitatively different ratio. It is this relation that is the true locus of the true (quantitative) infinite, because through it the finite Quantum $x$ is ceaselessly led beyond itself into the bad potential infinite. (Lacroix 2000: 311-315).

[128] Lacroix, Alain (2000). The Mathematical Infinite in Hegel. *The Philosophical Forum, 3-4, XXXI,* 298-327. DOI: 10.1111/0031-806X.00043

[129] Inwood, Michael (1992). *A Hegel Dictionary*. Oxford: Blackwell. John (unknown date). The Gospel according to Saint John. In Moses, Peter, Luke *et. al. The Holy Bible: American Standard Version*

[130] Lacroix, Alain (2000). The Mathematical Infinite in Hegel. *The Philosophical Forum, 3-4, XXXI,* 298-327. DOI: 10.1111/0031-806X.00043

[131] Paterson states that "Hegelian philosophy can be used to make conceptual sense of the development of non-Euclidian geometry and the nature of geometry in general." He adds that "..is not to say that Hegel gave any account of non-Euclidian geometry in his writings. Rather, he 'fully accepted the essential validity of the Euclidian approach', albeit that he criticized some of Euclid's proofs, especially when they involve superposition. His criticism was based on the fact that two distinct congruent triangles are conceptually the same." (Paterson 2004/2005: 46). Hegel's criticism of the Euclidian approach was based on the fact that two distinct congruent triangles are conceptually the same According to Hegel therefore, a pure mathematical triangle can only be congruent *with itself*. Hence, congruence must be proven from one triangle instead of from superposition of one triangle over another (Paterson 2004/2005: 37-39).

In 1994 Ellsworth de Slade wrote a study on the counterparts of Hegel's true infinity in his conception of infinitesimal mathematics.[132]

In 1932 Baer published an article on Hegel and Mathematics in general.[133] In their texts both hail the result of the last paragraph as one of the most important insights Hegel has to offer in the field of mathematics (Baer 1932: 112; Ellsworth de Slade 1994: 213).

Hegel's views on the infinite and infinitesimal mathematics are not intramathematical, but conceptual. However, as Wolff clearly shows in his 1986 text entitled *Hegel und Cauchy*, he was well versed in the research that mathematicians such as Lagrange and Cauchy have done on the subject.[134] In this text Wolff traces how Cauchy influenced Hegel regarding the mathematical infinite and infinitesimal mathematics and discusses similarities and differences between the two (1986: 197-263).[135]

Finally, in the first three papers in a series on Hegel's *Wissenschaft der Logik,* Carlson (2000, 2002, 2003a)[136] gives a complete account of Hegel's determination of the quantitative in pictographic terms. His treatment in these papers is very similar to the treatment of Damsma and mine. That is, all of Hegel's dialectical transformations and all of the important concepts

[132] Ellsworth de Slade, H. (1994). Das wahrhafte Unendliche und die Unendlichkeit der Mathematik; Eine Studie zu den Entsprechungen zwischen Hegels Bestimmung des wahrhaften Unendlichen in der "Wissenschaft der Logik" und seiner Auffassung der Infinitisimal-Mathematik. Heidelberg: Philosophisch-Historischen Fakultät der Ruprecht-Karls-Universität Heidelberg.

[133] Baer, Reinhold (1932). Hegel und die Mathematik. In Veröffentlichungen des Internationalen Hegelbundes, *Verhandlungen des Zweiten Hegelkongresses vom 18. bis 21. Oktober 1931*. Berlin & Tübingen: s.n., 104-120.

[134] -- Wolff, Michael (1979). Über das Verhältnis zwischen logischem und dialektischem Widerspruch. In: Hegel Gesellschaft, *Hegel Jahrbuch.* München: Hegel Gesellschaft, 340-348.---— (1986). Hegel und Cauchy; Eine Untersuchung zur Philosophie und Geschichte der Mathematik. In Horstmann, Rolf-Peter & Petry, Michael J. (Eds.), *Hegels Philosophie der Natur: Beziehungen zwischen empirischer und spekulativer Naturerkenntnis.* Stuttgart: Klett-Cotta, 197-263.

[135] Wolff, Michael (1979). Über das Verhältnis zwischen logischem und dialektischem Widerspruch. In: Hegel Gesellschaft, *Hegel Jahrbuch.* München: Hegel Gesellschaft, 340-348.---— (1986). Hegel und Cauchy; Eine Untersuchung zur Philosophie und Geschichte der Mathematik. In Horstmann, Rolf-Peter & Petry, Michael J. (Eds.), *Hegels Philosophie der Natur: Beziehungen zwischen empirischer und spekulativer Naturerkenntnis.* Stuttgart: Klett-Cotta, (1986: 197-263).

[136] Carlson, David G. (2000). Hegel's Theory of Quality. *Public Law Research Paper, 17*. New York: Cardozo Law School. Consulted version: http://ssrn.com/abstract=241950.
Carlson, David G. (2002). Hegel's Theory of Quantity. *Cardozo Law Review 24, 6*. Consulted version: http://ssrn.com/abstract=326822.188
Carlson, David G. (2003a). Hegel's Theory of Measure. *Public Law Research Paper, 66*. NewYork: Cardozo Law School. Consulted version: http://ssrn.com/abstract=413602. DOI: 10.2139/ssrn.413602
Carlson, David G. (2003b). The Antepenultimacy of the Beginning in Hegel's Science of Logic. *Public Law Research Paper, 74*. New York: Cardozo Law School. Consulted version: http://ssrn.com/abstract=425122.

in the *Wissenschaft* are discussed, explained and when appropriate, amended with modern-day insights.

All we can say at such an abstract level about the Quality of Being is that it consists of a manifold of indeterminate Ones upon which we can only externally and arbitrarily reflect, turning it into Quantity. To get rid of the arbitrariness, **a Qualitative Quantum is required: Measure** (Damsma, 2010 elaborates on this).

The grounds towards the merger between the categories of quality and quantity in the Hegelian Concepts, and the category theory in mathematics and categorical logic, could be found in the series of works of Alan L. T. Paterson, Adjunct Professor of Mathematics, University of Colorado, Boulder, the so called mathematical/Hegelian philosophy papers [137] and the works by Dirk Damsma [138].

Hegel's great work on Logic, the "Science of Logic" (*Wissenschaft der Logik)*, appeared in three volumes in 1812, 1813 and 1816, contains long discussions of mathematics. In particular in "Science of Logic" Hegel develops philosophies of the *natural numbers*, the *negative numbers* and also of the *rational numbers*. In Hegel's logic we will find an extensive and erudite account (with the real numbers presupposed for the sake of the argument) of how this philosophical understanding of mathematics ``makes sense'' of the (informal) infinitesmimal manipulations of the differential and integral calculus.

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[137] G. W. F. Hegel: Geometrical Studies - translated with Introduction and Notes, Bulletin of the Hegel Society of Great Britain 57/58, 2008, 118-153.; The Hegelian Concept and set theory, 15 pages, 2007.; A modern Hegelian Philosophy of Special Relativity, 32 pages, 2006.; Hegel's Early Geometry, Hegel Studien 39/40, 2004/2005, 61-124.; Does Hegel have anything to say to modern mathematical philosophy?, Idealistic Studies 32:2, 2002, 143-158.; The Successor Function and Induction Principle in a Hegelian Philosophy of Number, Idealistic Studies 30 (1) 2000, 25-61.; Frege and Hegel on concepts and number, 22 pages.; Self-reference and the natural numbers as the logic of Dasein, Hegel Studien 32(1997), 93- 121.; Alan Paterson, Towards a Hegelian philosophy of mathematics, Idealistic Studies, 27(1997), 1-10.
On Paterson's website (http://sites.google.com/site/apat1erson/) we will find five papers on the merits of a Hegelian philosophy of mathematics. Three of these (Paterson 1997b; 1999; 2000) are about the philosophy of Number, one is about the concept of the propositional calculus (Paterson 1994) and one is about the Hegelian philosophy of mathematics in general (Paterson 2002). In each of these, Hegelian philosophy is proposed as a solution to the problems 'which arise out of the existence in mathematics of self-referential, nonconstructive concepts (such as *class*)' (Paterson 2002: 143).

[138] Dirk Damsma: Set Theory and Geometry in Hegel (2011), In Hegel Gesellschaft, Hegel-Jahrbuch. Berlin: Akademie Verlag, http://www1.fee.uva.nl/pp/bin/311fulltext.pdf; Qualitative and Quantitative Analysis in Semantic Dialectics: Marx vs. Hegel and Arthur vs. Smith (2010); On the Dialectical Foundations of Mathematics.(2006);

Alan Paterson argues that Hegel has much to say to modern mathematical philosophy, although the Hegelian perspective needs to be substantially developed to incorporate within it the extensive advances in post-Hegelian mathematics and its logic. According to Paterson, the Hegelian approach provides a framework for answering the philosophical problems, discussed by Kurt Gödel in his paper on Bertrand Russell, which arise out of the existence in mathematics of self-referential, non-constructive concepts (such as class).

In his paper "Does Hegel have anything to say to modern mathematical philosophy?", Paterson insists that "it is time to grapple with what Hegel was trying to do in his treatment of mathematics. Of course a lot of the *details* of Hegel's view of mathematics belong to his time and sound *quaint*(!) (but no more so than Newton's discussions of fluxions), and Hegel did not have the benefit of coming after Weierstrass, Dedekind, Cantor and Frege. But Hegel's method of inquiry in terms of a conceptual development is the correct method for investigating the human activity that is called mathematics simply because it is intrinsic and lets the subject develop itself (as a genuine infinite, a circle closed on itself) rather than assuming an external ``objective'' pose in which it is put in front of the investigator and inspected ``from the outside'', only for it to creep in from the back in the very act of investigation."[139]

For the purpose of the question above and in particular for the issue of human infinity as conceptual infinity, I will focus on the discussion given by Alan Paterson in his paper "Towards a Hegelian philosophy of mathematics". According to Alan Paterson, as he establishes in "Towards a Hegelian philosophy of mathematics"[140] - "the well-known difficulties in modern mathematical logic require a Hegelian grounding for their resolution and that, in the terminology of Hegelian logic" (Hegel's "Science of Logic*").* In his paper Paterson describes how "the discussion of the problems with the formal approach to mathematics generates a Hegelian approach to mathematics, one in which the great results of mathematical logic are *aufgehoben*, sublated their integrity preserved, while at the same time,

[139] Alan Paterson, Does Hegel have anything to say to modern mathematical philosophy?, Idealistic Studies 32:2, 2002, 143-158.
[140] Alan Paterson, "Towards a Hegelian philosophy of mathematics", Idealistic Studies, 27(1997)

the rigid separation of mechanical logic and mathematical intuition - that is, the obsessive abstractive operation of *Verstand*, the Understanding – is overcome."[141]

Paterson establishes that "the Hegelian Concept, however, does not have just two moments. It has three, those of Universality, Particularity and Singularity (*Allgemeinheit, Besonderheit, Einzelheit*)." The same statement is valid in thinking of the categories as "quality" and "quantity".

According to Paterson, "in Hegelian logic, the Concept is self-identical in its three moments: roughly speaking, the thinking through of any one of them brings to light the others." Thus the thinking through "quality" brings to light the "quantity" and in reverse, the thinking of "quantity" brings us to "quality".

Paterson concludes that "since the intuitions are the concept of mathematics as Particularity, the Concept (in the Hegelian approach) will produce these "intuitions" underlying formal systems as its own and explicitly show how it is self-identical in them. In the process, the relations between these intuitions will be made explicit. In formal logic, by contrast, the intuitions just lie there unconnected."

Paterson progresses with the discussion on Hegel's infinities, pointing out that "in this self-identity of the Concept in its particularization (and its other moments) we have the full ``genuine infinite'' (*das wahrhafte Unendliche*) of the Concept contrasting with the ``bad infinite'' (*die schlechte Unendlichkeit*), the latter being the finitized form of the former in which is endlessly posited the same repeating process in the impossible task of ``attaining'' the genuine infinite. The correct picture of the genuine infinite is, for Hegel, that of a circle that closes back on itself while that of the bad infinite is that of an infinite line which goes on ``for ever'', or of an infinite sequence (in Hegelian terms, an infinite progress (*Progre*ß)) that never stops."

The notion of "genuine infinity" and the "bad infinity" is discussed in detail by Hegel in the chapter on *Dasein* in the *Wissenschaft der Logik*.

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[141] Alan Paterson, "Towards a Hegelian philosophy of mathematics", Idealistic Studies, 27(1997)

Paterson states that “in the externalizing of formal systems, we are ultimately trapped in the bad infinite almost by definition - the genuine infinite cannot be expressed in terms of sequences of meaningless wff's linking onto one another, the allowable linkings being checked mechanically.
The conceptual ``in-itself'' of the system, driving the system, which is a genuine infinite must eventually make explicit the bad infinite of the formal system, the latter's incongruity with its concept.”

Later on Paterson links the “bad infinity” with the the methods of mathematical logic itself – “The explicit exhibiting of this bad infinite - coming out of the methods of mathematical logic itself - is to be found in the fourth inadequacy discussed in § 5 (from Paterson’s paper), that concerning Gödel's second incompleteness theorem.”

Paterson explains how “a formal system tries to investigate itself in its own terms. This is nothing other than the reflexivity of the genuine infinite of Concept”.

The failure of the system to prove its own consistency is nothing but the illustration of the thesis that “the bad infinite of the formal system is not true (= not genuine in the Hegelian sense) - it cannot reach back on itself to establish that it does not contain a contradiction.

The bad infinite in the sense of Hegel is evident in the never stoping attempts and sequence of formal systems to prove its own consistency. According to Paterson, Hegelian logic provides an explanation for the five inadequacies of formal systems, which inadequacies Paterson himself discussed in § 5 of his paper. With the help of Hegelian logic, Paterson establishes that “these inadequacies are not to be treated as difficulties to be lamented but rather understood *positively*, as asserting the reflexivity and infinitude of mathematics. As such, the formalism of mathematical logic is *restored*, not adopting a pose external to mathematics and putting it to rights, as it were, but as part of the activity of mathematics itself which it presupposes and from whose meaningfulness it is derived.”

For Paterson, the “true mathematical logic is the self-determining of the concept of mathematics, in which one starts at the simplest, most abstract stage of intuitive mathematics and teases out what it implicitly contains in a logical development ( *Entwicklung*). In Hegelian logic, the starting point is, of course, abstract, indeterminate *Being*. A major difference from the formal approach to logic is that such a development is *conceptual* in character, the bringing out explicitly of the riches implicit in mathematical concepts and their interconnection.”

The topological notion of Hegel’s logic contains the seed of topological hermeneutics with “genuine infinite, a circle closed on itself” …The infinite that wants to be unlimited, because as Hegel points out – “there are two worlds, one infinite and one finite, and in their relationship the infinite is only the limit of the finite and is thus only a determinate infinite, an infinite which is itself finite." 48

What the understanding should not forget is that these alternating determinations of the finite and the infinite are ultimately determinations of one unified something. Topological cobordis is just another dialectical and hermeneutical view on the betweennes of these two somethings – the finite and the infinite.

## 2. Topological (in) Hegel - Previous Literature on the existed literature:

A few scholar have felt that Hegel’s logic and his views of space, time and matter anticipate some intuitive topological notions.

The thesis how Hegel’s method in the Science of Logic, or what is “methodological” refer to the “**topological”** is expressed by Angelica Nuzzo. (Nuzzo 2011). Nuzzo observes in her analysis on Hegel’s Science of Logic, that “in the Science of Logic, the problem of ‘method’ does not appear thematically until the last chapter. . . what ‘method’ is in Hegel’s speculative-dialectical logic deserves special attention.”[142] Nuzzo states that – “Following the suggestion of method’s ‘circle’, I propose to read the movement of the first sphere of logic starting from the end of the work (SL) or from the end of the work, or form the from the perspective of the

[142] Angelica Nuzzo, The Problem: Perspective on Method, Or, How to Approach Being, (111-139), p. 112, in Stephan Holgate, Michel Baur, ed., 2011, A Companion to Hegel, Blackwell Companion, Willey-Blackwell.Angelica Nuzzo (Thinking Being: Method in Hegel’s Logic of Being).

end having finaly been achieved – that is, with the consciousness that has been gained once the entire logical development has come to the conclusion. I will call such consciousness or knowledge methodological."[143] In the footnote of the above text, Nuzzo made the following remark: "In the first, general determination, "methodological" refer to the **topological** standpoint assumed by the interpretation. Accordingly, "methodological" is the view that is placed of the end of the work and that from this refers back to the beginning: 'immanent' is the perspective that follows the development step by step with no "whereto" in sight, the former is a circular, and the later is a linear reading." The grounds for this conclusion over Hegel's Wissenschaft Der Logic, Nuzzo finds in ". . . the general strategy of Hegel's dialectical logic, which always begins with the qualitative "principle" of a categorial determination in principle, and then constructs an external representation of the same (being-for-another) in order to identify the immanent principle of being with its formal representation. Second, the consistent application of this triadic scheme clearly shows that Hegel's approach to the construction of number is much closer to the Dedekind-Cantor approach than Pinkard would have us believe."[144]

Vagueness is philosophically important and can be traced in philosophy of Kant[145] and Hegel[146].

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[143] Angelica Nuzzo, The Problem: Perspective on Method, Or, How to Approach Being, (111-139), p. 112, in Stephan Holgate, Michel Baur, ed., 2011, A Companion to Hegel, Blackwell Companion, Willey- Blackwell.: P.112

[144] Angelica Nuzzo, The Problem: Perspective on Method, Or, How to Approach Being, (111-139), p. 112, in Stephan Holgate, Michel Baur, ed., 2011, A Companion to Hegel, Blackwell Companion, Willey- Blackwell.: P.112

[145] Boniolo Giovanni and Valentini Silvio, 2008, Vagueness, Kant and Topology: A Study of Formal Epistemology, *Journal of Philosophical Logic*, Vol. 37, No. 2 (April 2008), pp. 141-168, Springer. – Boniolo and Valentini's approach a vagueness as characterized by two features. The first one is philosophical following a Kantian path emphasizing the knowing subject's conceptual apparatus. The second one is formal, faceing vagueness, and a philosophical view on it, proposing the use of topology and formal topology. The authors show that the Kantian and the topological features joined together allow an atypical, but promising, way of considering vagueness.

[146] See: Angelica Nuzzo's Vagueness and meaning variance in Hegel's logic, in Nuzzo, A. ed., 2010, Hegel and the Analytic Tradition, London: Continuum, 2010, pp.208

Tyson Gofton in his *Analysis, Systematicity and the Transcendental in Hermann Cohen's System of Critical Idealism*. (Gofton 2013:205), asserts that there are strong and explicit suggestions about the presence of ‘topological’ in Hegel’s Logic. (Gofton 2013:205).

Tyson asserts “…..the process of determining numbers (i.e., instances of “the one”), for Hegel, begins not with the continuity of a quantitative continuum, but rather with a merely qualitative continuum: the general relational determinability of something in general, the principle of determinability as the a priori system of determinable relations.(14) (Tyson, Analysis, 2013:208) Under the reference note (14) following the text quoted above, Tyson states: that “Hegel is almost certainly thinking of quantity in terms of an implicit, phenomenological, continuous topology”. (Tyson, Analysis, 2013:209). For Tyson “the question is not whether Hegel can construct ordinal numbers, but whether he is in a position to do so at the point when the concept of quantity is first made explicit.” (Tyson, Analysis, 2013:207)

Robert Groome, the author of the original study “Formalization of Hegel’s Phenomenology of the Spirit”, (Groome 2012), proposed and developed the thesis of ‘topological’ Hegel, establishing that Hegel’s The Phenomenology of the Spirit [Phänomenologie des Geistes] first appeared in 1807 under the title System of Science. According to Groome, “Here, *Knowledge* [Wissen/Savoir] or *Science [Wissenshaft] has been called by Hegel* a *System or Manifold (Mannigfaltigkeit),* which has both a *topological* and *philosophical sense*.” (Groome 2012). Countering “the standard procedures that view the notion of a Manifold philosophically, if not metaphorically” (Groome 2012), Groome’s aim is to read and construct its place topologically, both globally and locally, in a Theory. In proceeding in such a literal way, Groome’s aim is both to introduce a Hegelian theory that is not another philosophical commentary on Hegel's philosophy, but first and foremost an isolation of its structure. Groome’s conclusion is that the Phenomenology of the Spirit and Hegel's conception of phantasy are constructed in a topological structure.

Certain topological features are presented by Ilmari Jauhiainen, in *Constructions and situations: a constructivist reading of Hegel's System.*[147] (Ilmari Jauhiainen 2011)

Far from being explicit in the use of term 'topology', Jauhiainen asserts that "Hegel's Logic is meant to be more than mere analysis of situations: it should describe also relations between different possible situations. Furthermore, it is an ontological investigation of the most general structures of possible situations and objects." (Ilmari Jauhiainen 2011:116)

Jauhiainen asks the question as How such an investigation is possible …and answers that "Hegel's suggestion is that we could perhaps use the general structures or categories themselves as one possible field of application of the categories: thus, we could justify the meaningfulness of the categories without any reliance on something other than the categories themselves." (Ilmari Jauhiainen 2011:116)

Jauhiainen approached Hegel's Logic as a constructivist study. The medium of Logic is seen as a particular language and its signs, and in order to rid even the final link to senses and intuition, the references of this language are to be taken merely from the words of this language and their characteristics: although linguistic signs are undoubtedly intuitable objects, they are such that we can ourselves manufacture. The aim of the Logic is to show how one can construct examples of more complex structures from given examples of simpler structures. The first structure from which the Logic begins is an empty situation, because it can be abstracted from any structure. (Ilmari Jauhiainen 2011:116)

Hegel's categories of qualities and quantities, and notions of space and time are the most important in constructing the manifold of situations in Hegel's logic and philosophy.

Ilmari Jauhiainen, describes "a situation" , according to his use of the term, as "something where there might be objects. The word "where" suggests something spatial, and indeed, spatial positions are perhaps the easiest example of situations: there is a cow in the barn, thus,

[147] Ilmari Jauhiainen, in *Constructions and situations: a constructivist reading of Hegel's System,* Faculty of Arts at the University of Helsinki, 2011

the barn is in this sense a situation, London is in Britain, and so Britain is also a situation. Jauhiainen extends the realm of situation "from spatial positions to temporal moments: Charlemagne lived in Middle Ages, and thus Middle Ages is one sort of situation." (Jauhiainen, 2011:13) He investigates "the leap from spatial to temporal situations", and uses the term "situation" to cover particular states of objects as "in autumn leafs are in a state of redness, thus, redness is a type of situation." From fleeting states of objects Jauhiainen, extends the term to cover "more essential qualities of objects", where for instance, "fragility is not a state of glass, because glass is always fragile.., still, if we classify things to fragile and nonfragile things, we might say that glass falls in the position of fragility – thus, we might (Ilmari Jauhiainen 2011:13) say that fragility is as well a sort of situation." (Ilmari Jauhiainen 2011:14).

For Jauhiainen, "spatial and temporal situations and qualities are relatively simple situations, but I assume that more complex structures could also be called situations: for example, the Earth is in the Solar System and so the Solar System should be a situation, although it is a highly complex system of bodies and their qualities and relations. (Ilmari Jauhiainen 2011:14)

The concept of situation as proposed by Jauhiainen in relation with Hegel's logic and philosophy is related not only with the concrete physical objects. It extends "from concrete objects to cover everything that could be spoken of, that is, everything which could be taken as a subject term of some statement or from which something is predicated. Thus, we could speak of a pack of wolves that is in a state of having twelve members: here the pack would be an object, while the state of "twelveness" would be a situation. In addition to sets of physical objects, we could also speak of abstractions like number two and hence they could be taken as objects in the extensive sense I have used the word: thus, because number two is in the system of whole numbers, this system could be taken as a situation." For Jauhiainen, "situations themselves can be spoken and could thus be taken as objects: hence, qualities like "being a spatial situation" could also be taken as situations." (Ilmari Jauhiainen 2011:14)

For Jauhiainen, the concept of situation is applicable also to the empty space, where there are no object presented, and this type of situation is quite typical for Hegel's Logic of Being, "an empty space between two objects or beyond any object would still be a situation. The possibility of empty situations is crucial for Hegel's philosophy". (Ilmari Jauhiainen 2011:14)

## Concept of Situation and Hegel's concept of Sein:

Jauhiainen asserts that his concept of situation is particularly related to Hegel's concept of *Sein, as "the* part of the Logic dedicated to the investigation of *Sein*, particularly, is filled with study of different types of situations and especially classifications of different situations: there are, for instance, qualitative classifications, like different colours, and quantitative classifications, like whole numbers. Indeed, the important point for Hegel appears to be that *all situations could be thus classified or represented as a sort of "space" of situations."*

Jauhiainen sees situations in Hegel's Logic as "possible worlds, because *these situations are truly like small worlds for the objects."...and "*The comparison with the notion of possible worlds might suggest that situations would be closed from one another like Leibnizian monads."

There are strong characteristics of some topological notions presented in Jauhiainen's concept of situation as we could see that "Situations are not meant to be closed realities, but more like snapshots of reality: partial views of what there is and thus connectible to one another. The cow is in the barn, but beyond the barn there is the field and other possible places where the cow could also be; the leaf is in state of redness in the autumn, but in summer it has been in state of greenness. (Ilmari Jauhiainen 2011:14)

## Chapter 3 Topological notion of Qualitative quantity – Plato – Aristotle – Hegel

Hans-George Gadamer states, in his essay *The Idea of Hegel's Logic*, that "*in Greek philosophy Hegel saw the philosophy of* ***logos***." For this realm Hegel coins a new expression, typical of him – the logic and "*the logical*."[148] Hegel's concept of spirit transcends the

[148] Hegel states that Fichte was the first to grasp the universal systematic implications of Kant's way of viewing things from the perspective of transcendental philosophy yet Fichte's own "Doctrine of Science" did not really finish this task of developing the entirety of human knowledge out of self-consciousness. Hans-George Gadamer interprets Hegel's critique on Fichte, with the claim that Fichte thought is subjective idealism and Hegel was the first to join this subjective idealism with the objective idealism of Schelling's philosophy of nature through the grand synthesis of absolute idealism. - "Die Idee der Hegelschen Logik (1971) in Hans-George Gadamer, "Hegel's Dialectic: Five Hermeneutical Studies", translated into English by P. Christopher Smith and collected in (New Haven: Yale University Press, 1976). These five essays are "Hegel and Heidegger,"; "Hegel's Dialectic of Self-consciousness"; "Hegel and the Dialectic of the Ancient Philosophers,"; "Hegel's 'Inverted World,'; and "The Idea of Hegel's Logic," 75-99

subjective forms of self-consciousness[149] and taking the tradition of Palato and Aristotle (*logosnous*). In this fashion, Hegel achieves his objective of re-establishing the Greek *logos* on the new foundation of modern, self-knowing spirit. (Gadamer 1971).

In his exploration of method (the method of dialectic) of Hegel's Logic, Gadamer establishes that "*a glance back at Greek philosophy is necessary, too, if one is to understand Hegel's conception of the method through which he converted the traditional logic into a genuine philosophical science*." [150] (Gadamer 1971).

Gadamer traces the seed of the Hegel's method back to the Plato in his dialogues with Socrates' discussions and logic, just to remind that for Hegel, Plato's dialectic does not reach any scientific insight.

There are two approaches to the concept of Quality of Numbers presented in ancient Greek philosophy - Platonic and Aristotelian.[151] (Tyson, 2013).

As Alfredo Ferrarin asserts "*Even if at first Hegel placed Plato higher than Aristotle but later reversed this order, he always coupled the two as 'teachers of mankind' and would have extended to Plato as well Dante's famous characterization of Aristotle as the "master of those who know*."[152] (Ferrarin, 2007:138)

## 1. Plato's dimensional mathematical model: point – line – surface – figure

[149] "Die Idee der Hegelschen Logik (1971) in Hans-George Gadamer, "Hegel's Dialectic: Five Hermeneutical Studies", translated into English by P. Christopher Smith and collected in (New Haven: Yale University Press, 1976). These five essays are "Hegel and Heidegger,"; "Hegel's Dialectic of Self-consciousness"; "Hegel and the Dialectic of the Ancient Philosophers,"; "Hegel's 'Inverted World,'; and "The Idea of Hegel's Logic," 75-99

[150] Hans-George Gadamer, The Idea of Hegel's Logic, in "Hegel's Dialectic: Five Hermeneutical Studies", translated into English by P. Christopher Smith and collected in (New Haven: Yale University Press, 1976). These five essays are "Hegel and Heidegger,"; "Hegel's Dialectic of Self-consciousness"; "Hegel and the Dialectic of the Ancient Philosophers,"; "Hegel's 'Inverted World,'; and "The Idea of Hegel's Logic," 75-99

[151] For these two moments described as 'Platonic' and 'Aristotelian' See Gofton B Tyson, Analysis, 2013, Systematicity and the Transcendenal in Herman Cohen, p.166.
www.tysongofton.com/s/Gofton_B_Tyson_2013_PhD_thesis.pdf:
Tyson discusses the two moments of the Hegelian dialectic related with the problem of the doctrine of the infinite and the infinitesimal, and states that "There are two ways, Platonic and Aristotelian, of regarding numbers lead to different number systems. What is a finite number for Platon is not necessarily a finite number for Aristotle." : p.166.

[152] Ferrarin, A., 2007, Hegel and Aristotle, Cambridge University Press, 2007

Back in the late eighties, in my first publication on Qualitative quantity and topology[153] (Dimitrov, 1989) my approach to Hegel's Qualitative quantity, from the diachronical tradition of ancient Greek philosophy, was based on Plato's dimensional model of the Being. Presently, especially from the standpoint of my current review of Hegel's rhetoric, metaphors, language and the role of topos in Hegel's syntax, I still support my previous suggestion that Hegel's concept of qualitative quantity is related with Plato's concept of quality of numbers, in particular Plato's dimensional mathematical model: point – line – surface – figure[154] (Dimitrov, 1989) and in the concept of Indefinite Dyad, known as "aoristas duas", which is a mathematical explanation of "forms" in Plato.

Aristotle makes it eminently clear that within the Academy, Plato professed Two Principles, principles that were involved in the construction of the Forms (Universals or Archetypal Numbers), as well as the Sensible (Particulars) of our Empirical World.

The First Principle is generally acknowledged. It is the Good of Plato's Republic, also referred to in the Academy in its more mathematical context as the One.

The other Principle was usually referred to as the Indefinite Dyad, and at times as the Greater and the Lesser, Excess and Deficiency, or the More and the Less.

Occasionally one would see these two principles contrasted in terms of the One as Equality and the Indefinite Dyad as embodying Inequality. The base of seeing the roots of Hegel's qualitative quantity in the Plato's quality of numbers is The Unwritten Doctrines of Plato. One of the most original works in the history of philosophy written in the 20th century is the *Arete bei Platon und Aristoteles* ("Arete in Plato and Aristotle", Heidelberg 1959) written by Hans Joachim Krämer.[155] (Krämer, 1959).

The title of Krämer's first book reflects an earlier stage of the dissertation project - starting from an analysis of the arete concepts in both thinkers. Krämer recognized the Platonic origin

[153] Dimitrov, B. (1989), "Philosophic Thought Magazine", March, 1989, the journal edition of Institute of Philosophical Sciences, Bulgarian Academy of Science.
[154] Dimitrov, B."Quality of the Quantity", "Philosophic Thought Magazine", March, 1989, journal edition of Institute of Philosophical Sciences, Bulgarian Academy of Science.
[155] Hans Joachim Krämer, "Arete in Plato and Aristotle" /Arete bei Platon und Aristoteles/, Heidelberg 1959

of the Aristotelean doctrine of arete as mesotes and the ontological foundations of this doctrine that Aristotle ascribes to Plato, but that are not found explicitly in the dialogues.

I agree with the suggestions and statements of the so called German Tubingen School that the debate between a systematic and a non-systematic Plato was alive at least since Hegel's time.

In his book “Plato and the Foundations of Metaphysics, A Work on the Theory of the Principles and Unwritten Doctrines of Plato with a Collection of the Fundamental Documents” (1990), Krämer unfolds the philosophical significance of the unwritten doctrines in their fullness. Krämer demonstrates the hermeneutic fruitfulness of the unwritten doctrines when applied to the dialogues, and how in these doctrines the revival of the presocratic theory is brought to a new plane through Socrates. In this way, Plato emerges as the creator of classical metaphysics. In the third part of the book, Kramer compares the structure of Platonism, as construed by the Tubingen School, with current philosophical structures such as analytic philosophy, Hegel, phenomenology, and Heidegger. Within the five appendices of the book, Kramer presents the most important English translations of the ancient testimonies on the unwritten doctrines. These include the "self-testimonies of Plato." There is also a complete bibliography on the problem of the unwritten doctrines.

One of the most ancient implementation of Qualitative quantity as quality of the number can be found in Plato’s dimensional mathematical model: point – line – surface – figure.[156] (Dimitrov 1989). The whole presentation of the idea of quality of number in Plato is embeded in his teaching about the "eidical number". The quality of the quantity emerges as criteria for recognizing the difference between the eidical numbers and natural arithmetical number.

Illustrating the ancient Greek idea of the cosmos, Plato’s concept represents hierarchical structure of the being as mathematical dimensional model of the cosmos. In this mathematical dimensional model Plato imply his ontological system but not just in plane and pure quantitative correlation.

[156] Dimitrov, B. “Quality of the Quantity”, “Philosophic Thought Magazine”, March, 1989, journal edition of Institute of Philosophical Sciences, Bulgarian Academy of Science.

The number in Plato is not only trivial mathematical construct but it turned into the internal contextual determiner of the being. [157] (Boyadhiev, 1984). The qualitative expression of the geometrical figure is the most suitable expression of the Plato's concept of "idea" known as "eidos". The geometrical figure points out of it's self to the symbolized in her structure of the being as logos. The plastic perception of even the most abstract and complex mathematical categories and correlation is one of the characteristic and type variety of ancient Greek mathematics. Qualitatively determined the representation of the number as figure is stimulated by the absence of the developed numerical signs. This definition by quality and characterization of one number as triangle or square, or with some other figure of the geometrical order is typical for the ancient Greek mathematical intend. The strong link and interaction between the quantitative value and it qualitative expression is typical for the ancient Greek mathematics. (Boyadhiev, 1984).

The language the number is not just simple abstraction of the quantitative characteristics of the things but always some qualitatively determined figure plastically expressed in texture and structure that stays in its own qualitatively defined and clearly determined boundaries. This interaction and mutual penetration between the quantitative relations and qualitative expressions is the cause for the too flexible and hard to find differentiation and resolution between the ancient geometry and arithmetic.

The mathematical constructs in Plato's philosophy, possesses the character of eidos samplers thus there are presented not only as quantitative abstractions but also as determined qualities.

The archaic presentation of the numbers as figures and letters noticed yet by Nicomacus is not sign for lack of development and naivete. The representation of the quantity by quality is not only typical for the ancient Greek mathematical tough but also for the dialectical nature of Plato's philosophical method, which relates both the ontological and gnoseological structure of the being. Dialectically interpreted and Implemented in ontological level the too abstract itself Plato's dimensional model of the cosmos obtains the character of universal explanatory principal. Plato's dimensional mathematical model develops character of nest of philosophical categories and dialectical expressions.

[157] Boyadhiev, T. (1984) "The Unwritten Doctrine of Plato", Sofia, Bulgaria, 1984.

This clearly mathematical model of the being consisting in point - line – surface – figure obtains the qualitative character when enters into the dialectic-ontological structure which includes as it components as follows: the primordial principles of eidos-cosmos - the ideas-numbers - the sensible soul, mathematical things-the sensible cosmos. There is importance of the conclusion that "mathematics becomes the science when link to its own logical problematic is established. The Platonic philosophy clear from mathematics never ever existed" [158] (Boyadhiev, 1984).

Plato's concept of quality of numbers as the the concept of Indefinite Dyad, known as "aoristas duas", which is a mathematical explanation of "forms" is implemented in the sudy of Aleksei Fedorovich Losev – "The Classic Kosmos" /Ancient Cosmos and Modern Science, 1927. [159] (Losev 1927).

Long before the late-period Foucault discovered the relevance of Platonist hermeneutics, and Badiou recognized the necessity of the Platonic gesture, a radical turn towards the Platonic dialectic occurred in the work of Alexei Losev. (Kosykhin 2013)

In *Philosophy of the Name*, written in 1923 and published in 1927 (roughly the same time as Heidegger's *Being and Time*), Losev provides the first sketch of his dialectical system. As Losev writes, "I understand the *dialectic* as the logical elaboration (i.e., the elaboration in logos) of being considered in its eidos" (Losev 1990: 167). (Kosykhin 2013)

For Losev, "the dialectic is the theory of the element of thought, which embraces all manner of eide in unified, integral being" (Losev 1990: 168).

Vitaly Kosykhin asserts that "the theory of the interaction of part and whole is this dialectic's constitutive aspect. Hence Losev's theory of ***topology*** as the ***quality*** of things forming a whole, i.e., the "theory of the eidetic morph, or the perfect space," and of *arithmology* as the theory "of the eidetic schema, or the perfect number" (Losev 1990: 346). (Kosykhin 2013)

[158] Boyadhiev, T. (1984) "The Unwritten Doctrine of Plato", Sofia, Bulgaria, 1984.
[159] Losev, A. (1927) Ancient Cosmos and Modern Science, 1927

Vitaly Kosykhin points out that "both ***topology*** and ***arithmology*** are theories of meaning, which is considered, to quote Losev, in terms of "self-identical difference" in the first case, and "mobile rest," in the second. We see in these expressions a dialectical interaction between the four major ontological categories of Platonism—namely, identity, difference, motion, and rest—which are the supreme forms of ideas in Plato's ontology and immediately follow the idea of being, for whose essential description they are, in fact, meant. In Losev's work, this description is eidetic. (Kosykhin 2013)

The eidos of a thing is its intellectually grasped concept, which supposes a given thing to be precisely this thing, different from other things (Losev 1990: 168); moreover, as Vitaly Kosykhin establishes "this supposition is topological and arithmological. (Kosykhin 2013) And, what is quite significant, the "eidos of a thing is precisely what never changes, as if the thing itself actually did not change, and thus the logos of the thing, as the schema of semantically apprehending the eidos, is also something immutable" (Losev 1990: 173).

Kosykhin assertrs that "the categories of whole and part are accorded a much more modest status in Hegel's dialectical system. The *Science of Logic* devotes only a few pages (in the second section, entitled "Appearance," of "The Doctrine of Essence") to the relations between the whole and its parts. Actually, whole and part are not even categories for Hegel, insofar as "essence," "appearance," and "reality," respectively, as the most common and fundamental concepts, function as categories in his philosophy." (Kosykhin 2013). According to Hegel, whole and parts presuppose each other; moreover, the whole figures as self-subsisting, whereas the parts, which subsist *immediately*, form the appearance of the whole. (Kosykhin 2013).

Kosykhin states that "Hegel is primarily interested in the *relation* between whole and parts" (Kosykhin 2013). In my wiew Kosykhin's conclusion is not correct and indeed his statement confirms the presence of 'topological' in Hegel as topology is concerned with the relationship between the parts within the whole.

Hegel *identifies* the whole and its parts in true topological mode:

In the relation of whole and parts, the two sides are these self-subsistences but in such a way that each has the other reflectively shining in it and, at the same time, only is as the identity of both. Now because the essential relation is at first only the first, immediate relation, the negative unity and the positive self-subsistence are bound together by the 'also'; the two sides are indeed both posited as *moments*, but *equally* so as concretely existing *self-subsistences*. (Hegel 2010b: 451)

The study of Aleksei Fedorovich Losev implemented in his book "The Clasic kosmos" could be described as inquiry into the spatial Formenbildung of Classical kosmos. According to Losev, who always sees the Classic kosmos from inside, it is important to use the unique tool of dialectics which for the Greeks has been the ultimate instrument to make the theory of kosmos. For Losev there are three categories indispensable for the reconstruction of Classic kosmos and the analogies between Classic and modern science. These three are: The Name, The Number, and The Thing. Dialectically, the Classical kosmos is Thing, constructed of Numbers, and appearing as its Name. These three are the main concepts of Losev study "Clasic Kosmos". [160] (Losev 1927).

According to Losev, the first dialectical system is the system of Plato and from there no more progress is made neither by Plotinus, nor Fichte and Hegel. For Losev, dialectic of Plato is the genuine dialectics in its purest form uncompromised by naturalistic logic of Aristotle. Losev claimed that only Hegel understood the dialectic of Plato, congenial to himself, and had drawn main principles: 1) antithetic of one and other; 2) construction of common; and 3) common in itself. When Hegel says that "das Allgemeine die Widerspruече in sich aufloest und aufgeloest hat, mithin als das in sich Concrete; so dass diese Aufhebung des Widerspruchs das Affirmative ist" there is not Hegelianism, but pure Platonism. [161] (Losev 1927).

This Losev' claim is shared by one of the most interesting contemporary philosophers working on Hegel, the Brazilian philosopher Carlos Cirne Lima. Carlos Cirne Lima supports the thesis that all science, including philosophy, has an ascending and descending path. In Plato's ancient greek terminology these movements contained in the trajectory of science are

[160] Losev, A. (1927), Ancient Cosmos and Modern Science, 1927
[161] Losev, A. (1927), Ancient Cosmos and Modern Science, 1927

called “anábasis” and “katábasis”. Carlos Cirne Lima is the author of one of the most intriguing comentary on Hegel’s Science of Logic – “Beyond Hegel – A Critical Reconstruction of the Neoplatonic System”, 2006.[162] (Lima 2006). Opening up his study with the claim “And I cannot accept that the overwhelming majority of today’s Analytical Philosophers are unable to read and interpret a single page of Hegel’s Science of Logic”, he establish his intention with the book “to build a bridge, insofar as possible, between the way of thinking of the Analytical Philosophers and Hegel’s system”.[163] This bridge is possible on the base of Carlos Cirne Lima’s claim that “Hegel’s Science of Logic is the last great Neoplatonic system”.[164] (Lima 2006).

Going back to Losev, we could see how the distance from Plato to Hegel is bridged. Difference between Platonism and Aristotelism lies here. Aristotle replaced Plato’s dialectics by apodeictics of formal logic. For Losev dialectics is probably the most accurate product of human thought. Dialectics is some kind of logos, mythologically invariant logical construction. It is independent of any contents of this construction, and amuses itself only with logical, cold, abstract, adiaphorous inquiry into things. Dialectics is logical construction of eidos. Here lies the difference between it and formal logic which is logos about logos. Whereas eidos is the noetic visage of thing in its absolute entirety, its agalma /Plotin/, given purely in thought,- logos is a set of rules for logical construction.

In his study “The Clasic Kosmos” Losev claimed that the main principle of formal logic, the law of contradiction, is not valid in the realm of dialectics, replaced by the principle of coincidentia oppositorum. Following the principle of coincidentia oppositorum Hegel constructed the relationships between qualitative and quantitative within the concept of qualitative quantity and measures. Dialectics of qualitative and quantitative is like the whole

[162] Carlos Cirne Lima, “Beyond Hegel – A Critical Reconstruction of the Neoplatonic System”, 2006 /http://www.cirnelima.org/Beyond-book.doc/ - See also: Carlos R. V. Cirne-Lima, Antonio C. K. Soares, “Being, Nothing, Becoming - Hegel and Us – A Formalization”, Translatated into English by Luís M. Sander, Niura Fontana and Beatriz Fontana.

[163] Carlos Cirne Lima, “Beyond Hegel – A Critical Reconstruction of the Neoplatonic System”, 2006 /http://www.cirnelima.org/Beyond-book.doc/ - See also: Carlos R. V. Cirne-Lima, Antonio C. K. Soares, “Being, Nothing, Becoming - Hegel and Us – A Formalization”, Translatated into English by Luís M. Sander, Niura Fontana and Beatriz Fontana.

[164] Carlos Cirne Lima, “Beyond Hegel – A Critical Reconstruction of the Neoplatonic System”, 2006 /http://www.cirnelima.org/Beyond-book.doc/ - See also: Carlos R. V. Cirne-Lima, Antonio C. K. Soares, “Being, Nothing, Becoming - Hegel and Us – A Formalization”, Translatated into English by Luís M. Sander, Niura Fontana and Beatriz Fontana.

dialectics a construction of special kind of eidos - categorical eidos. Dialectics manipulates not the whole thing but only its eidetic backbone and does not fill this logical and categorical skeleton with any flesh of contents. Mythology dives deeply into the primordial abyss of category, whereas dialectics depends upon the purity of these categories because any mythological premises obscure the crystal clear categorical system of eidos. Also, here dialectics differs from arithmology, which analyses eidos as some kind of numerical scheme, where only fact of presence or absence of categories makes a sense.

For Losev, Dialectics is a construction of categorical eidos as self-sufficient and self-based being. Eidos is the overall sense of thing, i.e., the whole of thing, i.e. eidos is self-sufficient. In contrast, logical constructs are simply the sets of instructions, and the only way to discern underlying eidos is simply to watch it, and this sight will be the only criterion of its reality. Dialectics of categorical eidos cannot t be based on any fact and would be dependent only upon some other eidos. This relationship is not factual, naturalistic, or metaphysical,- only noetic. Therefore dialectics is the specific categorical eidetics of these relationships because the salient feature of eidoi is their interdependence. Any eidos is dependent upon other and upon all of them. Phenomenology, in contrast, is insight into statically given eidos as sense with its own form. It could only construct some whole from parts, whereas dialectics is able to derive new wholeness, representing now category, from already given (phenomenologically) wholes.[165] (Losev, 1927).

Dialectics is a logical construction of self-based categorical eidos, and this construction is absolutely universal, it involves all actual and potential kinds of being, and all non eidetic, irrational, illogical,- all these also are in the specific eidetic relation with pure eidos! Here dialectics differs from any kind of rationalism which hypostasizes any logos it creates and pretends there are things. (Losev, 1927).

Dialectics gives eidetically bound system of categories - from primordial element to eidos as name,- because it is name when we first time really encounter any thing. Before it there is no noetic communication with any thing. Therefore, when we start dialectically analyzing some categorical eidos, we inadvertedly should proceed until we reach the stage of name. Principal backbone of entire scope of dialectics is the dialectics of name.

[165] Losev, A. (1927), Ancient Cosmos and Modern Science, 1927

According to Losev Dialectics is a soul of Classical philosophy, and the constructions of this philosophy have remained and will remain in any dialectics. Dialectics is the most exact knowledge, it is the very life given in concepts and therefore rather distant from the life itself which is not conceptual or exact. But according its object dialectics is the life, and there are no constructs closer to life. Therefore Greek dialectics has originated in the depths of myth /Pythagoreism/, evolved as the perception of myth (Plato and Plotinus), and concluded as the construction of ultimate dialectical mythology /Proclus/.

In Pythagoreism we clearly see the mythology as a womb of dialectics: Pythagoric Hieros logos namely because of its mythological operationalism requires for its categorialization through diaeresis into number (Noys) and into what later will become an apeiron. The doctrine of Number (Noys) in Plotin and Aristotle is doctrine of the quality of number. For Aristotle, Numbers thinks. Number thinks because of its nature. If Number would think about other Number would belittle itself and lessen its beauty because it is best of all. Therefore Numbers thinks about itself. Numbers and object of thinking are the same in the ideal sphere, but not in material. to ti hen einai incorporates this identity, whereas fact of world incorporates difference. Thinking of quality of numbers and the interplay of the categories quality and quantity we should recall this Losev statement that Dialectics is the logos of categorial eidos, and we should pay attention only to these categorical structures and be able to derive them one from another.

From the topo-logical model of the Being in Plato (point – line – surface – figure), through the topological notion of the logos of categorical eidos in Losev, I would recall the topological synergetics of Buckminster Fuller, who turned the cycle back to Plato, unfolding the issue of knowledge as system, as topological system.

The topology of the knowledge as geometry of thinking is presented in the synergetics of Buckminster Fuller, where the knower and the known are the observer and observed.

Plato's dimensional model of the being translated in mathematics as unity of four elements of point – line – surface – figure is actually known more as the planar triad associated with the "Good"/"Arete".

Rethinking on the Plato's triad Buckminster Fuller, in his "Synergetics – Explorations in the Geometry of thinking",[166] suggests that "this triadic concept is exclusively planar – ergo nonexistent. (542.01). Fuller is opening up his discussion with the quote from Plato's "Philebus":

". . . then, if we are not able to hunt the Good with one idea only, with three we may catch our pray: Beauty, Symmetry, Truth are the three . . . "

- Plato, Philebus

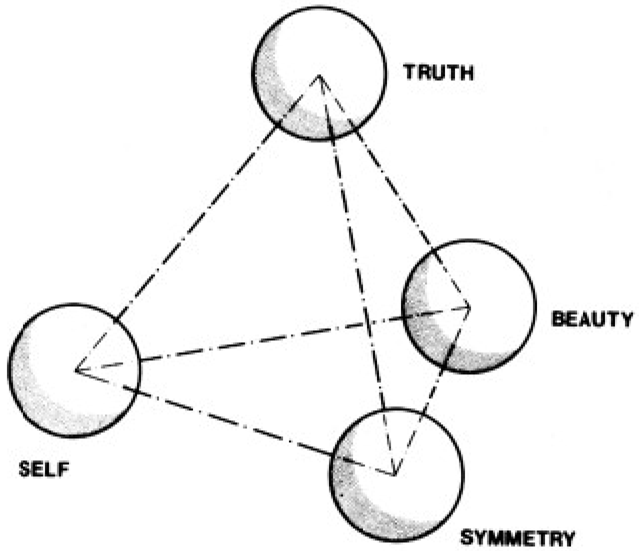


Fig. 542.02 from Buckminster Fuller's "Synergetics – Explorations in the Geometry of thinking" /Macmillan Publishing Co. Inc. 1975, 1979/ [167]

According to Fuller, "the observer position marks the fourth corner of the tetrahedron" which is "the minimum system" / 542.01/.

Fuller asserts:

[166] Fuller, Buckminster (1975/1979). "Synergetics – Explorations in the Geometry of thinking", Macmillan Publishing Co. Inc. 1975, 1979

[167] Fuller, Buckminster (1975/1979). "Synergetics – Explorations in the Geometry of thinking", Macmillan Publishing Co. Inc. 1975, 1979

"The observer – plus – the observed, Beauty, Symmetry, and the Truth are the four unique system-defining characteristics. It is possible that Plato might have approved a systematic reordering of his statement to read. The observer (as a truth) observing three other truths, constitutes system". [168]

Buckminster Fuller, in his "Synergetics – Explorations in the Geometry of thinking" states:

"It is possible that Plato might have approved a systematic reordering of his statement to read: The observer (as a truth) observing three other truths constitutes a system whose macro-micro-Universe-differentiating capability displays inherent symmetry and beauty–symmetry of four vertexes subtending four faces and symmetry of any two opposite pairs of its six edges precessionally subtending one another, together with the beauty of accomplishing such symmetry and Universe- differentiating with the minimum of structural system interrelationships.

The qualitative interrelationships of this beautiful and symmetrical system are expressed in the generalized formula

$$N = \frac{N^2 - N}{2}$$

in which N = 4, the number of vertexes of the minimum system constituting the tetrahedron; wherefore

42 = 16, 16 - 4 = 12, 12/2 = 6.

Six is the set of the uniquely symmetrical interrelationships of the minimum system."[169]

For Buckminster Fuller, "Beauty and symmetry are inherent and make superficially "good" the three additional interrelationships: thankfulness, maximum economy, and wisdom. They also make "good" all the remaining cases on balance–the 32 cases /Sec. 1044, "Synergetics –

[168] Ibid.

[169] Fuller, Buckminster (1975/1979). "Synergetics – Explorations in the Geometry of thinking", Macmillan Publishing Co. Inc. 1975, 1979

Explorations in the Geometry of thinking"/ of all the simplest cosmically conceptual and structurally realizable systems of Universe."[170]

Fuller set up the question: "Is there a *qualitative* systems attribute? Can special case system events and characteristics be appraised *qualitatively?* My answer is that you and I are always off- center. All special case realizations of generalized principles are always aberrated realizations. Since there is no simultaneity, there is always realization lag. Consciousness is aberration. But the lags are always accompanied by experientially evolved inventions of more efficient operational means of decreasing the magnitude of tolerated errors–i.e., off-center aberrations." /542.05/ And answer: "Quality describes the relative proximity–concentrically attained–of absolute congruence with convergently directioned, dimensionless truth."/542.07/

The missing fourth element of the Plato' triadic concept is regarded by Buckminster Fuller as the unity of the observer plus observed, thus the triad is to be transformed to the tetrahedron. The Plato's "good" should be read as arête in the original of Plato' Philebus.

Buckminster Fuller's tetrahedron should be linked with the concept introduced by Hans Joachim Krämer. Krämer claims, in accord with Vitruvious, that there are three characteristics of the Being, where Being is defined by Arete. These three are – symetry, harmony and structural stability. Symetry corresponds to the Vitruvian Utilitas (appropriate spatial accommodation), harmony corresponds to the Vitruvian Venustas (attractive appearance) and the third is the structural stability or Firmitas.

Hans Joachim Krämer, "Arete in Plato and Aristotle"[171] , is one of the most original works in the history of philosophy written in the 20th century. The title of Krämer's first book reflects an earlier stage of the dissertation project - starting from an analysis of the *arete* concepts in Plato and Aristotle. Krämer recognized the Platonic origin of the Aristotelean doctrine of *arete* as *mesotes* and the ontological foundations of this doctrine that Aristotle ascribes to Plato, but that are not found explicitly in the dialogues.

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170 Fuller, Buckminster (1975/1979). "Synergetics – Explorations in the Geometry of thinking", Macmillan Publishing Co. Inc. 1975, 1979

171 Krämer, Hans Joachim (1959). "Arete in Plato and Aristotle" /Arete bei Platon und Aristoteles/, Heidelberg 1959

According to Plato and Krämer Arête is the basic characteristic of the Cosmos or Being, [172] [173] presented within beauty, symmetry, and truth. According to Fuller the observer and observed are presented as one additional element to the other three elements beauty, symmetry, and truth. The active element of the unity of observer and observed is the dynamic point of the Being – Cosmos -Awareness.

The topology of truth is the topology of the self – the observer /as a truth/ observing three other truths /the observed Beauty, Symmetry, Truth/ and this system is constitutes. The topology of truth is presented in rhetorical practise of the sophists. Rhetorical Dialexeis of Kairos, Arete and Dissoi Logoi.

There are two notions of Time and two notion of Spece in the Ancient Greek concepts of time and space. The two temporal notion of Time are Chronos and Kairos. Chronos is the quantitative Time and Kairos is the qualitative Time. And there are two spatial notion of Space – Chora and Topos. Chora is the quantitative abstract notion of space and Topos is the qualitative notion of space. The abstract space is Chora and the concrete place is Topos. Aristotle defined Chronos quantitatively as the "number of motion with respect to the before and the after", which is a classical expression of the concept of (chronos) time as change, measure, and serial order. The definition of Chronos is focused on an exact quantification of time. Following Aristotele analysis there are temporal and spatial pairs of *chronos*/*kairos* and *chora*/*topos*, relationships. Kairos is the time that gives value thus quality. Kairos is qualitatively defined. [174]

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[172] Krämer, Hans Joachim (1959). "Arete in Plato and Aristotle" /Arete bei Platon und Aristoteles/, Heidelberg 1959

[173] For Arete – See also:
- Carmen Cozma, Introduction to Aretelogy. A Short Treatise of Ethics, „Al.I.Cuza" University Publishing House, Jassy, 2001; second edition, 2004;
- Carmen Cozma, Àreté, a Nucleus-Value of the Contemporary Ethics;
- Carmen Cozma, The Aretelogical Challenge;
- Carmen Cozma, Around the Aretelogical Challenge of the "ontopoiesis of Life";
- Carmen Cozma, Some Considerations Concerning the Question of Measure in the Phenomenology of Life;
- Carmen Cozma, In Quest of the Measure''s Restoration;
- Carmen Cozma, Mapping the Offer of the Phenomenology in Arts;
- Carmen Cozma, The Ethical Values of the Music Art of the Ancient Greeks: A Semiotic Essay

[174] Rämö, Hans: An Aristotelian Human Time-Space Manifold. From chronochora to kairotopos. In Time & Society VOL 8(2) 309-328 Sage 1999.

In **"**Bernhard Riemann's Conceptual Mathematics and the Idea of Space", [175] Arkady Plotinsky assetred that "One might argue that the ancient Greeks had ***philosophical*** **topology**, as is suggested by Plato's concept of *khora* in *Timaeus*, which may even be seen as already questioning the very concept of spatiality. But they did not have a mathematical discipline of topology; their only mathematical (exact and quantifiable) science of space was geometry. Anticipated by Leibniz's conception of "analysis situs" (the term used by Riemann and for a while after him), topological ideas were gradually developed by Riemann and others, especially Henri Poincaré, whose work was uniquely responsible for establishing topology as a mathematical discipline." [176]

Examining the "The Spaces of the Baroque (with Leibniz, Riemann, and Deleuze)",[177] Plotinsky links the space /topos/ in the Baroque with Plato's khora /Timaeus/.

In this link given by Plotinsky for the topology of Baroque and topos and chora, we could see the notion of Hegel's qualitative quantity – **the fourfold of infinities in Hegel: 1. the bad qualitative infinity; 2. the good qualitative infinity; 3. the bad quantitative infinity; 4. the good quantitative infinity.** This Hegel's forufold of infinities is related with the fourfold interplay of the two pair of Ancient Greek categories of time and space.

## 2. The Aristotelian Heritage in The Science of Logic

Approaching Hegel's Logic in our research for 'topological' both as philosophical topology in the sense of mathematics and as topoi in the rhetorical sense, the starting point shall be Aristotle.

According to Nicolai Hartmann, "Hegel perceived himself as the Aristotelian who . . . recognized and completed the work of the master."[178] Only in the remark to §204 of the Encydopiedia does Hegel mention Aristotle. But that he has nothing else in mind than the

[175] Plotinsky, Arkady (2009) Bernhard Riemann's Conceptual Mathematics and the Idea of Space, Configuration, Volume 17, Numbers 1-2, Winter 2009

[176] Ibid

[177] Plotinsky, A. The Spaces of the Baroque (with Leibniz, Riemann, and Deleuze)

[178] Ferrarin, A., 2007, Hegel and Aristotle, Cambridge University Press, 2007

Aristotelian natural beings is shown by the oral addition to the introductory section of the Philosophy of Nature. Hegel states that an adequate conception of spirit needs the revitalization of Aiistotle's De Anima. In the preface to the second edition of the Encycloptedia Hegel writes that understanding "Plato, and much more deeph' Aristotle [. . .] is at once not merely an understandingof that Idea, but an advance of science itself."(…)

Alfredo Ferrarin who examines 'The Aristotelian Heritage in The Science of Logic', in his study of 'Hegel and Aristotle' (Ferrarin, 2007) states that "For Hegel, the meaning of Aristotle's philosophy, as well as Greek philosophy from Anaxagoras's intellect to the Platonic and Aristotelian, is that the Idea, is "objective thought," or what he calls the soul of the world, or the Logical. Thinking in the Logic is free from the substrates of representation; that is, the Logic is not a thinking about something {WL 1: 44), a stable substrate whose existence is given and which forms the basis for our thinking, because here the Phenomenology of Spirit is presupposed, that is, the liberation from the oppositions of consciousness. The logic contains thought insofar as this is first as much the thing (Sache) in its own self or the thing in its own self insofar as it is equally pure though ( WL 1 : 43; SL)[179]

Ferrarin asserts that "what Hegel thus reads in Aristotle is that the form and actuality are the truth of finite things: the conformity of a substance to its concept, to its energeia, is the decisive truth of its being. Concept and actuality are prior to potency and matter, as we have seen; and for Hegel's interpretation of Aristotle this is the onlv possible criterion for the truth of things. This is Hegel's understanding of the meaning of Aristotle's first philosophy as the science of being qua thought: the concept is the true. (Ferrarin, 2007:135)

This identification of thing-form-truth does not only hold for the finite. The absolute Idea is nothing other than the noesis noeseos, so that the logic is, in Hegel's well-known metaphor, "the exposition of God as he is in his eternal essence before the creation of nature and of finite spirit" {WL 1: 44, .S'L 50, transl. modified).

[179] Ferrarin, A., 2007, Hegel and Aristotle, Cambridge University Press, 2007

Hegel’s view and positions are very similar to that of Aristotle. For Hegel philosophy does what the sciences cannot do, because these assume the existence of their object and an external method.

Aristotelian qualities are qualities of substances, and as such these are not treated in the Logic of Being but in the Logic of Essence, as properties of, in necessary relation to, a substrate. (Ferrarin, 2007:135)

For Kant, quantity is the first group of categories because it is a pure synthesis of the manifold of space and time, while quality is the real in sensation, that is, the degree to which an appearance affects our senses. Quality is therefore more complex and also more empirical than quantity, which is in turn more universal and fundamental for appearances (everything is a quantity, but not everything is a quality - for example, geometrical figures constructed in intuition are not. (Ferrarin, 2007:135)

Hegel has a different principle for the deduction or order of categories than Kant. Hegel proceeds not from what is first for us but from what is first in the thing or Sache. Thus quality is the immediate determination of something, while quantity is indifferent to the thing, being only the quantity of a quality.

When quantity is posited as limited it is a quantum, and the quantum is expressed in number. Hegel's notion of number is peculiarly Greek in its definition: it is the union of amount or annumeration and unit {WL 1: 232, SL 203; ENZ.C §102), and is the resolution of the contradiction between continuity and discreteness. As in this tradition, number is defined as the how-many-times the unit is repeated; but the unity of units, the thought of the many as one, is a break of continuity or the discontinuity of determinate pluralities, and is thus a limit of the many. (Ferrarin, 2007:135)

In Aristotle in particular, arithmnos was never understandable in separation from what it numbered; number is relative to a definite collection of items which it measures. Aristotle's word for quantity, means "the quantitative". It is a predicate — that is, of substances in intelligible matter. Further, number is negation and delimitation of the contininim.

Hegel, whose knowledge of mathematics is rather impressive, shares with Aristotle not only this concept of number, but also the resistance to treating mathematics as a separate formalism. For Hegel mathematics is fundamentally a theory of ratios; as such, it is not an independent construction with its own requirements and language.

Wlien quantity' and quality determine each other, as in the category of measure, Hegel mentions the "Greek awareness that everything has a measure" {WL 1: 394, SL 329) and often uses examples from Aristotle.

In the Politics (VII 4) Aristotle writes that the best city has to be able to be overseen in a single glance to guarantee the economic autarchy and the mutual acquaintance of citizens; a change in the dimensions of the state disturbs the balance of this ratio and consequently of the constitution. This statement implies certain topological view or topological map.

Hegel’s discussion on the Paradoxes such as that of the heap or the bold man {when does the repeated removal of a grain from a heap stop being simply quantitative and equal to the disappearance of the heap?), which show how a quantitative change results in a qualitative one, are taken from Aristotle's Sophistic Refutations (which Hegel quotes at WL 1: 397, SL 335).

Hegel picks up the definition of essence expressed by the Aristotelian, which we can understand as "what being was before its existence" (where the imperfect is not a past tense but is exempt from time).”The intelligible determinations of substance resolve the thing in its intelligibilitv; yet essence exists only in relation to the composite, as its explanation. In Hegel, the Concept at the level of essence is the negative relation to the immediacy of being. In reflection the essence is posited in relation to its unity, and designates the sublated being as an intemporal having-been {WL 2: 15, .SL 391). (Ferrarin, 2007:137)

## 3. The presence of ‘topological’ in Aristotle

Following the above discussion about the Aristotelian heritage in The Science of Logic, in my approach to Hegel’s topology, I would like to focus on presence of ‘topological’ in Aristotle. The presence of ‘topological’ in Aristotle is emphasized by Michael White in his article ‘On continuity: Aristotle versus topology?' (1988) [180] and Michael Eldred in his 'Digital dissolution of Being" (2010).[181]

Aristotelian conception of continuity *(synecheia)* and the contemporary topological account share the same intuitive, proto-topological basis: the conception of a 'natural whole' or unity without joints or seams.[182] (Michael White 1988) Michael White discusses Aristotle's argument that 'continuous' cannot be constituted of 'indivisibles' (e.g., points), asserting that this argument can be examined from a topological perspective. White concludes that "from that perspective, the argument fails because Aristotle does not recognize a *collective* as well as a *distributive* concept of a multiplicity of points." (White, M., 1988).

This *collective* as well as a *distributive* concept of a multiplicity of points in Aristotle, allows contemporary topology to identify some point sets with spatial regions (in the proto-topological sense of this term). This identification, in turn, allows contemporary topology to do what Aristotle was unwilling to do: to conceive the property of continuity, as well as the properties of having measure greater than zero and having *n*- dimension, as *emergent* properties. Thus, a point set can be continuous (connected) although none of its subsets of sufficiently smaller cardinality can be. White discusses 'the manner in which a topological principle, *viz.,* the principle that none of the singletons of points of a continuum can be open sets of that continuum, captures certain aspects of the Aristotelian proto-topological conception of the relation between points and continua. e.g., for both Aristotle and contemporary topology, points in a continuum exist simple as limits of the remainder of the continuum: their singletons have empty 'interiors' and, hence, they are not 'chunks' (topologically, regular closed set) of the continuum. (White, M., 1988).

In Aristotle's thinking, number is abstracted from physical beings. [183] This abstracting consists for Aristotle in a being becoming **place-less**. For

[180] White, Michael J. (1988/2007), On continuity: Aristotle versus topology?, History and Philosophy of Logic,Volume 9, issue 1, 1988

[181] Eldred, Michael (2010), Digital Dissolution of Being, Published in Left Curve no. 34 (2010)

[182] White, Michael J. (1988/2007), On continuity: Aristotle versus topology?, History and Philosophy of Logic,Volume 9, issue 1, 1988

[183] Eldred, Michael (2010), Digital Dissolution of Being, Published in Left Curve no. 34 (2010)

Aristotle the ‘placeless-ness’ consists in separation of being from its surroundings. In such way the number distilled from the physical being become a number in the abstraction. Why this abstraction (distillation) is important? The numbers mediate between the continuity and discreteness of the being. **Numbers** originally arise by an iterative procedure such as counting thus they are separated from each other and relates to the **discreteness**. This is the reason for Aristotle to separate the numbers from the **physical beings** characterized by **continuity.** For Aristotle, the very same abstraction is applicable to the geometrical figure of physical being. The points (sti/gmai) of the geometrical figure (within the Ancient Greek geometry **(based on points, lines, planes and solids),** are also ‘placeless’ yet this placeless-ness has a topos (place), position, thus the geometrical figure like physical being itself is **continuous**.

**Continuity** consists, as Michael Eldred asserts, “in the way the points of a figure or the parts of the underlying physical being, which all have a position and are thus posited, hold and hang together. The points hang together by touching each other at their extremities (e)/sxata).’ The points even share their extremities. The points are all identical yet differentiated through their differing positions.” (Eldred, M., 2010)

The abstraction of and placeless-ness of numbers and points in geometrical figure is different placeless-ness and abstraction. The difference of the numbers is ‘visible’ and easily one can distinguish the number 3 from 5 or 9. This is the ground for Aristotle’s claim that numbers have to be plural, i.e. at least 2, in order to be numbers. For Aristotle, the the principle of unity (monad) is the starting point of arithmetic. A number answer the question ‘how many’? The distinction of numbers is only sensible when one proceeds from the *counting process*. Proceeding from monad, one comes to two as the first successor in the counting process, and this may be taken as the first counting number. But the monad itself must already distinguish itself from something else, from nothing, a nil number, i.e. there must be a difference between 1 and 0 which corresponds to the difference between a unified something and nothing, emptiness. Only from the principle of unity (monad) can arithmetic, i.e. numbers in the Greek sense, be built up one by one through the iterative counting process. In a further development, and because the base for counting, in principle, is arbitrary, today, all numbers can be represented, manipulated and calculated on a binary basis. The Greeks thought number from the counting process and therefore had no zero, which prevented the assimilation of geometry to arithmetic. To do so would have required the insight into the correspondence between the geometric point and the number 0.1 (Eldred, M., 2010)

Although the numbers bear the difference within themselves, two points on a line are identical and hardly one can distinguished one point from the other without the difference of position. Through a difference in position only we distinguish the difference of the points. The placeless-ness of the numbers is 'placeless-ness without position', the placeless-ness of points on the line are 'placeless-ness with position'. Here we can find the root of the difference between arithmetic (calculus) and geometry. (Eldred, M., 2010)

The difference and similarity between the arithmetic and geometry has the core of the interplay between quality and quantity, between discreteness and continuity, in particular the quality of number, the existence of the smallest number in arithmetic and the lack of the smallest magnitude in geometry. For Aristotle there is a smallest number, and, proceeding from the geometric line, also that there is no smallest magnitude, but does not resolve the disparity.

The abstraction of numbers out of physical beings opens up the possibility of calculating with numbers. The numbers are open to logismo/j, but at the price (or the advantage) of becoming placeless and positionless.

Such a lack of place and position, Michael Eldred asserts, it seems, characterizes also the **digital beings** (subject of Eldred's work and discussion) which we deal with today. For them (digital beings), matter in its continuity and its fixedness of place becomes indifferent. Same proposition is relevant for the numbers, numbers are **without place and also without position but are differentiated within themselves.** (Eldred, M., 2010)

Topological notions, in the sense that topology is about the relationship between figures, could be found in the discussed above specifics of the points of a figure or the parts of the underlying physical being, which all have a position and are thus posited, hold and hang together. Here, Michael Eldred remark that ‘the points hang together by touching each other at their extremities emphasize on this topological specifics on ‘position’, ‘relation’ and ‘between-ness’, the coined above term ‘placeless-ness with position’. This ontological topology or topological ontology is emphasized by Michael Eldred with his assertion about the ontological discreteness and continuity, about the arithmetic and geometry, the discrete nature of our sensuous perception and the continuity of being, that “What is ontologically most complex in the way it hangs together, i.e. the continuous geometric figures and physical beings, is most simple for sensuous perception, but is very unwieldy for calculation. And conversely: what is ontologically more simple, i.e. the arithmetic entities in their ordered, countable succession, is not easily accessible to sensuous perception but can be calculated (logismo/j) without any difficulty.” (Eldred, M., 2010)

The abstraction of numbers from physical being, the discrete nature of the arithmetic entities and their interrelations can be more easily brought to presence by the lo/goj or the logic than geometric entities which, in turn, are closer to sensuous experience, i.e. not so abstract, and here resides the calculative power of mathematical analysis which reduces the geometric to the arithmetic, the continuous to the discrete, irrational (real) number to rational number, the quality to quantity, by conceiving real numbers as (Dedekind) cuts or partitions in the (infinite, but countable) sequence of rational numbers. This reduction facilitates calculation in the mathematical language of algebra, and, conversely, the results of the calculation can be translated once again back into the sensuously aisthaetic intuitions of geometry which have a representation in the imagination. “With the arithmetization of geometry - Michael Eldred asserts - the mathematico-logical manipulation of beings thus attains a hitherto unprecedented power.” (Eldred, M., 2010)

Michael Eldred discusses the possibility of ‘bridging the gulf between the discrete and the continuous, stating that:

"One of the main issue in history of mathematics, since Ancient Greek mathematical thought, is gulf between the discrete and continuous, between the arithmetic (based on counting starting with the monad and unit) and geometry (based on points, lines, planes and solids), or the question how was it possible to gain a mathematical hold on real, physical beings? From logical side, from the side of logoi, there is no difficulty in representing any statement in numbers, and, in particular, in numbers to the base 2, i.e. binary code, since both number and logoi are discrete, but how was it possible to gain a mathematical hold on real, physical beings? For this, the geometric (based on points, lines, planes and solids) and the arithmetic (based on counting starting with the unit) had to be brought together." (Eldred, M., 2010)

This search for common point or topos in between-ness is truly topological in metaphorical, rhetorical and logical way of thoughts. . . this search was search for the quality of numbers and found in some numbers that are missing from the countable integers and fractions, those numbers 'in between' the fractions that could not be brought into the form of a fraction, i.e. a ratio of two whole numbers.[184] These numbers were called irrational numbers or surds. The simplest irrational number arises already in considering the diagonal of the unitsquare, whose length is the square root of two. Theseirrational numbers are the magnitudes arising from geometric figures which, in turn, are obtained by abstracting the contour outlines of continuous, physical entities. (Eldred, M., 2010)

Geometric figures clearly (i.e. for the visual imagination) hold themselves together; they are continuous. How are all the points on the fundamental geometric figures of a line or a plane to be captured numerically if number is conceived as fundamentally countable? (Michael Eldred)

[184] As Michael Eldred asserts "from the geometric side, however, the Greeks were aware that somehow there were some numbers missing from the countable integers and fractions, namely, those numbers 'in between' the fractions that could not be brought into the form of a fraction, i.e. a ratio of two whole numbers. - Michael Eldred, 2010, Digital Dissolution of Being, Published in Left Curve no. 34 (2010)

This countability, in turn, derives ontologically from the implicit Greek preconception of being as presence-at-hand: a definite number arises from actually counting the things lying present at hand. For Greek thinking, that which lies present at hand is the u(pokei/menon, and such u(pokei/mena in a multitude are countable. In his *Physics*, Aristotle thinks the phenomenon of continuity ontologically progressing from discrete beings which touch, to those lined up in succession, that hang together and, finally, hang tightly together.

The counting unit is indivisible, whereas the unit line is infinitely divisible. Not all the possible magnitudes contained in the unit line can be captured by countable, i.e. rational numbers. The rational numbers have to be complemented by the irrational numbers to attain the entirety of a continuous line with all the possible magnitudes it contains.

Although rational numbers can be made to approximate each other as closely as one likes, between any two rational numbers whatever there is an irrational number, i.e. a magnitude that cannot be expressed as a fraction of two integers.

How are the countable, rational numbers to be completed to get the real numbers?

Real number is an appropriate term because only by means of these real numbers can *all* the magnitudes of sensually perceptible, real, physical bodies be assigned a number. The task is how physical *res* can be captured mathematically by number, and not merely by geometry. Only number opens the possibility of calculation, whereas geometry has to rely on intuitive proofs for which the geometrical objects have to be imagined sensuously in an immediate intuition.

To be continuous, and thus to capture all physical magnitudes of any kind, number has to become real, uncountable. Uncountability implies that, since the rational numbers are countable, between any two rational proportions of integers, no matter how minimal the difference between them, there are always non-rational numbers, i.e. rational numbers can come infinitely close to one another without ever gaining continuity, i.e. there is always a gap between them that is not rational (i.e. irrational), and in this sense they do not hang tightly together like the geometric line. Richard Dedekind's small but crucial step in the second half of the nineteenth century was to fill in the gaps between the rational numbers by conceiving the real numbers as the limits of infinite, but countable sequences of rational numbers.

## 4. Continuity and discreteness, the infinite and the infinitesimal in Aristotle and Hegel

Aristotle was the first who undertook the systematic analysis of continuity and discreteness. He maintained that physical reality is a continuous plenum, and that the structure of a continuum, common to space, time and motion, is not reducible to anything else. Aristotle's answer to the Eleatic problem is a refinement of that of Anaxagoras, namely, that continuous magnitudes are potentially divisible to infinity, in the sense that they may be divided *anywhere*, though they cannot be divided *everywhere* at the same time. (Bell, John L, 2005:20)

In Book VI of the Categories, under the title of Quantity, Aristotle identified continuity and discreteness as attributes applying to the category of Quantity. For him the category of Quantity as associated with the question 'how much' and quantities exhibits continuity and discreteness. Quantities are distinguished by the feauture of being 'equal' or 'unequal'.

As examples of continuous quantities, or *continua*, Aristotle offers lines, planes, solids (i.e., solid bodies), extensions, movement, time and space; among discrete quantities he includes 'number' and 'speech'. Aristotle points out that (spoken) words are analyzable into syllables or phonemes, linguistic "atoms" themselves irreducible to simpler linguistic elements. For Aristotle, as for ancient Greek philosophers generally, the term 'number' – 'aritmos – means just 'plurality'. He also lays down definitions of a number of terms, including 'continuity':

Things are said to be "together" in place when the immediate and proper place of each is identical with that of the other and "apart" (or "severed") when this is not so . They are "in contact" when their extremities are in this sense "together". One thing is "in (immediate) succession" to another if it comes after the point you start from in an order determined by position, of "form", or whatsoever it may be, and if there is nothing of its own kind between it and that to which it is said to be in immediate succession … "Contiguous" means in immediate succession and in contact. Lastly, the "continuous" is a subdivision of the contiguous; for I mean by one thing being continuous with another that those extremities of the two things in virtue of which they are in contact with each other become one and the same thing and (as the very name indicates) are "held together", which can only be if the two limits do not remain two but become one and the same. From this definition it is evident that continuity is possible in the case of such things as can, in virtue of their natural constitution, become one by coming into contact; and the whole will have the same sort of union as that which holds it together, e.g. by rivet or glue or contact or organic union. 39 (Aristotle (1980), V, 3., in (Bell, John L, 2005:21)

In the above quoted text, Aristotle defines continuity as a *relation* between entities rather than as an *attribute* appertaining to a single entity. Aristotle does not provide an explicit definition of the concept of *continuum.* At the end of this passage he indicates that a single continuous whole can be brought into existence by "gluing together" two things which have been brought into contact, which suggests that the continuity of a whole should derive from the way its *parts* "join up". That this is indeed the case is revealed by turning to the account of the difference between continuous and discrete quantities offered in the *Categories:*

Discrete are number and language; continuous are lines, surfaces, bodies, and also, besides these, time and space. For the parts of a number have no common boundary at which they join together. For example, ten consists of two fives, however these do not join together at any common boundary but are separate; nor do the constituent parts three and seven join together ay any common boundary. Nor could you ever in the case of number find a common boundary of its parts, but they are always separate. Hence number is one of the discrete quantities… . A line, on the other hand, is a continuous quantity. For it is possible to find a common boundary at which its parts join together—a point. And for a surface—a line; for the parts of a plane join together at some common boundary. Similarly in the case of a body one would find a common boundary—a line or a surface—at which the parts of the body join together. Time also and space are of this kind. For present time joins on to both past time and future time. Space again is one of the continuous quantities. For the parts of a body occupy some space, and they join together at a common boundary. So the parts of the space occupied by various parts of the body themselves join together at the same boundary as the parts of the body do.

Thus space is also a continuous quantity, since its parts join together at one common boundary.40 (Aristotle (1996a), *Categories,* VI., in (Bell, John L, 2005:21)

Accordingly for Aristotle quantities such as lines and planes, space and time are continuous by virtue of the fact that their constituent parts "join together at some common boundary". By contrast no constituent parts of a discrete quantity can possess a common boundary.

One of the central theses Aristotle is at pains to defend in Physics VI is the irreducibility of a continuum to discreteness—that a continuum cannot be "composed" of indivisibles or atoms, parts which cannot themselves be further divided.

He begins his reasoning as follows:

Now if the terms "continuous", "in contact", and "in immediate succession" are understood as defined above—things being "continuous" if their extremities are one, "in contact" if their extremities are together, and "in succession" if there is nothing of their own kind intermediate between them—nothing that is continuous can be composed of indivisibles: e.g. a line cannot be composed of points, the line being continuous and the point indivisible. For two points cannot have identical extremities, since in an indivisible there can be no extremity as distinct from some other part; and (for the same reason) neither can the extremities be together, for a thing which has no parts can have no extremity, the extremity and the thing of which it is the extremity being distinct. Yet the points would have to be either continuous or contiguous if they were to compose a continuum. And the same reasoning applies in the case of any indivisible. As to the impossibility of their being continuous, the proof just given will suffice; and one thing can be contiguous with another only if whole is in contact with whole or part with part or part with whole. But since indivisibles have no parts, they must be in contact with one another as whole with whole. And if they are in contact with one another as whole with whole, they cannot compose a continuum, for a continuum is divisible into parts which are distinguishable from each other in the sense of being in different places. (Bell, John L, 2005:23)

In this last sentence Aristotle appears to be arguing that a number of indivisibles wholly in contact with one another would constitute another indivisible, and not a continuum, since a continuum is always divisible.

In his discussion on Aristotle, John L Bell emphasize on the *isomorphism thesis*:

Aristotle sometimes recognizes *infinite divisibility*—the property of being divisible into parts which can themselves be further divided, the process never terminating in an indivisible—as a consequence of continuity as he characterizes the notion in Book V. But on occasion he takes the property of infinite divisibility as *defining* continuity. It is this definition of continuity that figures in Aristotle's demonstration of what has come to be known as the *isomorphism thesis*, namely:

The same argument applies to magnitude, time and motion: either they are all composed of indivisible things and divided into indivisible things, or none of them is.

Briefly, the isomorphism thesis asserts that either magnitude, time and motion are all continuous, or they are all discrete. Aristotle's demonstration of this thesis rests on two key postulates concerning motion:

1. When motion is taking place, something is moving from here and *vice-versa.*

2. A moving object cannot simultaneously be in the act of moving towards a given point and in the state of being already at it.

In *Physics* Book IV, 11, Aristotle had defined time as "the number of motion in respect of 'before' and 'after' "—a definition to which his pupil Strato later objected, not unreasonably, on the grounds that the use of the term "number", as a discrete quantity, is inappropriate in connection with time, which is continuous.

The question of whether magnitude is perpetually divisible into smaller units, or divisible only down to some atomic magnitude leads to the *dilemma of divisibility,* a difficulty that Aristotle necessarily had to face in connection with his analysis of the continuum. In the dilemma's first, or *nihilistic* horn, it is argued as above that, were magnitude everywhere divisible, the process of carrying out this division completely would reduce a magnitude to extensionless points, or perhaps even to nothingness. The second, or *atomistic,* horn starts from the assumption that magnitude is not everywhere divisible and leads to the equally unpalatable conclusion (for Aristotle, at least) that indivisible magnitudes must exist.

### 4.1. The concepts of continuity and discreteness in Aristotle

Aristotle's famous claim established in his Physics **- infinitum actu non datur** - declares that there is no actual mathematical infinity. This dogma remained until 1880's when George Cantor began arguing for an actual infinity of actual infinities. Aristotle distinguishes between different types of infinities, some of which may be potential, and others are straightforwardly impossible.

In the *Physics*, Aristotle identifies (among others), two essential types of infinities: infinite numbers and infinite extensions. Furthermore, number and extension may be thought of as infinite in two different ways: as an additive infinity or as a divisive infinity. The reason for Aristotle's claim, that there is no actually infinite number, but there is a potentially infinite number, is that for any (actual) number that can be given, another (potential) number that is greater than it can be found, simply by adding one to the actual number. Aristotle also claims that there is neither an actual infinite extension, nor a potential infinite extension. In other words, all (particular) extensions are finite.

Aristotle is interested in the existence of particulars, the primary ontological presupposition underlying his doctrine of the infinite. The concept of an infinite extension implies the existence of a particular that is greater than all possible particulars (including itself), which is a contradiction. Similarly, there can be no infinitely divisible number, because only whole numbers are proper particulars. Rational numbers, for Aristotle, are ratios between whole numbers; that is, they are relations, not numbers. Second, the modality of which actuality and potentiality are quantifications is time, which, for Aristotle, conditions the existence of the particular.

Aristotle denies the actuality of infinite numbers because the larger infinite number does not exist — yet. And so, while the concept of an infinite number is, for Aristotle, well-defined, it cannot be said to exist actually since the condition of existence is the operation through which a particular is constructed. The same consideration explains why there can be no actual infinitesimal extension, although its concept is well-defined and it clearly exists as part of the actual extension in question.

The concept of infinitesimal was long considered to be problematic and contradictory[185] [186] due to the fact that infinitesimal is treated as finitely-valued, zero valued and depending on the requirements of calculation due to it negligible value. As Gofton Tyson asserts, it was the mathematics of the infinite and the infinitesimal that ultimately showed the inadequacy of the Aristotelian conception of the infinite.[187] Tyson provides an example with the infinitesimal method for calculating, where the tangent of a curve at a point consists of assuming that very small intervals of a curve may be (a) thought of as finitely valued for the purposes of calculating the differential quotient, even though (b) the punctual contact of the curve and its tangent implies that the interval has no extension (i.e., has a zero value).

As a mathematical technique, the calculus quickly achieved results that were wideranging in implications and applications.45 However, the physical interpretation of the calculus — and its mathematical object — was beset with paradoxes.46

### 4.2. The concepts of continuity and discreteness in Hegel

The concepts of continuity and discreteness, albeit in a unorthodox and esoteric form, play an important role in the **philosophy of Hegel**. Hegel saw continuity and discreteness as being

[185] See George Berkely, The Analysis, 1754. Berkely objection to the infinitesimal.

[186] Isaac Barrow (1735), the English mathematician who developed infinitesimal calculus based on the discovery of the fundamental theorem of calculus. Isaac Newton, who went on to develop calculus in a modern form, was student of Barrow.

[187] Gofton B Tyson, Analysis, 2013, Systematicity and the Transcendenal in Herman Cohen, p.29. *www.tysongofton.com/s/Gofton_B_Tyson_2013_PhD_thesis.pdf*

locked in an indissoluble dialectical relationship—a "unity of opposites". (John L Bell 2005:124)

Continuity and discreteness are the "moments", that is, the defining or constituting attributes, of the category of Quantity; the latter is itself a "simple unity of Discreteness and Continuity".

In Hegel's conception continuity is, as it was for Parmenides, first and foremost a form of unity or identity; in the *Science of Logic* (1812–16), § 396, Hegel characterizes continuity as:

simple and self-identical self-relation, interrupted by no limit or exclusion; not however, an immediate unity, but a unity of the Ones which are for themselves. The externality of plurality is still here contained, but as something undifferentiated and uninterrupted. In continuity, plurality is posited as it is in itself; each of the many is what the others are, each is equal to the other, and hence plurality is simple and undifferentiated equality.[188]

Here, continuity is characterized as simple, self−same self−relation, which is not interrupted by any limit or exclusion; it is not, however, an immediate unity, but a unity of ones which possess being−for−self. The asunderness of the plurality is still contained in this unity, but at the same time as not differentiating or interrupting continuity, the plurality is posited as it is in itself; the many are all alike, each is the same as the other and the plurality is, consequently, a simple, undifferentiated sameness. Continuity is this moment of self−sameness of the asunderness, the self−continuation of the different ones into those from which they are distinguished. [189](John L Bell 2005:124)

Continuity still entails plurality, but as "something undifferentiated and uninterrupted." In continuity, Hegel says:

plurality is posited as it is itself; each of the many is what the others are, each is equal to the other, and hence [the] plurality is simple and undifferentiated equality.[190]

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[188] Hegel, G. (1961). *Science of Logic.* Tr. Johnston & Struthers. Allen & Unwin., p.200

[189] Bell, John L (2005) The Continuous and the Infinitesimal in Mathematics and Philosophy. Polimetrica, 2005, p. 124

[190] Hegel, G. (1961). *Science of Logic.* Tr. Johnston & Struthers. Allen & Unwin., p.200

Hegel remarks that imagination “easily changes Continuity into Combination, that is, into an external relation of the Ones to one another”. But on the other hand, “continuity is not external but peculiar to [the One], and founded in its essence”. Atomism, says Hegel, “remains entangled” in this “externality of continuity”. By contrast, mathematics:

rejects a metaphysic which should be content to allow time to consist of points of time, space in general (or as a first step, the line) of points in space, the plane, of lines, and the whole of space, of planes; it allows no validity to such discontinuous Ones. And although it determines, for instance, the magnitude of a plane as consisting of the sum of an infinity of lines, yet this discreteness is taken only as a momentary image; and the infinite plurality of lines implies, since the space which they are meant to constitute is after all limited, that their discreteness has already been transcended. (Remark 1: The Conception of Pure Quantity).”

This antinomy consists solely in the necessity of asserting Discreteness as much as Continuity. The one-sided assertion of Discreteness gives an infinite or absolute division (and thus something indivisible) for principle; and the one-sided assertion of Continuity, infinite divisibility. [191]

Hegel finds Kant’s analysis of the antinomy wanting in that an absolute separation is made, inadmissibly, between continuity and discreteness. He writes:

Looked at from the point of view of mere discreteness, substance, matter, space and time, and so on, are absolutely divided, and the One is their principle. From the point of view of continuity, this One is merely suspended: division remains divisibility, the possibility of dividing remains possibility, without ever actually reaching the atom. Now ... still continuity contains the moment of the atom, since continuity exists simply as the possibility of division; just as accomplished division, or discreteness, cancels all distinctions between the Ones—for each simple One is what every other is, —and for that very reason contains their equality and therefore their continuity. Each of the two opposed sides contains the other in itself, and neither can be thought of without the other; and thus it follows that, taken alone, neither

[191] Hegel, G. (1961). *Science of Logic.* Tr. Johnston & Struthers. Allen & Unwin., p.206

determination has truth, but only their unity. This is the true dialectic consideration of them, and the true result. 317[192]

Hegel next embarks on a discussion of continuous and discrete magnitude. Having observed that Quantity, which embodies both continuity and discreteness, continuous magnitude is identified as Quantity "posited only in one of its determinations, namely, continuity,"[193] so that *continuity is only coherent and homogeneous unity as unity of discrete elements, and posited thus, it is no longer mere moment but complete Quantity: this is Continuous Magnitude.*[194]

Quantity is identified by Hegel as "externality in itself", and Continuous Magnitude is "this externality as propagating itself without negation, as a context which remains at one with itself." By contrast Discrete Magnitude is "this externality as non-continuous or interrupted". If continuity is identity, then discreteness is distinguishability.

Discrete Magnitude is Quantity; and for that very reason [its] discreteness is continuous. The continuity in discreteness consists in the fact that the Ones are equal to one another, or have the same unity. Discrete magnitude, then, is the externality of much One posited as the same, and not of the many Ones in general; it is posited as the Many of one unity.[195]

Hegel distinguishes between *extensive* and *intensive* magnitude. When a magnitude is regarded as a multiplicity, it is extensive; regarded as a unity, it is intensive.

For Leibniz, the Many in One was manifested in continuous extension, but Hegel saw it in discrete magnitude. Dauben suggests that Hegel's conception of *discrete* magnitude may be seen as corresponding to Cantor's famous definition of *set*: *By a "set" we mean any collection M into a whole of definite distinct objects m... of our perception or thought.* [196]*(*Dauben 1979:170)…And Hegel's conception of *continuous* magnitude would further correspond to Cantor's notion of *power* or *cardinal number.* (Dauben 1979:221).

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192 Hegel, G. (1961). *Science of Logic.* Tr. Johnston & Struthers. Allen & Unwin., p.211

193 Hegel, G. (1961). *Science of Logic.* Tr. Johnston & Struthers. Allen & Unwin., p.211

194 Hegel, G. (1961). *Science of Logic.* Tr. Johnston & Struthers. Allen & Unwin., p.212

195 Hegel, G. (1961). *Science of Logic.* Tr. Johnston & Struthers. Allen & Unwin., p.212

John Bell also raletes Hegel's concept of continuous with Cantor's infinity, stating that "It is delightfully dialectical that Hegel seems to have identified as continuous what Cantor held to be discrete. (John L Bell, 2005:127)

The *Science of Logic* contains an extensive discussion of the ideas underlying the calculus. Like Berkeley, d'Alembert and Lagrange, Hegel was critical of the use mathematicians had made of infinitesimals and differentials. But far from rejecting the infinitesimal, Hegel was concerned to assign it a proper location within his philosophical scheme, whose reigning principle was the division of reality into the triad of Being, Nothing, and Becoming. For Cavalieri **infinitesimals** possessed Being, and for Euler they were Nothing, but for Hegel they fell under the category of **Becoming**. [197]

Hegel writes:

In an equation where x and y are posited primarily as determined through a ratio of powers, x and y as such are still meant to denote Quanta [198]: now this meaning is entirely lost in the so-called infinitesimal differences. dx and dy are no longer Quanta and are not supposed to signify such; they have a significance only in their relation, a meaning merely as moments. They no longer are Something (Something being taken as Quantum), nor are they finite differences; but they are also not Nothing or the indeterminate nil. Apart from their relation they are pure nil; but they are meant to be taken as moments of the relation, as determinations of the differential coefficient .325 dx dy. [199] [200]

*Now when the mathematics of the infinite* [i.e., the infinitesimal] *still maintained that these quantitative determinations were vanishing magnitudes, that is, magnitudes which no longer are any Quantum but also not nothing, it seemed abundantly clear that that such an intermediate state, as it was called, between Being and Nothing did not exist. ... The unity of Being and Nothing is indeed not a* state; *for a state would be a determination of Being and*

[196] Dauben, J. (1979). Georg Cantor: His Mathematics and Philosophy of the Infinite. Harvard University Press.
[197] John L Bell, The Continuous and the Infinitesimal in Mathematics and Philosophy. Polimetrica, 2005, p. 128
[198] By *Quantum* Hegel means determinate Quantity, that is, Quantity of a definite size.

[199] Hegel, G. (1961). *Science of Logic.* Tr. Johnston & Struthers. Allen & Unwin., p.269
[200] Hegel, G. (1961). *Science of Logic.* Tr. Johnston & Struthers. Allen & Unwin., p.270

*Nothing such as might have been reached by these moments only contingently, as it were through disease or external influence, and through erroneous thinking; but, on the contrary, this mean and unity, this vanishing and, equally, Becoming is, in fact, their only truth.* [201]

In Hegel's subsequent review of **how the infinitesimal has been conceived by mathematicians of the past,** those who regarded infinitesimals as fixed quantities receive short shrift, while those who saw infinitesimals in terms of the limit concept (which in Hegel's eyes fell under the appropriate category of Becoming) are praised. Thus, for example, Newton is praised for his explanation of fluxions not in terms of indivisibles, but in terms of "vanishing *divisibilia*", and, further, "not [in terms of] sums and ratios of determinate parts, but [in terms of] the limits (*limites*) of the sums and ratios."[202]

Newton's conception of generative or variable magnitudes also receives Hegel's endorsement. He quotes Newton to the effect that "[Any finite magnitude] is considered as variable in its incessant motion and flow of increase and decrease, and so its momentary augmentation [I give] the name of Moments. These, however, must not be taken as particles of determinate magnitude (particulae finitae). They are not moments themselves, but magnitudes produced by moments; the generative principles or beginnings of finite magnitudes must here be understood. [203]

And then comments:

*—An internal distinction is here made in Quantum; it is taken first as product or Determinate Being, and next in its Becoming, as its beginning and principle, that is , as it is in its concept or (what is the same thing) in its qualitative determination. In the latter the qualitative differences, the infinitesimal incrementa or decrementa, are moments only; and it is only in*

[201] Hegel, G. (1961). *Science of Logic.* Tr. Johnston & Struthers. Allen & Unwin., p.270

[202] Hegel, G. (1961). *Science of Logic.* Tr. Johnston & Struthers. Allen & Unwin., p.274
[203] Hegel, G. (1961). *Science of Logic.* Tr. Johnston & Struthers. Allen & Unwin., p.274

*what has been generated that we have Quantum or that which has passed over into the indifference of determinate existence and into externality.*[204][205]

The use of fixed infinitesimals, on the other hand, Hegel deplores:

*The idea of infinitely small quantities (latent also in increment and decrement) is far inferior to the mode of conception* [just] *indicated. The idea supposes them to be of such a nature that they may be neglected in relation to finite magnitudes; and not only that, but also their higher orders relative to the lower order, and the products of several relative to one.—With Leibniz this demand to neglect (which previous inventors of methods referring to this kind of magnitude also bring into play) becomes more strikingly prominent. It is this chiefly which gives an appearance of inexactitude and express incorrectness, the price of convenience, to this calculus in the course of its operation.* [206]

Nor does **Euler**'s view of infinitesimals as formal zeros fare much better:

*In this regard Euler's idea especially must be cited. On the basis of Newton's general definition, he insists that the differential calculus considers the ratios of incrementa of a magnitude, while the infinitesimal difference as such is to be regarded wholly as nil.—It will be clear from the above how this is to be understood: the infinitesimal difference is nil only quantitatively, it is not a qualitative nil, but, as nil of quantum, it is pure moment of a ratio only. There is no magnitudinal difference; but for that reason it is, in a manner, wrong to express as incrementa or decrementa as as differences those moments which are called infinitely small magnitudes. ... the difficulty is self-evident when it is said that for themselves the incrementa are each nil, and that only their ratios are being considered; for a nil is altogether without determinateness. Thus this image, although it reaches the negative aspect of Quantum and expressly asserts it, yet does not simultaneously seize this negative in its positive meaning of qualitative determinations of quantity, which, if torn away from the ratio and treated as Quanta, would each be but a nil.* [207]

[204] Hegel, G. (1961). *Science of Logic.* Tr. Johnston & Struthers. Allen & Unwin., p.275-276

[205] Hegel, G. (1961). *Science of Logic.* Tr. Johnston & Struthers. Allen & Unwin., p.275

[206] Hegel, G. (1961). *Science of Logic.* Tr. Johnston & Struthers. Allen & Unwin., p.274

[207] Hegel, G. (1961). *Science of Logic.* Tr. Johnston & Struthers. Allen & Unwin., p.275-276

Hegel goes on to discuss some of the methods that mathematicians have employed to resolve the conceptual difficulties caused by the use of infinitesimals. He pays particular attention to Lagrange's attempt to eliminate infinitesimals from the calculus through the use of Taylor expansions. Hegel considers that the Taylor expansion of a function "must not only be regarded as a sum, but as qualitative moments of a conceptual whole."

Hegel is often regarded a philosopher who did not take mathematics very seriously. The fact that he devoted a substantial portion of *The Science of Logic* to the infinitesimal calculus speaks to the contrary.[208]

## 5. The dialectical determination of thought: the Aristotelian (finite) and the Platonic (infinite)

There are two moments of the Hegelian dialectic related with the problem of the doctrine of the infinite and the infinitesimal, as presented in Plato and Aristotle. As Gofton Tyson states that "a systematic dialectic is both a priori *and* immament, presented in thought in all of its moments: formal, subjective, universal, and phenomenological", and "none of these may be privileged above the others".

Gofton characterizes these two moments as 'Platonic' and 'Aristotelian'. There are two ways, Platonic and Aristotelian, of regarding numbers lead to different number systems. What is a finite number for Plato is not necessarily a finite number for Aristotle. In contemporary mathematics, the notion of finite is defined in terms of the completed infinity N. There is no clear concept of the finite in terms of which the infinite can be defined as not-finite. One goes in the opposite direction in contemporary, Platonic, mathematics and defines the finite as not-infinite. (Tyson, Analysis, 2013:116)

The dialectical determination of thought consists in the systematic relation of **the Aristotelian (finite) and the Platonic (infinite)** modes of *rational compossibility* and *determinability* as a system of immanent grounds (the 'Aristotelian' *hypokeímenon*) and transcendent grounds (the

[208] Bell, John L (2005) The Continuous and the Infinitesimal in Mathematics and Philosophy. Polimetrica, 2005, p. 132

‘Platonic’ *hypothesis*). More precisely, it consists in their immanent identity within the finite subject; that is, their ideal identity and real contradiction: the reality and actuality of the idea (*eidos*). (Tyson, 2013:116)

Tyson Gofton, asserts that **“**The Aristotelian moment begins with the *finite hypokeímenon* (*das Zugrundlegende*) and attempts to *ground* the *actuality* of the *infinite* in the *compossibility* of the finite, i.e., through a *compositum ideale*, an absolute infinite *in potens*. It is thus the actuality of (finite) thought trying to reach beyond itself (the infinite). But, insofar as it just as this thinking of what is beyond its own finitude, it is at the same time this very beyond. That is, it is also the Platonic moment of ideal intelligibility, *noésis* or the *hypothesis* (*das Zugrundliegende*) through which attempts to *ground* the *reality* of the *finite* in the *determinability* of the *infinite*, i.e., through a *totum reale*, an absolute unity *in potens*.” (Tyson, 2013:116)

What is remarkable about Hegel’s mathematics is the attempt to make the Aristotelian conception of “predicative mathematics” immanently (i.e., in thought) equivalent to the Platonic conception of “impredicative mathematics”. This distinction echoes the problem Kant addresses in the Mathematical Antinomies, and therefore points to the importance of Hegel’s reconstrual of the Antinomies as the central interpretive problem for grasping not just Hegel’s philosophy of mathematics, but his conception of the infinite (as both predicative and impredicative) as the central tension of dialectical logic.[209] (Tyson, 2013:116)

There, we think of the world whole through the category of unity (and thus as the result of a constructive algorith) and through the category of totality (and thus as the idea of the whole set of possible numbers). The difference lies in the interpretation of the quantifier as applying to all cases of number (or magnitude) that have been constructed (*8x; x* is a *y*, and *y* is a determinate construction) and the quantification across all numbers in the absence of a

[209] Tyson, Gofton B. (2013) Analysis, Systematicity and the Transcendenal in Herman Cohen, p.166. www.tysongofton.com/s/Gofton_B_Tyson_2013_PhD_thesis.pdf

determinate predicate (*8x; x* is not a *y*, where *y* is a determinate construction).[210] (Tyson, 013:116)

The ideality — and thus the intelligibility — of mathematics lies in the perfection of the Platonic moment of thought.

The actuality — and thus the objectivity — of mathematics lies in the perfection of the Aristotelian moment of thought.

Thought is *both* of these moments; it is both infinite and ideal, and finite and real. It is both *hypothesis* and *hypokeímenon*.
The concept of a magnitude — and *a fortiori* the concept of the infinite magnitude — therefore invokes two definitions of the infinite. The diachronic concept of infinite compossibility (i.e., the imperfect infinite) must contain in its definition its own supercession. This is the case, for example, in the definition of the *syncategorematic infinite*: *8x9y* = *x* + 1*; x* < *y*.

Accordingly, Hegel will say that the the first definition "contains in itself ... externality", that is, the concept of the infinite contains a number that is outside the scope of the universal quantifier.

The synchronic concept of infinite divisibility, however, is "the negation of the same", that is, the negation of externality, or the assertion that *9x8y* = *x* + 1*; :*(*x* < *y*). This, however, just is the definition of the *categorematic infinite*: *9x8y* = *x*+1*; x _ y*. Here, then, is Hegel's full definition of the infinite: "The infinite quantum rather contains in itself first externality [i.e., *y* = *x* + 1, which is greater than any actual *x* — BTG] and second the negation of the same [i.e., *:*(*x* < *y*) — BTG]" (WL1; HW 21:306-7).

The determination of the individual (*das Etwas*) — as either finite or infinite — rests on possibility of its real limitation, i.e., on the real, bounded totality. The concept of such an individual, however, consists of its infinite composition . . . (Tyson, 2013:116)

[210] Tyson, Gofton B. (2013), Analysis, Systematicity and the Transcendenal in Herman Cohen, p.166-167. www.tysongofton.com/s/Gofton_B_Tyson_2013_PhD_thesis.pdf

Hegel's first transformation to the table of judgments implies the priority of the judgments of general logic (the principles of thought) over the quantitative judgments of mathematics (the synthetic principles). More importantly, it implies the priority of a constructable manifold of determinables over the quantification of a *factical* manifold of sensibility.

For, Kant's judgments of quantity are only able to appear first in the order of explanation (and thus of apprehension, reproduction, recognition and cognition) insofar as they take up a manifold that is (paradoxically) already given *as a manifold*. (Tyson, 2013:180)

Hegel, claims that the qualitative moment of judgment precedes the quantitative moment: "quality leads to quantity". For Hegel the qualitative moment of judgment is taken to imply an immediate, conceptually indeterminate mode of cognition (intuition).

For Hegel, the first judgment is a positive judgment: the "positing" of being as a total determinable. This determinable is a *totality*, rather than a mere whole, since its qualitative moment, or the implicit manifold of its parts, is already implied in the mere positing of being as such. Hegel understands logic to include the production of intentional content and this is the reason why the qualitative judgment precedes the qualitative judgment.

If Kantian logic of judgment derives its objective orientation from the "given", for Hegel content is derived from the activity of consciousness, as an act prior to the quantitative judgment, whereby the intentional content of thought may be determined. (Tyson, 2013:116) . . "…the central task of the Doctrine of Being in Hegel's *Objective Logic* is the determination of the objectivity of the object, i.e., the possible (transcendental) relation between the totality of the manifold (being) and the concept of the individual.Through this determination, the individual is posited as a finite concept that is immanently correlated with its infinite composition as idea. (Tyson, 2013:116)

One aspect of the Doctrine of Being that is too often overlooked is that it is first and foremost a philosophy of mathematics, or more precisely, an analysis of the concept of quantity as the finite part of infinite continuum of the manifold. That is, every determinate (subject) is an infinitely determinate totality (in space and time) that is nonetheless a finite modification of an infinite manifold (being). (Tyson, 2013:116)

Accordingly, the (infinite) determination of (finite) totalities within the infinite continuum of the manifold is the primary task of the Doctrine of Being; this is not the ontological or metaphysical determination of material reality, but rather the determination of the concept of a finite quantity, i.e., the concept of the discrete individual, or what Kant calls the *quantum discretum*. (Tyson, 2013:116)

However, it is Hegel's Doctrine of Being, and the purported philosophy of mathematics contained therein, that has been the object of perhaps the most vigorous attacks on Hegel's system — even from friendly quarters.() [211] (Tyson, 2013:116)

### 6. Hegel's thinking of quantity in terms of an implicit continuous topology

If David Carlson's approach in his 'Hegel's Theory of Quantity', implements the claims that Hegel provides "philosophy's most rigorous definition of quantity" (Carlson 2001, 2027), Tyson Gofton in his PhD thesis - *Analysis, Systematicity and the Transcendental in Hermann Cohen's System of Critical Idealism*. (Tyson, 2013:205), asserts that there are strong and explicit suggestions about the presence of 'topological' in Hegel's Logic. (Tyson 2013:205).

Tyson asserts "…..the process of determining numbers (i.e., instances of "the one"), for Hegel, begins not with the continuity of a quantitative continuum, but rather with a merely qualitative continuum: the general relational determinability of something in general, the principle of determinability as the a priori system of determinable relations.(14) (Tyson, 2013:208)

Under the reference note (14) following the text quoted above, Tyson states: that "Hegel is almost certainly thinking of quantity in terms of an implicit, phenomenological, continuous topology". (Tyson, 2013:209).

[211] Gofton, Tyson. 2013. *Analysis, Systematicity and the Transcendental in Hermann Cohen's System of Critical Idealism*. PhD Thesis. University of Toronto., p. 205

For Tyson “the question is not whether Hegel can construct ordinal numbers, but whether he is in a position to do so at the point when the concept of quantity is first made explicit.” (Tyson, 2013:207)

The qualitative continuity of the number is what Hegel calls a “principle”, but this is merely the categorial intelligibility of a unity in general, and is not the assertion of a quantitative or determinate continuity within the one. Indeed, the continuity of number is the *third* moment of quantity: the infinite.

Hegel’s triadic structure is at work in the concept of number as discrete magnitude has first the one as principle, and is second the plurality of the ones [the one of the one (*der Eins)*], third it is essentially continuous. While the one remains the “principle” of number, this categorial thought really is “nothing” unless it can be given an internal articulation, and this articulation must come not from continuity, but rather from discrete enumeration. Indeed, for Hegel “quantity is only concrete unity insofar as it is the unity of different moments” (WL1; HW 21:234), i.e., analytical composition.

What we require then, is a means whereby a system of differences may be constructed insofar as it relates the the infinite totality of a particular number. For, it is only insofar as strictly constructive procedures can be given for the determination of differences that they may be conceptually interpreted, and correlated with the idea of the number as an infinitely determinate totality. This just is the *principle of intelligible totality*.

While the qualitative “discretion” of the one into the ones is given in the Book I (‘Determinacy’), this determinacy is merely qualitative and immediate. Emphatically, “the quantum, at first quantity with a determinacy or a limit in general, — is in its complete determinacy number” (WL1; HW 21:240). The concrete or conceptual determinacy of number thus relies on the construction of a system of relational differences that is not given a priori or a posteriori, but which must be constructed. Similarly, the continuity of number is given qualitatively as the determinability of unity, yet we require a constructive procedure in order to render the difference of individual moments intelligible. For Hegel, “the **first** producing of number is the taking-together of many as such, i.e., that in which a many is posited as only one, — enumeration.". That is, number itself arises from counting units and their assimilation to a set or collection that we take to be a unity, not from the "principle" of a continuous unity, which is nothing more than the categorial determination of number as such.

In his general discussion on mathematics and continuum, Tyson states that "the rise of mathematical physics transformed the language in which truths about ordinary objects were expressed. Just as important as the transformation of the language in which epistemic claims are framed is the object implied by mathematical physics. Classical (Aristotelian) physics is concerned with particulars and their interactions. This model of investigation is well-suited to classical epistemology, with its syllogistic logic and substance ontology. After the Copernican revolution, however, it is no longer the particular which is the proper object of physical inquiry, but rather appearance as such. That is, natural physics is not the investigation of objects or bodies, but the investigation of the manifold of space and time, or, what is the same, the real continuum (R4): "Nature". The new physics requires, as we have seen, a different mode of expression and a different mode of justification. "(Tyson, 2013:207)

Same is relevant for the new fields of mathematics – topology…the modern topology requires a different mode of expression and different mode of justification, also a new object: the real continuum. Topology with her various fields introduces into ontology, logic and epistemology new problems within the problem of continuum. The continuum which as ultimate objects of philosophical investigation was first introduced by Leibniz, who developed calculus and first introduced topology – analysis situs.

# Chapter 4 Topological (in) Hegel

## 1. The Qualitative quantity in Hegel

### 1.1. Qualitative quantity in Hegel's "Encyclopedia of Philosophical Sciences", Part One, referred to as The Lesser Logic

Hegel discusses Qualitative quantity in his Logics – "The Science of Logic" /Wissenschaft der Logik, referred to as the Greater Logic /1812, 1813, 1816/ and The Lesser Logic /Part One of the “Encyclopedia of Philosophical Sciences” / Enzyclopädie der philosophischen Wissenschaften im Grundrisse /1817/.

Qualitative quantity in Hegel’s “Encyclopedia of Philosophical Sciences”,
Part One, referred to as The Lesser Logic

In § 105 and § 106 of the Lesser Logic (Hegel, G.W.F. 1817: p.170/171-353/354)[212], Hegel defines “Qualitative quantity, or Measure” in the following lines:

“The two sides of the ratio are still immediate quanta: and the qualitative and quantitative characteristics still external to one another. But in their truth, seeing that the quantitative itself in its externality is relation to self, or seeing that the independence and the indifference of the character are combined, it is Measure.

Thus by means of the dialectical movement which has now been discussed, the movement of quantity through its several stages, quantity turns out to be a return to quality. The first notion of quantity presented to us was that of quality abrogated and absorbed. That is to say, quantity seemed an external character not identical with Being, to which it is quite immaterial. This notion, as we have seen, underlies the mathematical definition of magnitude, as what can be increased or diminished. At first sight this definition may encourage a belief that quantity is merely whatever can be altered: " increase and diminution alike implying determination of magnitude otherwise " and may tend to confuse it with determinate Being, the second stage of quality, which in its notion is similarly conceived as alterable. We can, however, complete the definition by adding, that in quantity we have something which alters, but which in spite of its changes still remains the same. The notion of quantity, as it thus turns out, implies an inherent contradiction.

This contradiction is what forms the dialectic of quantity. The result of the dialectic however is not a mere return to quality, as if that were the true and quantity the false notion, but an

[212] “Hegel’s Logic”, translated by William Wallace, with Foreword by J N Findlay, Clarendon Press 1975. First published 1873

advance to the unity and truth of both, to *qualitative quantity*, or Measure." (The Lesser Logic, § 106), (Hegel, G.W.F. 1817: p.170/171-353/354).[213]

According to David Gray Carlson, "Measure is the third and last province in the kingdom of Quality, which itself comprises the first third kingdom in the empire of the Science of Logic. When Measure concludes, we will have arrived at the portal of the negative, correlative underworld of shadowy Essence." (Carlson, D.G. 2003). Carlson states that "Hegel proclaims the development of Measure to be 'extremely difficult,' and many commentators have concurred. We can nevertheless describe the theme of Measure easily enough—change; more precisely, an exploration of the difference between qualitative and quantitative change." (Carlson, D.G. 2003.)

The quality of the quantity is derived from §105 of The Lesser Logic, (Hegel, G.W.F. 1817: p.170-353) where Hegel states: "That the Quantum in its independent character is external to it self, is what constitutes its quality" and "In that externality it is itself and referred connectively to itself. There is a union in it of externality, i.e. the quantitative, and of independency (Being-for-self)-the qualitative." Also "The Quantum when explicitly put thus in its own self is the Quantitative Ratio, a mode of being which, while, in its Exponent, it is an immediate quantum, is also mediation, viz. the reference of some one quantum to another, forming the two sides of the ratio. But the two quanta are not reckoned at their immediate value: their value is only in this relation." (Hegel, G.W.F. 1817: p.170-353)

In § 106, Hegel claims that "This contradiction is what forms the dialectic of quantity. The result of the dialectic however is not a mere return to quality, as if that were the true and quantity the false notion, but an advance to the unity and truth of both, to qualitative quantity, or Measure.

It may be well here to draw attention to the circumstance, that if we employ quantitative terms in our observation of the world of objects, it is in all cases the Measure \which we have in view, as the goal of our operations. This is hinted at even in language, when the ascertainment of quantitative features and relations is called measuring." (Hegel, G.W.F. 1817: p.171-354)

[213] "Hegel's Logic", translated by William Wallace, with Foreword by J N Findlay, Clarendon Press 1975. First published 1873/§ 106

For Hegel, Measure "has within itself the difference from itself", and Carlson's comment is that "when difference was simply external, we had before us quantitative difference. But now, having been captured by Measure, this difference is a qualitative moment." (Carlson D.G. 2003). Regarding Essence, Measures are brought together by an external measurer. The Measures are *ready* to be brought together. Measure therefore is, as Hegel will later say, "**the *immanent* quantitative relationship of *two* qualities to each other."** Each Measure imposes quantitative change on the other Measure. Each Measure has a qualitative resilience against the change imposed upon it from the outside. If this resilience is isolated and considered on its own, we have the Measureless or Essence. (Carlson D.G. 2003). As Carlson puts it "for the moment, Quality and Quantity are still with us, but in mediated form. Each of these extremes in the syllogism of Measure is equally the one and the other. This was not so before. In Quality, the Understanding grasped Being as an affirmative immediacy. In Quantity, the Understanding learned that the negative, quantitative moment of Continuity was the truth of Being. Now the Understanding sees that the qualitative and the quantitative are two Verona houses both alike in dignity. The difference between them is indifferent and so is no difference." (Carlson D.G. 2003).

Carlson state that "The difference between Quality and Quantity has been sublated. In Ratio, Quantity showed itself to be a return-into-self. This very reflection-into-self *is* Quality." (Carlson D.G. 2003).

For Carlson, "Quantity is open to mere qualitative change. Quantitative change is change imposed from the outside. The very quality of quantum was that it was indifferent to change imposed upon it from outside. Qualitative change is self-imposed change from the inside. We will learn, however, that genuine qualitative change depends on quantitative change." (Carlson D.G. 2003).

The elaboration between qualitative and quantitative in the Measure was not a single act of writing but complex task for Hegel. Hegel warns that the development of Measure is "extremely difficult" ("Eine der schwierigsten Materien."[214] (Carlson, 2003:2). This remark Hegel added to the version of the Science of Logic from 1831. Hegel renounced this

[214] David G. Carlson, 2003, Hegel's Theory of Measure, . . .note 4.

conclusion in the 1817 "Heidelberg" Encyclopedia of Philosophical Science, and then omitted the renunciation in the later Berlin editions of the Encyclopedia. In the Heidelberg version from 1817 Hegel used a "single" transition from Quality to Quantity, and a "single" transition back. In this single transition, only vanishing was emphasized. In Berlin editions of the *Encycolpedia* from 1827 and 1830, Hegel realized that there was a "double" transition, where each side of the syllogism vanishes *and* sustains itself. This leads Hegel to withdraw his renunciation of his earlier work due to the fact that empirical quanta are not *entirely* unrelated to Logic. [215] Errol Harris asserts that Measure is “extraordinarily difficult . . . so obscure as to be, for the most part, hardly intelligible, and, while it contains some astonishingly prescient scientific comments, it also indulges in what, to us in the twentieth century, must appear ill-informed and perverse polemic against sound scientific insights.”[216] (Harris, 1983:149)

The theme of Measure can be described with one word, easily enough as David Carlson suggests. This word is change, change seen as “an exploration of the difference between qualitative and quantitative change.” (David G. Carlson, 2003, Hegel’s Theory of Measure).

As I stated above, for Hegel the relationship of concepts in judgement is a form of transition (*Übergehen*) and this transition is related with ‘becoming’ and ‘change’. Hegel uses the word “transition” together with words such as ‘becoming’ and ‘change’. The subject and the predicate are different phases in the judgement. The judgement is a transition and the syllogism is transition as well, but unlike the judgement, it is a mediating transition.

Here I shall emphasize that the words used above, namely: ‘exploration of difference between’ suggest topological notions. The nature of the ‘change’ changed itself through the journey of development of quality and quantity in the Science of Logic. The face of Quality and the face of Quantity change as well through the development of Hegel’s logic. Their role is shifting as well. There are three main forms, faces, roles that both Quality and Quantity shift through this development. The first stage of they change is the Understanding, then the next stage is Dialectical Reason and the third stage is the Speculative Reason. Understanding

[215] Cinzia Ferrini, Framing Hypotheses: Numbers in Nature and the Logic of Measure in the Development of Hegel's System, in Hegel, and the Philosophy of Nature, 283 (Stephen Houlgate ed., 1998)
[216] Errol E. Harris, An Interpretation of the Logic of Hegel, 1983: 143

– Dialectical Reason – Speculative Reason, are the three stages of exploration of the difference between qualitative and quantitative change.

As David Carlson elaborates "Change has itself *changed (*over our journey). (David G. Carlson, 2003, Hegel's Theory of Measure). Carlson call this transition of the transition - "was*transition*." Being became Nothing, Determinate Being became Negation, The Finite ceased to be. Starting with the True Infinite, however, change itself changed.

The True Infinite did not cease to be. It stayed what it was even while it became something different. This was the beginning of *ideality*. In the True Infinite, immediate Being ceased to be *and* preserved itself in an idealized form. When Being ceased to be (while surviving as the mere memory of immediacy), we entered the realm of Quantity, which was Being with all its content outside of itself. Whatever Quantity is, it is by virtue of outside force designating what it is. Quantity is open to mere *quantitative* change. Quantitative change is change imposed from the outside. The very quality of Quantum was that it was indifferent to change imposed upon it from the outside. Qualitative change is self-imposed change from the inside. We will learn, however, that genuine qualitative change depends on quantitative change. Nature does make great leaps, but only after nature indifferently undergoes incremental quantitative change. (Carlson, 2003:2).

Perhaps the most common illustration for the transition of quantitative change to the qualitative change within the measure although from the point of Understanding is how the liquid water gets colder due to outside force, indifferently stays liquid, but, at 0E centigrade, liquid, radically and all at once, becomes a solid. (Carlson, 2003:3). Here, as Carlson suggests "Measure emerged in the Ratio of Powers (*e.g.*, *x2 = y*), which showed itself to be "self-related externality." (327). Here by "ratio" Hegel means *any* relation between two quanta, including *xy*, *x2*, or *x/y*. In *x2 = y*, the identity of the first (internal) *x* is determined by the second (external) *x*. Hence the first *x* is in the thrall of externality. Nevertheless, *x* = *x*, and so it is self-related, even while externally determined. As self-related, the Ratio of Powers (Measure) is "a *sublated* externality." (327) Under the law of sublation, externality is canceled *and* preserved. Hence, Measure "has within itself the difference from itself." (327). (Carlson, 2003:3). Quantitative difference appears when difference is simply external. Captured by Measure, this difference is a qualitative moment. The topological mode of this development of syllogism is presented in the act of relating, in the act of mutual reference, the homology between two different (indifferent to each other) realities. As John Burbidge remarks: "*Measuring* . . . introduces an explicit act of relating. It brings together two realities, indifferent to each other… Since mutual reference is now an inherent characteristic of the concept, one passes beyond simple immediacy.[217] (Burbridge, 1981: 63) In his book on chemistry, Burbidge remarks that: "Measuring uses a quantity to specify a quality. That definition sets the logical task."[218] (Burbridge, 1996: 53) Carlson remarks that "this formulation threatens to obscure the fact that, for Hegel, a Measure's quality *is* its quantity-accurate reportage of what the thing is." (Carlson, 2003:4, endnote 14).

### 1.2. Qualitative quantity in Hegel's "The Science of Logic" (Wissenschaft der Logik), referred to as the Greater Logic

The notion of Qualitative quantity appears in Hegel's early work "The Science of Logic" /The Greater Logic (Hegel, G.W.F. 1812, 1813, 1816. tr. /1969. Miller*,* A. V., foreword by Findlay J. N.)

In the first chapter, the "Specific Quantity", § 711 – "Qualitative quantity in the *first* place an immediate, specific *quantum*.", also in § 731, § 774 /in "Nodal Line of Measure Relations" /B/ in Chapter 2 /"Real Measure"/. The specific notion of the "qualitative quantity" Hegel explores in the Second Chapter "Real Measure" - "The Relation of Self-Substistent Measures" /A/ in "Combination of Two Measures" /a/, in "Measure of a series of Measure Relations" /b/, and in "Elective Affinity" /c/. The notion of "qualitative quantity" is the reason for Hegel to quote Carl Linnaeus's "Nature Does Not Make Leaps"/ § 774/.

In the first chapter, the ―Specific Quantity, in § 711 (Hegel, G.W.F. 1812, 1813, 1816. tr. / 1969. Miller*,* A. V., foreword by Findlay J. N.), Hegel states that "Qualitative quantity in the *first* place an immediate, specific *quantum*." And then continues "*Secondly,* this quantum as relating itself to another becomes a quantitative specifying, a sublating of the indifferent quantum. This measure is so far a *rule* and contains the two moments of measure distinguished; namely, the intrinsic quantitative determinateness and the external quantum. In

[217] John W. Burbridge, On Hegel's Logic: Fragments of the Comentary, 1981:63.
[218] John W. Burbridge, Real Process: How Logic and chemistry combine in Hegel's philosophy of nature, 1996:53.

this distinction, however, these two sides become qualities and the rule becomes a relation between them; consequently measure exhibits itself *thirdly,* as a relation of qualities. These at first have a single measure, but this is further specified within itself into distinct measures." (Hegel, G.W.F. 1812, 1813, 1816. tr. /1969. Miller*,* A. V., foreword by Findlay J. N.)

In § 731, Hegel concludes that "Measure is thus the *immanent* quantitative relationship of two qualities to each other." (Hegel, G.W.F. 1812, 1813, 1816. tr. /1969. Miller*,* A. V., foreword by Findlay J. N.) Hegel's examinination of the Relation of the two Sides as Qualities, in § 731, goes as follow:

"The qualitative, intrinsically determinate side of the quantum exists only as a relation to the externally quantitative side; as a specifying of the latter it is a sublating of its externality through which quantum as such is. This qualitative side thus has a quantum for its presupposition and its starting point. But this quantum is also qualitatively distinguished from the quality itself; this difference between them is now to be posited in the *immediacy* of being as such, in which determination measure still is. The two sides are thus qualitatively related and each is on its own account a qualitative determinate being; and the one quantum which at first was only a formal quantum indeterminate in itself, is the quantum of a something and of its quality, and also — now that the connection between them is determined as a measure — the specific magnitude of these qualities. These qualities are related to each other according to their determination as measures which determination is their exponent. But they are already implicitly related to each other in the *being-for-self* of measure; the quantum in its dual character is both external and specific so that each of the distinct quantities possesses this twofold determination and is at the same time inseparably linked with the other; it is in this way alone that the qualities are determined. They are therefore not only simply determinate beings existing for each other but they are posited as inseparable and the specific magnitude connected with them is a qualitative unity — a single determination of measure in which, in accordance with their Notion, they are implicitly bound up with each other. Measure is thus the *immanent* quantitative relationship of two qualities to each other." (Hegel, G.W.F. 1812, 1813, 1816. tr. /1969. Miller*,* A. V., foreword by Findlay J. N.)

The specific notion and role of the "Qualitative quantity" Hegel explores in the Second Chapter "Real Measure" –"The Relation of Self-Substistent Measures" /A/ in "Combination of Two Measures" /a/, in "Measure of a series of Measure Relations" /b/, and in "Elective Affinity" /c/. The notion of "Qualitative quantity" is the reason for Hegel's famous quote "Nature Does Not Make Leaps" ( § 774) in "Nodal Line of Measure Relations" /B/ in Chapter 2 ("Real Measure"). (Hegel, G.W.F. 1812, 1813, 1816. tr. /1969. Miller*,* A. V., foreword by Findlay J. N.)

In § 774, Hegel explains that:

"The system of natural numbers already shows a nodal line of qualitative moments which emerge in a merely external succession. It is on the one hand a merely quantitative progress and regress, a perpetual adding or subtracting, so that each number has the same *arithmetical* relation to the one before it and after it, as these have to their predecessors and successors, and so on. But the numbers so formed also have a *specific* relation to other numbers preceding and following them, being either an integral multiple of one of them or else a power or a root. In the musical scale which is built up on quantitative differences, a quantum gives rise to an harmonious relation without its own relation to those on either side of it in the scale differing from the relation between these again and their predecessors and successors. While successive notes seem to be at an ever-increasing distance from the keynote, or numbers in succeeding each other arithmetically seem only to become other numbers, the fact is that there suddenly emerges a *return,* a surprising accord, of which no hint was given by the quality of what immediately preceded it, but which appears as an *actio in distans,* as a connection with something far removed. There is a sudden interruption of the succession of merely indifferent relations which do not alter the preceding specific reality or do not even form any such, and although the succession is continued quantitatively in the same manner, a specific relation breaks in *per saltum.*" (Hegel, G.W.F. 1812, 1813, 1816.

Emphasizing the transformation of quality to quality by leaps "per saltum" in the "nodal Line of Measure Relations" (§ 777), Hegel defines "the attempt to explain coming-to-be or ceasing-to-be on the basis of gradualness of the alteration" as **"tedious like any tautology".** (Hegel, G.W.F. 1812, 1813, 1816. tr. /1969. Miller*,* A. V., foreword by Findlay J. N.) And again, very next in the text Hegel defines as tautology the attempted explanation of the gradual notion (the "gradualness"): "what comes to be or ceases to be is assumed as already complete and in existence beforehand and the alteration is turned into a mere change of an external difference, with the result that the explanation is in fact a mere **tautology**." (Hegel, G.W.F. 1812, 1813, 1816. tr. /1969. Miller*,* A. V., foreword by Findlay J. N.) Hegel warns about "the intellectual difficulty" for the "attendant on such an attempted explanation comes from the qualitative transition from something into its other in general, and then into its opposite; but the *identity* and the *alteration* are misrepresented as the indifferent, external determinations of the quantitative sphere." (Hegel, G.W.F. 1812, 1813, 1816. tr. /1969. Miller*,* A. V., foreword by Findlay J. N.)

Hans-George Gadamer in his essay "The Idea of Hegel's Logic" [219], discusses the logical instinct of language in Hegel's Logic and the notion of Hegel's of "speculative logic" and "speculative statement" that goes beyond Kantian transcendental analytics and the formal logic. Gadamer remind us that our human nature is so much determined by finitude that the phenomenon of language and the thinking wherein we seek to get hold of it must always be viewed as governed by the law of human finitude. For Gadamer, Hegel's "speculative statement" is not so much a statement as it is language. Gadamer states that the speculative statement maintains the mean between the extremes of tautology on the one hand and self-cancellation in the infinite determination of its meaning on the other.

In his "Hegel's Dialectic" /1976/[220], Gadamer asserts that Hegel's approach to logic and indeed his whole style of writing is ''tautological''. In the essay "Hegel and the Dialectic of the Ancient Philosophers" [221] in his "Hegel's Dialectic", Gadamer maintains that a philosophical statement is very different from ordinary empirical statements. In the empirical statements, the predicate always leads us to ''something new or different'', but in a philosophical statement, the predicate always leads us back to a deeper reflection of the subject itself, such that ''to ordinary 'representative' thinking a philosophical statement is always something like a tautology; the philosophical statement expresses an identity''.

[219] "Die Idee der Hegelschen Logik (1971) in Hans-George Gadamer, "Hegel's Dialectic: Five Hermeneutical Studies", translated into English by P. Christopher Smith and collected in (New Haven: Yale University Press, 1976). These five essays are "Hegel and Heidegger,"; "Hegel's Dialectic of Self-consciousness"; "Hegel and the Dialectic of the Ancient Philosophers,"; "Hegel's 'Inverted World,'; and "The Idea of Hegel's Logic," 75-99

[220] "Die Idee der Hegelschen Logik (1971) in Hans-George Gadamer, "Hegel's Dialectic: Five Hermeneutical Studies", translated into English by P. Christopher Smith and collected in (New Haven: Yale University Press, 1976). These five essays are "Hegel and Heidegger,"; "Hegel's Dialectic of Self-consciousness"; "Hegel and the Dialectic of the Ancient Philosophers,"; "Hegel's 'Inverted World,'; and "The Idea of Hegel's Logic," 75-99

[221] "Die Idee der Hegelschen Logik (1971) in Hans-George Gadamer, "Hegel's Dialectic: Five Hermeneutical Studies", translated into English by P. Christopher Smith and collected in (New Haven: Yale University Press, 1976). These five essays are "Hegel and Heidegger,"; "Hegel's Dialectic of Self-consciousness"; "Hegel and the Dialectic of the Ancient Philosophers,"; "Hegel's 'Inverted World,'; and "The Idea of Hegel's Logic," 75-99

In contemporary philosophical research the creative power of tautology was definitely recognized. [222] [223]

Hegel's emphasis on the leap and nodal line in the relationship of qualitative and quantitative and the reason to set in brakets the Carl Linnaeus aphorism "Nature Does Not Make Leaps" /natura non facit saltus/, comes from the point of observeability and perceptability. This consideration is clear in Hegel:

"In thinking about the **gradualness** of the coming-to-be of something, it is ordinarily assumed that what comes to be is already sensibly or *actually in existence*; it is **not yet perceptible only because of its smallness.** Similarly with the gradual disappearance of something, the *non-being* or other which takes its place is likewise assumed to be *really there*, only not observable, and *there*, too, not in the sense of being implicitly or ideally contained in the first something, but *really there*, only not observable." /§ 777/. (Hegel, G.W.F. 1812, 1813, 1816. tr. /1969. Miller, A. V., foreword by Findlay J. N.)

Hegel's discussion of measure is presented as the third part of the Logic of Being, in three chapters. The first presents "Specific Quantity", the next offers "Real Measure", and the third delineates "The Becoming of Essence". Measure follows from quality and quantity because it involves determinacy that is both qualitative and quantitative. Measure is an immediate unity of quality and quantity. It is qualitatively specific quantum, where quantity is bound with quality without further qualification.

The term "Specific quantity" presents the qualitative quantum as it is immediately given. By contrast, "real measure" involves "The Relation of Self-Subsistent Measures," and these relations will take various forms. [224]

[222] For Hegelian and Heidegerian Tautologies, see: Tze-Wan Kwan, Hegelian and Heidegerian Tautologies, Analecta Husserlliana, The yearbook of Phenomenological research, Logos of Phenomenology and Phenomenology of Logos, Volume LXXXVIII, 2005, The World Institute for Advanced Phenomenological Research and Learning

[223] In the works of Gregory Bateson, also see: Allen Thiher, The power of tautology: The roots of leterary theory, Associated University Press, 1997

[224] Winfield, Richard Dien, 2012, Hegel's Science of Logic: A Critical Rethinking in Thirty Lectures, Rowman & Littlefield Publishers, Inc., p.144

About "real measure", the emphasis is on measure that is determinate, whereas specific quantity merely compromises measure per se. Just determinacy as such becomes a determinate determinacy in the contrast of something and other, so measure becomes real in relations between measures, relations between specific qualitative quanta.[225]

Topological notions here are presented with the emphasis on the "relations" and "between" measures, "relations between specific qualitative quanta" due to the fact that the main concern of "topology" are such notions as "relationships" and "between-ness", thus we can suggests the term 'topological notion of Qualitative quantity/ Specific quantity/ Qualitative quantum .

There are topological characters in the situation of the Specific quantum (Qualitative quantity) that contains both quality and quantity. Although quality and quantity are immediately united in Specific quantum (Qualitative quantity), they are also distinguishable. In topological transformation, the two different geometrical figures (such as circle and square) can be transformed gradually one into another (without cutting) and the two will be united due to the homology yet these two are still distinguishable. The duality of homeomorphic transformation is presented in the Hegel's notion of Specific quantum (Qualitative quantity). Due to this distinguishability between quality and quantity in the Specific quantum, "the specific quantum presents a duality that presents a quandary to those who are baffled by whether the removal of one hair makes one bald or whether the removal of a single grain of sand from the heap makes it cease to be heap."[226]

The duality yet the distinguishability between quality and quantity in the Specific quantum and the conjunction of these two sides is exhibited in the examples given by Hegel, the paradoxes of the Heap and the Bald, where the nodal alteration in these examples is bond up with the very character of measure, due to the Vague nature of quantity and quality. As Winfield asserts, "there is an aspect of indifference that cannot be completely eliminated, but on the other hand, there is a limit in measure, where something at a certain point will lose its quality, seasing to be what it is when a certain quantitative threshold is reached."[227] Once the measure relationship applies to quantities and presents a rule for them, a specific quantum can

[225] Winfield, Richard Dien, 2012, Hegel's Science of Logic: A Critical Rethinking in Thirty Lectures, Rowman & Littlefield Publishers, Inc., p.145
[226] Winfield, Richard Dien, 2012, Hegel's Science of Logic: A Critical Rethinking in Thirty Lectures, Rowman & Littlefield Publishers, Inc., p.145

serve as a standard for measuring other things, whose quantities have a qualitative side different from that of their standard of measure. We end up with relations between different measures. . .[228] If you keep on plucking your hair, you will eventually become bald, just as if you remove one grain of sand after another from a pile, you will finally have nothing left. Yet in both cases of the head of hair and the heap of sand, the qualitatively defining quantity is indefinite.[229]

This problem arises from the two sides of measure. With measure, we are dealing with quantity that is no longer indifferent to quality, but has quality associated with it, and with a quality that depends upon quantity for its qualitative character. One commonly speaks about something gradually altering its quality through a gradual increase or decrease in some quantitative dimension. This construal focuses purely on the quantitative dimension, yet qualitative change is not just quantitative. It is something different.[230]

When water freezes, the qualitative change does not happen gradually. It It happens all at once. There is a sudden transformation. There is a nodal alteration as the degree of temperature changes. This nodal transformation is build into measure. Hegel points out, given that measure involves an immediate conjunction of quantity and quality. On the one hand, because quantity is continuous, it points beyond itself and is subject to alteration. There is always a quantity beyond it with which it is continuous. On the other hand, measure imposes some limit to how much the specific quantum can undergo an increase or decrease without altering its quality. At some point there is a limit.[231]

[227] Winfield, Richard Dien, 2012, Hegel's Science of Logic: A Critical Rethinking in Thirty Lectures, Rowman & Littlefield Publishers, Inc., p.145

[228] Winfield, Richard Dien, 2012, Hegel's Science of Logic: A Critical Rethinking in Thirty Lectures, Rowman & Littlefield Publishers, Inc., p.145

[229] Winfield, Richard Dien, 2012, Hegel's Science of Logic: A Critical Rethinking in Thirty Lectures, Rowman & Littlefield Publishers, Inc., p.145

[230] Winfield, Richard Dien, 2012, Hegel's Science of Logic: A Critical Rethinking in Thirty Lectures, Rowman & Littlefield Publishers, Inc., p.145

[231] Winfield, Richard Dien, 2012, Hegel's Science of Logic: A Critical Rethinking in Thirty Lectures, Rowman & Littlefield Publishers, Inc., p.145

Measure has generated a process into which measure has absorbed itself, a process in which measures cancel themselves, moving beyond themselves only to return to measure, letting thir own vanishing vanish. This process contains the determinations of quality, quantity, and measure as mere states of itself, states mediated by this encompassing process. Why would this provide closure for the Logic of Being, and take us to something in this categorical arena? How are we now contending with what is no longer characterizable simply qualitatively or simply quantitatively or as a qualitative quantity? Why do we have something that requires new categories to be or be thought?[232]

The emergent process cannot be reducible to any of the terms it contains precisely because it contains them all with further qualification. You cannot use categories of quality, quantity, or measure to categorize what involves them all in an additional relationship, a relationship that takes the totality of their relations and renders them internal differentiations of something that robs them of their immediacy by being their underlying mediation.

What we have is both their determinations and that which generates them withut having any further character. Hegel points out that this is precisely what essence has always been understood to be (p.379). Traditionally, essence is construed as what underlies being. You arrive at the essence of being by somehow getting at what being really rests upon. Being has an essence insofar as it is mediated by something else, without which it cannot be.

Typically, Hegel observes in introducing essence, when we knowing seeks the truth of being beyond, what is  immediately given, it attempts to penetrate being by going behind it to its determining source. It is understood that underlying being is something else on which it depends.

[232] Winfield, Richard Dien, 2012, Hegel's Science of Logic: A Critical Rethinking in Thirty Lectures, Rowman & Littlefield Publishers, Inc., p.148

Cognition here presumes that being has a ground, that it has a foundation, that there is a privileged essence mediating what appears to be. Essence is here treated as what is ultimate, and if essence is what is ultimate, being is just an appearance of essence, something whose presence is really not immediate, because it is grounded on something else. Being's immediacy is illusory.[233]

It looks like Hegel's emphasis on the leap and quality (that breaks in) *per saltum* is considerd with our human ability to percept and observe easy qualitative difference of something apparent, that lack "smallness" – large enough to be noticed, something visible and observable. It seems that qualitative quantity is ignored because of its notion of gradualness and lack of ability to leap as quality *per saltum.* Emphasizing the transformation of quality to quality by leaps "per saltum" in the "nodal Line of Measure Relations," in § 777 Hegel defines *"the attempt to explain coming-to-be or ceasing-to-be on the basis of gradualness of the alteration" as "tedious like any tautology."*

Hiding the qualitative quantity behind the curtains of "**tautology,**" (Hegel, G.W.F. 1812, 1813, 1816. tr. /1969. Miller*,* A. V., foreword by Findlay J. N.) in his liturgy of quality *per saltum*, Hegel is "misleading" the philosophers following him, especially the dialectical materialism of Engels and Marx, who created the cliché of the dialectical law of transition of quantity into quality and *visa versa*. Dialectical materialism is dialectic *per saltum,* dialectic concerned only with…and *by a leap, by fits and starts.* The concern of dialectics per saltum is the leap and revolution and there is not place for the intermediation of the "Qualitative quantity."

Peter Caws in his study Yorick's World: Science and the knowing subject (1993), provides an intriguing discussion on the dialectical law of the passage of quantity into quality, emphasizing indeed on these small changes that are 'not yet perceptible only because of its smallness' (Hegel). The subchapter of the book is entitled by Caws 'Notable and Just Noticeable Differences'. (Caws, P. 1993:248-249) Caws employ the notion of 'concomitant change', after Mill's phrase "concomitant variation".[234]

[233] Winfield, Richard Dien, 2012, Hegel's Science of Logic: A Critical Rethinking in Thirty Lectures, Rowman & Littlefield Publishers, Inc., p.148

[234] *Early 17th century*: from late Latin concomitant- 'accompanying', from concomitari, from con- 'together with' + comitari, from Latin comes 'companion'.

For Caws “the idea of concomitant change ("concomitant variation," to use Mill's phrase) is basic to the scientific enterprise: we want to know, if we make some change in the world, what else will also change, so that we can achieve or avoid it.” (Caws, P. 1993:248-249). According to Caws, “changes can be large or small, dramatic or marginal and group sizes change by the addition or subtraction of members, other properties by augmentation or diminution, intensification or dilution, etc., or by outright metamorphosis, one property being replaced by another. Cumulative marginal changes, each of which is hardly noticed, may eventually result in states so altered that they require altogether different descriptions. But this phenomenon is context-dependent and works on both sides of the qualitative-quantitative boundary. If a large surface, a wall for example, has always been red, but suddenly overnight is painted yellow, the change is startlingly obvious, but if its red color is modified very slowly, through an imperceptible shift in the direction of orange and progressively through lighter and lighter shades, until finally the last trace of red has vanished and the wall is pure yellow, the fact that it has changed at all may dawn only slowly, and then only on an observant witness with a good memory (imagine the change stretched out over centuries, so that in any one witness's life it was just an orange wall). Psychologists speak of "jnd's" or "just noticeable differences" as a measure of the refinement of perception (similar to "resolving power" in optics), a threshold below which changes cannot be perceived, so that several subliminal moves may be possible before anything is noticed—and indeed if they are made at suitable intervals nothing may ever be noticed.” (Caws, P. 1993:248-249)

As Caws states “something very similar happens on the quantitative side if the sets in question are sufficiently large. If one person is in a room and another enters, the change is obvious enough, and similarly if a third joins a couple, but if forty people are watching a parade, let us say, the arrival of the forty-first may go entirely unremarked. Still if people keep coming, one by one, sooner or later we have a huge crowd, a demonstration, a triumph—and when exactly did this happen? “(Caws, P. 1993:249) Here Caws recalls the ancient paradox of The Heap, where a grain of wheat is set down, then another grain, and so on; eventually there is a heap, but which grain was it that turned a scattering of grain into a heap. Caws highlights the fact that “this paradox was presumably intended to remain paradoxical—no empirical research was done, as far as I know, to find out when impartial observers would start to use the term "heap" without prompting. (My guess is that four grains, in a tight tetrahedral array, would qualify as a very small heap, whereas if the procedure were to scatter randomly over a given area, say a square yard, there would be a range of many thousands of grains over which the status of the accumulation as a heap could be disputed.) The point the paradox makes is that categoreal boundaries, for example, between "scattering" and "heap," are fuzzy, but that surely comes as no surprise and hardly makes a very convincing foundation for philosophical doctrine, whether metaphysical or revolutionary." (Caws, P. 1993:249)

Most of the Zeno's paradoxes (the dichotomy paradoxes) exhibit the notion of gradualness of 'qualitative quantity' transformation dealing with counterintuitive aspects of continuous space and time.

There are cases, concludes Caws, in which "cumulative imperceptible changes in *x* lead to the emergence of *y* , and there are cases in which they just lead to more *x* —and either *x* or *y* can be indifferently qualitative or quantitative predicates; everything depends on the particular case, and can only be learned by looking. Adding atom after atom to a lump of uranium 235 eventually produces an atomic explosion and an assortment of vaporized fission products; adding atom after atom to a lump of gold just produces a bigger lump of gold. Water when refrigerated changes into ice; iron when refrigerated gets colder but doesn't change into another form." (Caws, P. 1993:250)

Caws is radical in his conclusion that "the dialectical law of the passage of quantity into quality, like its companions, the law of the interpenetration of opposites and the law of the negation of the negation, is thus seen to be an entertaining but nonessential red herring." (Caws, P. 1993:250)

In contemporary philosophy and science thinking about the notion of gradualness and the significance of the 'small changes', which are 'not yet perceptible only because of its smallness' (Hegel), thinking about the 'notable' and 'just noticeable' differences, one shall recal the Lorenz and Poincare's discovery that small changes in initial conditions of the system can lead to larger changes (classic example of The Chaos Theory).

From his reading of Hegel, Frederick Engels elucidated in in his work, commonly known as the "Anti-Duhring" and "Dialectic of Nature" the three laws of dialectics utilized by Marx.

The second law of dialectics, the law of transformation and the passage of quantitative changes into qualitative changes is based on Hegel's discussion of "nodal lines" in nature, where the nodes are qualitative shifts accomplished by the incremental quantitative changes. Such a shifts are  seen as "a sudden revulsion of quantity into quality", based on Hegel's example about "the qualitatively different states of aggregation water exhibits under increase or diminution of temperature." (G. W. F. Hegel, *Hegel's Logic, Being Part I of the Encyclopedia of the Philosophical Sciences ([1830] translated by William Wallace)* (Oxford: Clarendon Press, 1975), 160.

Engels cites this as "one of the best-known examples—that of the change of the state of water, which under normal atmospheric pressure changes at 0°C from the liquid into the solid state, and at 100°C from the liquid into the gaseous state, so that at both these turning-points the merely quantitative change of temperature brings about a qualitative change in the condition of the water." (Engels, F. 1939:138)

Peter Caws in his study *Yorick's World: Science and the knowing subject* (1993) and *Reform and Revolution* (1972) provide stong arguments challenging the explanation and example of Engels. (Caws, P. 1993:246-247) (Caws, P. 1972:78)

Caws state that the example with boiling/freezing water is 'highly misleading' and 'it gives the impression that temperature is a property of water that is causally related to its state: change the (quantitative) temperature, and the (qualitative) state will change." According to Caws, "the fact is that at the boiling and freezing points the temperature *can't* be changed *until* the state has changed.What happens is this (I will take the case of boiling, which applies *mutatis mutandis* to freezing also): steadily supplying enough heat energy to water will raise its temperature to 100°C; at this point supplying further energy will not change the temperature but will dissociate the molecules from one another so that they become steam at 100°C; when all the water has been changed to steam then, assuming a closed system, the supply of still further energy will raise the temperature of the steam above 100°C. But if the process begins at room temperature it will take about seven times as long to change all the water into steam as it took to raise the water to the boiling point." (Caws, P. 1993:246-247)

Caws conclude that “there are two things wrong with the Hegel-Engels account: first, it isn't changing the temperature that changes the state, and second, the change is not sudden. As I have pointed out elsewhere, (Caws, P. 1972:78) when water boils because it is heated from the bottom, the change of a small amount of it into steam makes dramatic bubbles, and this is not a bad analogy for *repressed* change, which was one of the popular senses in which the dialectical principle of quantity and quality came to be understood: history will accumulate exploitation and repression incrementally, until crisis and revolution suddenly ensue. And this may indeed happen—only the quantity/quality distinction has nothing to do with it. Water froze and boiled long before temperatures were thought of, and when we talk about "the boiling point" and attach a number to it (note by the way that it is impossible to *measure* the boiling point at standard atmospheric pressure in degrees Celsius, since 100°C is *defined* as the boiling point of water at standard atmospheric pressure), the number by itself does not refer to anything that is true of the water, but (as before) only to the cardinality of a collection of units.” (Caws, P. 1993:247)

Caws goos on asserting that “One of the remarkable and useful features of the exact sciences is that quantities can be measured and the measurements plugged into computations. The qualities whose degrees are attended to in the process of measurement (or predicted by the outcome of the computation) are sometimes thought to enter into the computations. Thus in the most elementary case of a freely falling body initially at rest we have the equation:

$s = \frac{1}{2} g t^2$ which means "the distance fallen is equal to half the acceleration of gravity multiplied by the square of the time elapsed." But a moment's thought will show that this can't possibly be what is meant: times can't be squared; only numbers can. Nothing can be multiplied by an acceleration. The expression is only a shorthand way of saying that measurements of the distance, the acceleration, and the time, using compatible units, will yield numbers that stand in the required arithmetical relation. In the algebraic expression given above *s* isn't a distance at all, it's a variable that can take numerical values, and so for the other elements.” (Caws, P. 1993:247-248)

Engels’s law of transformation and the passage of quantitative changes into qualitative changes, as all of the three laws of Engels’s dialectics become cliché in the mode of thinking of quality and quantity. The three laws of dialectics are not only oversimplified, but also misleading at best, establishing something quite self-evident, trivial and common. Gradualness and gradual changes are assumed as not leading to turning points, where one force overcomes the other and quantitative change appears in the new measure. Due to this assumption, the notion of Qualitative quantity in Hegel's dialectics, remained inapparent.

### 1.3. Qualitative quantity: D'Arcy W. Thompson's *On Growth and Form* and Hegel's gradualness, and "*the attempt to explain coming-to-be or ceasing-to-be on the basis of gradualness of the alteration*"

"Matter as such produces nothing, changes nothing, does nothing…[it] can never act as matter alone, but only as seats of energy and as centres of force."

(D'Arcy W. Thompson, *On Growth and Form,* 1942: 14).

My recollection of the Topological Notion of Qualitative quantity in Hegel, goes back to the year 1989, while working on my thesis (Dimitrov, B. 1989a) in my first two articles, (Dimitrov, B. 1989), (Dimitrov, B. 1990), where I argued that topology is the field of Hegel's qualitative quantity and topological homeomorphism is the exhibit form of this category.

This claim I have based on exploration of D'Arcy W. Thompson's work "Growth and Form" (1917) and some findings and examples, extracted from Hermann Haken. (Haken, H. 1983). Illustrating the qualitative quantity notion with the concept of structural stability, I have suggested that topology is the field of qualitative quantity and topological homeomorphism is the exhibit form of this category.

The concept of structural stability is related with the topological homeomorphism, thus topological homeomorphism was proposed as exhibit form of the category qualitative quantity.

In his book "Synergetics: Introduction and Advanced Topics", 41 in the Chapter 1.13. "Qualitative Changes: General approach", Hermann Haken explores and illustrate the structural stability with an example /figure 1.13, p.434 in Haken/ given by the Scottish biologist, mathematician and classics scholar D'Arcy W. Thompson, the author of the book, On Growth and Form, /1917/. My assertion is that Hegel's category of qualitative quantity is illustrated with Herman Haken's citation of D'Arcy W. Thompson. Exploring the invariance in deformation and transformation of the forms against spatial or temporal deformation, Haken wrote:

"Figure 1.13, p.434 /"Synergetics: Introduction and Advanced Topics"/ shows two different kind of fish, namely, porcupine fish and sun fish. According to the studies by D'Arcy W. Thompson of the beginning of the twentieth century, the two kinds of fish can be transformed into each other by a simple grid transformation. While from the biological point of view such a grid transformation is a highly interesting phenomenon, from the mathematical point of view, we are dealing here with an example of structural stability. In a mathematician's interpretation the two kinds of fish are the same. They are just deformed copies of each other. A fin is transformed into a fin, an eye into an eye and etc. In other words, no new qualitative features such as a new fin, occur. In the following we shall have structural changes /in the widest sense of word/ in the mind." (Haken, H. 1983)

Under the illustration set in Figure 1.13, p.434 /"Synergetics: Introduction and Advanced Topics"/, Haken wrote – "the porcupine fish and the sun fish can be transformed into each other by a simple grid transformation. After D'Arcy W. Thompson: On Growth and the Form, ed. By J.T. Bonner, University Press, Cambridge, 1981/." (Haken, H. 1983)

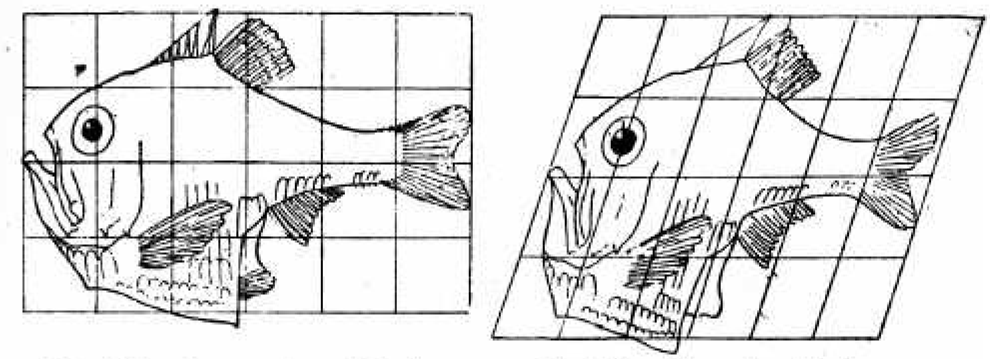

**Fig. 517.** *Argyropelecus Olfersi.* **Fig. 518.** *Sternoptyx diaphana.*

The original Thompson's illustration of the transformation of the fish Argyropelecus olfersi into the fish Sternoptyx diaphana by applying a 70° shear mapping.

Hermann Haken's example we are illustrating here with the original Thompson's illustration of the transformation of the fish Argyropelecus olfersi into the fish Sternoptyx diaphana by applying a 70° shear mapping. The reverse transformation is possible simply with manipulating the grid and shear mapping.

The example illustrated this transformation actually is a good example of homeomorphism. Two objects are homeomorphic if they can be transformed /or deformed/ into each other by a continuous inverible mapping, continuous one-to-one and having continuous inverse. The two fish are two objects with the same topological properties. They are said to be homeomorphic. There are properties that are not destroyed by stretching and distorting an object.

After the distance of these twenty five years, since my first exploration on Hegel's qualitative quantity and Hermann Haken/D'Arcy Thompson's thesis, I could see the growing relevance of D'Arcy Wentworth Thompson, especially in support of my research on qualitative quantity.

Under the arch of the "inapparent" nature and notion of Hegel's "qualitative quantity" and accusation of "tautology", labeled by Hegel himself for "the attempt to explain coming-to-be or ceasing-to-be on the basis of gradualness of the alteration", stand the figure of the "father of the gradualness" - D'Arcy Wentworth Thompson with his masterpiece "On Growth and Form". This book is called by Stephen Jay Gould "the greatest piece of scientific writing of the twentieth century"[235] (Gould, 1971) and G.E. Hutchison regarded as "one of the very few books on a scientific matter written in this century which will, one may be confident, last as long as our too fragile culture." The year of 2010 marked 150 years since the birth of D'Arcy Thompson /108/ and I may see Thompson's book all through these years, marching on what Alain Badiou called, the "fragile verbal footbridge" of Hegel's Logic, waking up the "Hegelian halutination" (Badiou's own accusation of Hegel) with D'Arcy Thompson's famous – "Everything is the way it is because it got that way."

Let we see again the passage of § 777 of Hegel's "The Science of Logic" /The Greater Logic/, and then see what D'Arcy Thompson see.

[235] Gould, Stephen Jay. (1971), D' Arcy Thompson and the Science of Form, New Literary History Vol. 2, No. 2, Form and Its Alternatives (Winter, 1971), pp. 229-258, Published by The John Hopkins University Press

Hegel states that “In thinking about the gradualness of the coming-to-be of something, it is ordinarily assumed that what comes to be is already sensibly or *actually in existence*; **it is not yet perceptible only because of its smallness.** Similarly with the gradual disappearance of something, the *non-being* or other which takes its place is likewise assumed to be *really there*, **only not observable**, and *there*, too, not in the sense of being implicitly or ideally contained in the first something, but *really there*, **only not observable**.” /§ 777/ - /*bold by me B.D*/.

A brief sample from D’Arcy Thompson’s “*On Growth and Form”* (Thompson, D.W., 1917) from Chapter II, "Of Magnitude") may be in order:

“We are accustomed to think of magnitude as a purely relative matter. We call a thing *big* or *little* with reference to what it is wont to be, as when we speak of a small elephant or a large rat; and we are apt accordingly to suppose that size makes no other or more essential difference, and that Lilliput and Brobdingnag are all alike, according as we look at them through one end of the glass or the other. Gulliver himself decalred, in Brobdingnag, that 'undoubtedly philosophers are in the right when they tell us that nothing is great and little otherwise than by comparison': and Oliver Heaviside used to say, in like manner, that there is no absolute scale of size in the Universe, for it is boundless towards the great and also boundless towards the small. It is of the essence of the Newtonian philosophy that we should be able to extend our concepts and deductions from the one extreme of magnitude to the other; and Sir John Herschel said that 'the student must lay his account to finding the disction of great and little altogether annihilated in nature.” (Thompson, D.W., 1917)

And

“All this is true of *number,* and of *relative magnitude.* The Universe has its endless gamut of great and small, of near and far, of many and few. Nevertheless, in physical science the scale of absolute magnitude becomes a very real and important thing; and a new and deeper interest arises out of the changing ratio of dimensions when we come to consider the inevitable changes of physical relations with which it is bound up. The effect of *scale* depends not on a thing in itself, but in relation to its whole environment or milieu; it is in conformity with the thing's 'place in Nature', its field of action and reaction in the Universe. Everywhere Nature works true to scale, and everything has its proper size accordingly. Men and trees, birds and fishes, stars and star-systems, have their appropriate dimensions, and their more or less narrow range of absolute magnitudes. The scale of human observation and experience lies within the narrow bounds of inches, feet or miles, all measured in terms drawn from our own selves or our own doings. Scales which include light-years, parsecs, Angström units, or atomic and sub-atomic magnitudes, belong to other orders of things and other principles of cognition.

A common effect of scale is due to the fact that, of the physical forces, some act either directly at the surface of a body, or otherwise in proportion to its surface or area; while others, and above all gravity, act on all particles, internal and external alike, and exert a force which is proportional to the mass, and so usually to the volume of the body.

A simple case is that of two similar weights hung by two similar wires. The forces exerted by the weights are proportional to their masses, and these to their volumes, and so to the cubes of the several linear dimensions, including the diameters of the wires. But the areas of cross-section of the wires are as the squares of the said linear dimensions; therefore the stresses in the wires per unit area are not identical, but increase in the ratio of the linear dimensions, and the larger the structure the more severe the strain becomes: $\frac{\text{Force}}{\text{Area}} \propto \frac{l^3}{l^2} \propto l$ and the less the wires are capable of supporting it.

In short, it often happens that of the forces in action in a system some vary as one power and some as another, of the masses, distances or other magnitudes involved; the 'dimensions' remain the same in our equations of equilibrium, but the relative values alter with the scale. This is known as the 'Principle of Similitude', or of dynamical similarity, and it and its consequences are of great importance. In handful of matter cohesion, capillarity, chemical affinity, electric charge are all potent; across the solar system gravitation rules supreme; in the mysterious region of the nebulae, it may haply be that gravitation grows negligible again.” (Thompson, D.W., 1917)

D'Arcy W. Thompson’s “On Growth and Form” has had a profound influence on present day science, art, architecture, engineering and anthropology. D'Arcy Wentworth Thompson was an inspiring visual thinker not just for scientists but also for artists. Major artists and thinkers from Henry Moore, Le Corbusier and Jackson Pollock to Claude Levi-Strauss, Alan Turing and Stephen Jay Gould have drawn on his work. The book's fascinating diagrams have become icons of visual thinking.

Topological presence in D'Arcy W. Thompson's science, in particular the discussed above transformation sprigs perhaps from the mutual influence shared between Thompson and Dorothy Maud Wrinch, mathematician, topologist and biochemical theorist best known for her attempt to deduce protein structure (protein folding) using mathematical principles. Wrinch had been greatly influenced by the ideas of D'Arcy Thompson.

Deeply inspired by D'Arcy Thompson's ideas on form, Wrinch capitalized on topological considerations. She proposed during the mid-1930s a honeycomb-like cage structure, a cyclol, for native globular proteins. That the cyclol consisted of 288 amino acid residues - and thus supposedly offered yet another independent source of evidence for the Svedberg and Bergmann-Niemann units - only served to enhance the 'hypnotic power of numerology."[236] (Kay, 1993)

Between 1918 and 1932 Wrinch published 20 papers on pure and applied mathematics and 16 on scientific methodology and on the philosophy of science. Bertrand Russell had a strong influence on Wrinch's philosophical work. In 1932 Wrinch was one of founders of the Biotheoretical Gathering, an inter-disciplinary group that sought to explain life by discovering how proteins work. Wrinch worked together with Joseph Needham, C.H. Waddington, J.D Bernal.

D'Arcy Wentworth Thompson pioneered the science of biomathematics. His book *On Growth and Form* is regarded as the first biomathematics treatise that has ever been written. In particular this is the chapter in which Thompson describes how differences in the forms of related animals can be formalized by means of simple mathematical transformations.

[236] Kay, Lily E. (1993), The Molecular Vision of Life: Caltech, The Rockefeller Foundation and the Rise of the New Biology (New York: Oxford University Press). 1993.

As  Brian D. Sleeman puts on, “The torch lit by D’Arcy Thompson has been taken up, in the last quarter of the twentieth century, by a growing band of mathematicians and theoreticians to the extent that mathematical or theoretical biology is well recognised as an important discipline in many undergraduate and graduate schools in universities and colleges.” [237] (Sleeman, 2003)

D’Arcy W. Thompson investigations of pattern formation in nature let to the introduction of the mathematical models for reaction-diffusion systems by A.M. Turing in 1936. Turing worked from 1952 until his death in 1954 on mathematical biology, specifically morphogenesis. He published one paper on the subject called *The Chemical Basis of Morphogenesis* in 1952, putting forth the Turing hypothesis of pattern formation. Alan Turing’s article *The Chemical Basis of Morphogenesis* [238] (Turing, 1952) is the second seminal work on biomathematics after Thompson’s *On Growth and Form.* In it Turing explores biological pattern formation by means of systems of reaction-diffusion equations. The *Turing instability* along with modelling in terms of reaction-diffusion processes are nowadays considered as one of the most important mathematical tools in the field of theoretical morphogenesis.

The biological tradition of D’Arcy W. Thompson and Alan Turing goes back to the Empedocles and Democritus, and their followers such as Lucretius, who rebuked intentional, functional, and teleological biological explanations. Growth and biological form, insofar as they can be scientifically characterized, arise through physical and structural necessity and chance, leaving talk of proper function, purpose, goal, and design aside. Both Thompson and Turing regard teleology, evolutionary phylogeny, natural selection, and history to be irrelevant distractions from fundamental biological explanation. Turing, rather, endorses D’Arcy Wentworth Thompson’s view that the teleological Aevolutionary explanations endemic to Darwinian Adaptationist’s biology are non-fundamental, fragile, misdirected, and at best mildly heuristic (Thompson, 1917).

[237] Brian D. Sleeman’s review on “Mathematics in Nature: Modeling Patterns in the Natural World”, John A. Adam, Princeton University Press, 2003, Reviewed by Brian D. Sleeman

[238] Turing, Alan M. (1952), The chemical basis for morphogenesis, Philos. Trans. Roy. Soc. London Ser. B 237 (1952), 37–72.

Justin Leiber examines the fragility of evolutionary explanations addressing Turing after Thompson in his paper “Turing and the fragility and insubstantiality of evolutionary explanations: a puzzle about the unity of Alan Turing’s work with some larger implication”. According to Leiber, in his foundational work "On the chemical basis of morphogenesis” - “Turing insisted that biological explanation clearly confine itself to purely physical and chemical means, eschewing vitalist and teleological talk entirely and hewing to D'Arcy Thompson's line that "evolutionary `explanations, "' are historical and narrative in character, employing the same intentional and teleological vocabulary we use in doing human history, and hence, while perhaps on occasion of heuristic value, are not part of biology as a natural science.”[239] (Leiber, 2001)

As Turing wrote: “Unless we adopt a vitalistic and teleological conception of living organisms, or make extensive use of the plea that there are important physical laws as yet undiscovered relating to the activities of organic molecules, we must envisage a living organism as a special kind of system to which the general laws of physics and chemistry apply. And because of the prevalence of homologies of organization, we may well suppose, as D’Arcy Thompson has done, that certain physical processes are of very general occurrence [these are the general properties of organic systems to which Chomsky refers; Turing follows with a specific instance]... What is novel in the theory is the demonstration that, under suitable conditions, many diffusion reaction systems will eventually give rise to stationary waves; in fact to a patterned distribution of metabolites.” [240] (Turing and Wardlaw, 1953/1992)

After D’Arcy Thomson, Alan Turing is next who walks on the “fragile verbal footbridge” (Alain Badiou expression about Hegel’ logic), as we may see in Justin Leiber’s “Turing and the fragility and insubstantiality of evolutionary explanations: a puzzle about the unity of Alan Turing’s work with some larger implication”. (Leiber, 2001)

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239 Leiber, Justin. (2001), Turing and the fragility and insubstantiality of evolutionary explanations: a puzzle about the unity of Alan Turing’s work with some larger implication, Philosophical Psychology, Vol.14, NO.1, 2001

240 Quoting from Turing, A..M., Saunders, P. T. Ed., (1953/1992), Collected works of A. M. Turing: Morphogenesis. Amsterdam: North-Holland. - See: Turing and Wardlaw, C. W. (1953/1992, p. 45 - TURING, A. (1952/1992). On the Chemical Basis of Morphogenesis. *Philosophical Transactions of the Royal Society of London*, Series B, 237, 37-72.

Hegelian “fragile verbal footbridge” comply with the fragility and insubstantial nature of evolutionary explanations. As we can see:

“the fragility and insubstantial nature of evolutionary explanations. received another ample demonstration recently. For several decades the eye has been evolutionary biologists' stock example of analogical development, a device so nifty that nature supposedly has "re-invented" it several times, for example, in mollusks, insects, and vertebrates if not, as some enthusiasts would have it, scores of times. Recently, molecular biologists showed that the same *Pax-6* DNA makes eyes in squids, fruit flies, and mice, so economical and homological nature apparently "invented" eyes only once.” (Leiber, 2001)

Based on these substantial differences in morphology and mode development the biologist Ernst Mayr has argued that different types of eyes evolved as many as forty times independently in the animal kingdom. Because the evolution of the prototype eye, at a stage before selection can exert its effect, must be a rare event, the independent evolution of so many prototypes represents a serious problem that is difficult to reconcile with Darwin's theory. ... [Our] findings lead to the further conclusions that the prototypic eye may have originated only once, rather than some forty times, and that the large variety of eye types found in the animal kingdom is derived from this prototype by divergent, parallel, and convergent evolution. [241] (Tomarev, S., Callaerts, P., Kos, L., Zinovieva, R., Halder, G., Gehring, H. & Piatigorsky, J., 1997).

D‘ArcyThompson’s transformations are remarkable illustration of topological relationship within the notion of Hegel’s “qualitative quantity”. As Arthur Stepanov puts in “Recursion in Natural Language: A Biolinguistic Approach” - “of particular significance to computational systems such as language are the work of D‘Arcy Thompson and Alan Turing on form and growth/morphogenesis. Natural selection can only function withina „channel“ of options afforded by natural law, including properties of complex systems.” And “D‘ArcyThompson‘s method of transformations illustrates the topological relationships between certain forms and

[241] Tomarev, S., Callaerts, P., Kos, L., Zinovieva, R., Halder, G., Gehring, H. & Piatigorsky, J., (1997). Squid Pax-6 and eye development. Proceedings of the National Academy of Sciences, 94,2421-2426.

how certain differences in structure may be accounted for by a single principle, usually changes in growth gradients during ontogeny."[242] (Gehring, W.J., 1998)

Rene Thom developed and used topology and singularity theory to investigate problems in developmental biology[243] (Thom, 2001) He took the approach of deriving and analyzing canonical mathematical models that capture certain fundamental aspects of developmental phenomena, which in return clarified understanding of the underlying biological processes.

D'Arcy W. Thompson's transformations and investigations of pattern formation in nature let to the introduction of Turing's mathematical models for reaction-diffusion systems and Rene Thom's use of topology and singularity theory to investigate problems in developmental biology. D'Arcy W. Thompson' theory of transformation is a scientific theory which impacted the many of the next and recently emerged scientific theories.

I will review few examples proving the relevance of the model introduced in D'Arcy W. Thompson theory and illustrating my claim that D'Arcy W. Thompson' model of transformation should be regarded as exhibit form of Hegel's "qualitative quantity" existing in the nature. I will apply in my approach here the concept of Patrick Suppes[244] introduced in his paper "Representation and Invariance of Scientific Structures", 2002.[245] (Suppes, 1988) The reason for this is that Suppes's concept is very suitable as application to D'Arcy W. Thompson's "law of transformation", alometry and isomorphism as well as to my claim that homeomorphism is the exhibit form of qualitative quantity.

For Patrick Suppes, analysis of structure of scientific theories is a central topic of philosophy of science. Suppes advocates the thesis that the analysis of the structure of a theory should be done in term of models of theory.

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[242] Gehring, 1998, pp. 204-209ff - GEHRING, W.J. (1998). *Master control genes in developmental evolution.* New Haven and London: Yale University Press.)" quoted by Justin Leiber, Turing and the fragility …

[243] R. Thom (2001), Structural Stability and Morphogenesis, Westview Press; New Ed R. Thom (2001).

[244] Patrick Colonel Suppes, the Lucie Stern Professor of Philosophy Emeritus at the Stanford University School of Education, is American philosopher who has made significant contributions to philosophy of science, the theory of measurement, the foundations of quantum mechanics, decision theory, psychology, and educational technology.

[245] Suppes, Patrick Colonel (1988). "Representation theory and the analysis of structure." Philosophia Naturalis 25: 254-268.

In his paper “Representation and Invariance of Scientific Structures”, Suppes applies the concept of isomorphism and homeomorphism of models of a theory. Definition is representation and according to Suppes, “the great successful example of representation by definition is the defining of all standard mathematical concepts within the set theory, beginning with just the single primitive concept of set membership.” (Suppes, 1988:51).

In the chapter “3.2. Isomorphism of models” from his paper, Suppes claims that “one of the most general and useful set – theoretical notions that may be applied to a theory is the concept of two models or structure of a theory being isomorphic.” (Suppes, 1988:51)

For Suppes, “Roughly speaking, two models of a theory are isomorphic when they exhibit the same structure from the standpoint of the casic concepts of the theory. The point of the formal definition of isomorphism for a particular theory is to make this notion of same structure precise.” (Suppes, 1988:51).

Suppes examines also the “homeomorphism of models” and the “embedding of models”, concluding that “we have seen that the notion of two models being homeomorphic is generalization of the notion of two models being isomorphic.” (Suppes, 1988:51).

D’Arcy W. Thompson investigations of pattern formation in nature and his grid transformation are itself probably one of the best illustration of Patrick Suppes concept.

Both the “representation” and “invariance” are of critical importance to D’Arcy Thompson’s “law of transformations”, as well as to the notion of qualitative quantity. Some brilliant illustration of the notion of qualitative quantity and the relevance of D’Arcy could be seen in the pattern theory, formulated by Ulf Grenander. [246] (Grenander and Miller, 2007)

[246] Grenander, Ulf; Miller, Michael (2007). Pattern Theory: From Representation to Inference. Oxford University Press.

**Hegel's speculative thinking and development of categories and concepts, demonstrates in the concept of regularity as invariance of topological transformation in our contemporary context.**

Today, the concept of regularity as invariance of transformation constitutes the core of the mathematical apparatus of pattern theory. In the work of Yuri Tarnopolsky and Ulf Grenander, History as Points and Lines (Tarnopolsky, Y and Grenander, U, 1989/2003), we could see how different (from Engels's interpretation) is the dialectics of qualitative and quantitative, indeed much closer to Hegel's. Discussing and illustrating the topological homeomorphism, the conclusion of Tarnopolsky and Grenander is that "we can regard stagnation, equilibrium, evolution, and revolution as **something that does not change during transformation from one society to another.** This concept of regularity as invariance of transformation constitutes the core of the mathematical apparatus of pattern theory and establishes a close relation between pattern theory and geometry." (Tarnopolsky, Y and Grenander, U, 1989/2003: 100-101) This "**something" is indeed Hegel's qualitative quantity – "that does not change during transformation from one society to another."**

In Tarnopolsky and Grenander's "History as Points and Lines" the main emphasis is put on the topology:

"The relation of being neighbors, or, more generally, being in a relation, is the essential subject of the mathematical discipline called topology. This term has two meanings: the name of the discipline and a property of a set of objects. Topology as science takes a set of objects, for example, points on a surface, and for every pair decides whether they are neighbors or not. Among other problems, the science of topology studies which points remain close when the surface is deformed, i.e., whether topology as property is preserved. Thus, when we have an orange made of modeling clay and make a cucumber out of it, topology is preserved, but if we make a donut, formerly close neighbors become separated. Taking another example, basketball and American football have different shape but the same topology, while bagel and muffin are worlds apart, although they are both as American as apple pie." (Tarnopolsky, Y and Grenander, U, 1989/2003: 95-96)

Also

“We can stretch and twist the rubber band, but this does not change its topology: any two infinitely close points will remain close. If we cut it, the relation of closeness between some points will be lost, unless we glue them back. In political terms, topology is preserved in a transformation without revolutionary scissors and authoritarian glue.

Graph is an example of a topological set because it is a binary relation between every two nodes that are either connected or not at all. The arc in a graph symbolizes exactly the relationship of neighborhood or vicinity: each node is the neighbor of all other nodes connected to it with their arcs.

Triangle also has a topology, and it is preserved when we deform it. However, Euclidean geometry, unlike topology, takes to account angles and distances, and if those are changed, the new triangle is different from the old one. What is of interest for us here is the common property of all triangles, their pattern of “triangularity” or “triangleness,” preserved under all transformations.” (Tarnopolsky, Y and Grenander, U, 1989/2003: 95-96)

Tarnopolsky and Grenander provide an example illustrating how we can transform a triangle without violating it “triangleness.” We can move it in its plane (A), rotate it, and stretch it uniformly (C) or unevenly (D). All this does not change the property of being triangle.” (Tarnopolsky, Y and Grenander, U, 1989/2003: 100-101, Figure 6.4. Geometrical transformations of triangles). The second example (Tarnopolsky, Y and Grenander, U, 1989/2003: 100-101, Figure 7.2) is borrowed from a serious book on mathematics – Isaak Moiseevich Yaglom’s Felix Klein and Sophus Lie (1988), an investigation of the evolution of the idea of symmetry in the 19th century. The picture illustrates one-to-one correspondence which allows for establishing similarity between different sets. This figure shows two extreme types of transformation. Transformation in (a) changes one discrete set of figures into another one, so that there is one-to-one correspondence between the two sets. One example of this transformation is a remake of a classical play where characters of a Greek or Shakespearean drama are transferred into contemporary surroundings. As another example, we would mention didactic folktales and fables whose creators—from Aesop (620-560 BC) in Greece to anonymous Sanskrit authors of Panchatantra (approximately 300 BC to 400 AD) to La Fontaine (1621-1695) of France—transposed typical human collisions into situations among animals. Every epoch, up to our times, made the reverse transpositions of the immortal patterns.

The figures of set α have a difficult-to-define similarity to their counterparts in set α1 when only considered through visual associations between the pawn and the slouching man, the bishop and the woman in a skirt, the rook and the square outline of a jacketed man, and the king and symmetric wide sleeves of a woman in a robe. Transformation in (b) is continuous: it changes the distances and angles yet preserves the general shape in the same manner transformation in (a) preserves functional roles. To be more precise, it preserves the topology of the figure: two close points in item F are as close in item F1. A small violation of this rule is noticeable as an extra vein on the left side, and we do not know whether it was intentional or just the manifestation of a universal chaos." (Tarnopolsky, Y and Grenander, U, 1989/2003: 100-101)

Yuri Tarnopolsky and Ulf Grenander' conclusion is "Thus we can regard stagnation, equilibrium, evolution, and revolution as something that does not change during transformation from one society to another. This concept of regularity as invariance of transformation constitutes the core of the mathematical apparatus of pattern theory and establishes a close relation between pattern theory and geometry. Geometry is not merely lines, just as mathematics is not only numbers. In this sense, pattern theory is a geometry of everything." (Tarnopolsky, Y and Grenander, U, 1989/2003: 100-101)

The "inapparent" notion of the qualitative quantity could be illustrated with the figure P.2 named "Find a star" - from the book "History as Points and Lines" – "where the viewer is invited to find a star in the showed drawing. First, all we see are black and white polygons. Suddenly, we notice the star and after that we can never "unsee" it. (Tarnopolsky, Y and Grenander, U, 1989/2003: 100-101)

The domain of qualitative quantity is the core of topology – the topological property or topological invariant - a property of a topological space which is invariant under homeomorphism. A property of spaces is a topological property if whenever a space *X* possesses that property every space homeomorphic to *X* possesses that property. Informally, a topological property is a property of the space that can be expressed using open sets. Qualitative quantity's notion is the Hegel's gradualness and qualitative quantity could be regarded as topological quality or invariant quality. The concept of qualitative quantity could be seen as the core of "the concept of regularity as invariance of transformation", which "constitutes the core of the mathematical apparatus of pattern theory and establishes a close relation between pattern theory and geometry."

In chapter 8 (Groups) of Yuri Tarnopolsky and Ulf Grenander's "History as Points and Lines" (Tarnopolsky, Y and Grenander, U, 1989/2003:111-113), we could see the discussion reffered to the relevance of D'Arcy Thompson and what I will call the "topological D'Arcy":

"Deformations and symmetry operations with triangles point to another important field of mathematics, reaching far beyond geometrical figures.

The concept of deformation or, more general, transformation, or, even more general, operation, attracted a lot of mathematical attention and is the subject of group theory. The founder of group theory, Évariste Galois (1811-1832) tried in vain to publish his discovery. On the eve of a duel he wrote notes about his work and was killed next day at the age of twenty-one. One of the pioneers in exporting the ideas of group theory to the world outside classical mathematics was D'Arcy Thompson, author of the influential and widely quoted book On Growth and Form. He thus explained the concept of representing a property by a deformation preserving this property:

This process of comparison, of recognizing in one form a definite permutation or deformation of another, apart altogether from a precise and adequate understanding of the original "type" or standard of comparison, lies within the immediate province of mathematics, and finds its solution in the elementary use of a certain method of the mathematician. This method is the

Method of Coordinates, on which is based the Theory of Transformations . . . which is part of the Theory of Groups (Thompson, 1961)."

Figure 8.1 illustrates his idea. The shape of the scull of a primate is the property in question. We start with any primate and use the shape of its scull as a template, i.e., a selected starting point. We can obtain any other scull shape by deforming the template while preserving its topology. Moreover, as follows from the concept, we can infer the scull shapes of primates that have never existed and, by employing a template with a different topology, we could infer entirely new species and families. Concerning the soft tissues, such details as ears, eye sockets, etc., have their own groups of transformations, so that we can fantasize over the shape of an eye and overall appearance.

Scores of sci-fi movies and animations, for example Star Wars, old and new, show various aliens and creatures that look very much unlike our earth forms or their hybrids with inanimate things. If we look closer, however, all we can see is a deformation of the human form.

This is the only template available to us without the actual contact with other worlds. If an artist invents something completely out of this world, the viewer may not even recognize it as a form of life.

The computer-generated aliens undergo further transformations that make them turn and move around without losing the identity.

As far as the triangles are concerned, it means that we can define the property of "triangularity" through a description of a deformation or symmetry operation that preserves this property, for example, stretching in any direction.

Group theory is a basic component of mathematical apparatus of pattern theory. We could do without it in our treatment of history, but it would not be fair: humanities owe a considerable debt to group theory which inspired the entire movement of structuralism that swept large areas of humanities in most of the twentieth century." (Tarnopolsky, Y and Grenander, U, 1989/2003:111-113)

Tarnopolsky and Grenander concludes with the questions addressing the structuralist movement and Claude Lévi-Strauss's topology /the structure of kinship/:

"The structuralist movement posed the following question. Can we define persistent cultural and historical phenomena not only by their description but also by transforming, deforming, and translating them through times, spaces, and cultures?" And "The idea of structure borrowed from the group theory invaded anthropology, linguistic, sociology, and other humanities, and, as any aggressive invader, created a resistance movement in the competitive atmosphere of humanities". .. "That was the idea of structure as system of relationship that was borrowed by classical structuralism of Claude Lévi-Strauss from mathematics. Structure in structuralist sociology and anthropology consists of binary relations called oppositions. Structure is a combination of relations and it can be traced trough various cultures." (Tarnopolsky, Y and Grenander, U, 1989/2003:111-113)

Walking on the Hegel's "fragile verbal footbridge" of qualitative quantity, the guru of the pattern theory Ulf Grenander with his book "The Patterns in Growth" (2004)[247] follows the footsteps of D'Arcy Thompson. The first chapter of Grenander's book is entitled exactly "In the footsteps of d'Arcy Thompson". We read the following:

"In the year 1917 an event occurred that must have seemed like a watershed to the small community of biologists that believed that mathematics had the potential to become a useful tool in their discipline. In that year d'Arcy Wentworth Thompson published his pioneering work "Growth and Form" in which he attempted to explain biological (and other)

[247] Ulf Grenander, 2004, The Patterns in Growth", source *www.dam.brown.edu/ptg/REPORTS/GRID_TMI_07.pdf*

patterns by appealing to mathematical principles. In particular he argued, in the last chapter of the book, that transformations of coordinates could make clear how the resemblence of structures were expressed in a striking and often convincing manner.

He based his arguments on the dictum that ”the search for differences or fundamental contrasts between the phenomena of organic and inorganic, of animate and inanimate things, has occupied many mens minds, while the search for commonality of principle or essential similarities, has been pursued by few; the contrasts are apt to loom too large, great though they be” and that ”in general, no organic forms exist, save as are in conformity with physical and mathematical laws”. (Grenander, 2004:1)

Thompson's ”Growth and Form” was received with enthusiasm, at least among the openminded public, but, surprisingly, it was admired by many but emulated by few. Although his way of thinking can be traced back to antiquity, for example to Aristotle, and while related ideas were pursued after his death by other scholars, it is only recently that they have been studied and extended systematically by mainstream biologists. A representative list of such attempts can be found in Bonner ed. (1982).

Among mathematicians one finds even fewer researchers following d'Arcy Thompson untill recently. The pathbreaking work by Bookstein beginning with his monograph appearing in 1978 and continuing forcefully in a huge stream of publications argued for using statistical ideas to elucidate problems of biological growth. The somewhat related work by Kendall ,see Kendall et al (1999) for an up to date presentation, was concentrated on the purely mathematical aspects but was also used in applications, among others archeology. Pattern theoory as presented in Grenander (1993) and earlier emphasized the compositional and  transformational properties of patterns of general form. This led to computational anatomy, see Grenander, Miller (1998) and in its most recent form in Beg, Miller, Trouve, Younes.” (Grenander, 2004:1)

There is significant impact of D'Arcy Thomson on many branches of sciences, including physiology, ecology, evolution, medicine and surgery, mathematical modeling. The mathematical and statistical modeling of biological growth is an important problem in medical diagnostics. In this area of research, Ulf Grenander, Anuj Srivastava and Sanjay Saini /134/ introduced a structural model called GRID from Growth by Random Interated Diffeomorphisms, that treats a cumulative growth deformation as a composition of several elementary deformations. The authors named the coordinate system that changed with anatomy – Darcyan Coordinate System – after the celebrated author **D'Arcy W. Thompson**. The authors proposed a mathematical framework for modeling growth of biological objects, such as anatomical parts, with a focus on locally active growth. This paper introduced a mathematical representation of deformation /resulting from anatomical growth/ that seems natural for growth analysis.[248] (Grenander, Srivastava, Saini, 2007)

As Grenander remarked, among mathematicians and researchers following d'Arcy Thompson is Fred L. Bookstein.[249] The pathbreaking work by Fred Bookstein and Ulf Grenander that build on D'Arcy Thompson, is an brilliant illustration of qualitative quantity the implementation.

In his "Foundation of Morphometrics" /136/ Fred L. Bookstein states: "Whether broadly or narrowly construed, morphometrics clearly has something to do with the assignment of quantities to biological shapes." (Bookstein, 1982:451:470)

"In most fields, the advent of quantification is followed a few years later by a systematization of the explanatory quantitative styles. At that time one encounters studies of the nature of

[248] Ulf Grenander, Anuj Srivastava, Sanjay Saini, A Pattern – Theoretic Characterization of Biological Growth, IEEE TRANSACTIONS ON MEDICAL IMAGING, VOL. 26, NO. 2, MAY 2007, 657-658 pp.

[249] Fred L. Bookstein is Professor of Morphometrics, Department of Anthropology, University of Vienna, Group Leader: Bioanthropology, Department of Anthropology. Professor of Statistics, University of Washington, USA, and Distinguished Research Professor emeritus, University of Michigan, USA. Biomathematician Fred Bookstein is among the small number of scholars seeking to operationalize and extend D'Arcy Thompson's construct. Bookstein has conducted a research program to reshape the branch of biometry known as morphometrics. Working predominantly in the area of human craniofacial growth, Bookstein has championed the development of his own extensions of Thompson's synthesis with dozens of mathematically complex and often contentious expositions since 1978.

information captured and discarded by the various conventions, general families of mathematical or statistical models mimicking relevant behaviors of the natural phenomena under study, and so forth: in short, the contemplation of foundations. In morphometrics this passage to introspection has not occurred. There is one classic in the field, **D'Arcy Thompson's "On Growth and Form",** which argues that form should be modelled as the expression of physical laws. This stance, now badly dated, has not been replaced by any other consistent point of view."[250] (Bookstein, 1982:451:470)

Fred L. Bookstein concludes that "(Otherwise), the morphometric literature is entirely application oriented rather than methododlogical." (Bookstein, 1982:451:470) The quality quantititative approach could be useful as bridge to this methodological lack.

Bookstein discussed morphometrics as biology and geometry:

"Morphometrics, in my view / Fred L. Bookstein/ is the empirical fusion of geometry with biology. It methods must explicitly take cognizance of two wholly distinct sources of information – geomethric location and biologic homology." (Bookstein, 1982:451:470)

Bookstein's definition of Homology is "a smooth correspondence that transforms any two related forms one onto another in accord with appropriate ontogenetic of phylogenetic criteria." …Morphometric quantifications arise from the interaction of these two sorts of information. Practitioners of morphometrics should be extracting infromation from the geometry of biologic shape for particular comparative purposes, such as the study of growth, abnormality, or taxonomic differences." (Bookstein, 1982:451:470)

"In this context of the interplay between geometric location and biologic homology, consider Thompson's method of Cartesian transformation - the fundamental construct of morphometrics." (Bookstein, 1982:451:470)

"In this strategy one mathematical object, a deformation, is used to represent explicitly the relation of a pair of forms. Although we may view these objects in pairs of sketches each

[250] Fred L. Bookstein, Foundation of Morphometrics, Annual Reviw of Ecology and Systematics, Vol.13, (1982), pp.451-470

displaying one form, in essence the method relates two coordinate systems upon the same figure /in the classic example, upon the deep-bodied fish, Mola/. One of the coordinate systems is the ordinary Cartesian one on the form – it is not ordinarily drawn, of course, the other wich *is* drawn, is the coordinate system for homologous points in the comparison form. By this devise the essentially biological transformation, about homology, is communicated most efficiently." (Bookstein, 1982:451:470)

Fred L. Bookstein's approach is based on the discussion - geometry without homology and homology without geometry. Morphometrics, for Bookstein is the extraction of information from the interplay between geometric coordinates and biological homology.

Bookstein proposed the so called "The Tensor Method". "The method grows out – he asserted – of my earlier technique of biorthagonal analysis, a quantification of D'Arcy Thompson's old method of Cartesian transformation. It was Thompson – wrote Bookstein – who realized that shape change was not to be measured by numerical differences among measures of shape separateelly." (Bookstein, 1982:451:470)

I would emphasize on the above quote – not quantity – measure, but quality of the quantity.

"Rather, shape change is a geometric object in its own rights, the deformation taking one form into the other in accord with biologic homology." …"Thompson suggested this object be depicited by its effect on a grid laid over one form; he always began with a Cartesian /square/ grids. The figures produced in this style are endlessly intiguing but do not directly lead to any effective quantification or feature extraction." (Bookstein, 1982:451:470)

And we should ask the question here - what is "effective quantification?" To my understanding "effective quantification" is quality of quantity-fication. Why Fred L. Bookstein suggests new approach to Thompson's original method, the so called "Tensor Method"?

Bookstein explains the Tensor Method as follow:

“The fundamental problem of Thompson’s original method is the selection of that starting grid, which leads to an asymmetry: A grid strictly square over one form is transformed into a curvilinear grid having in general no mathematical regularities. Bookstein suggests drawing instead a grid that has the same geometric properties in both forms: the grid along the pairs of directions that begin and end at 90 degrees to each other. One of these directions manifests the least dilation /specific rate of change of length between forms/ and the other the greatest of all directions locus by homologous locus.” …”In its original form, this technique applies to forms only in pairs. It executes its geometric computations in full spatial details, but it is unable to aggregate over sets of starting forms. It was the specific aim of the present method to lift this limitation, at the lowest cost in lost information so that a representative average transformation could be computed over diverse population and undergoing similar shape changes.” (Bookstein, 1982:451:470)

The inherited from Thompson, interplay of geometry and biology leads Fred L. Bookstein to the first systematic survey of morphometric methods for landmark data - “Morphometric Tools for Landmark Data: Geometry and Biology”.[251] (Bookstein, 1991)

Morphometrics is the statistical study of biological shape and shape change. Its richest data are landmarks, points, such as the bridge of the nose, that have biological names as well as geometrical limitation. The qualitative and quantitative interplay is evident in morphometrics due to the involvement of both statistical approach /quantitative/ and biological names /qualitative/. The geometrical location as an result of these two is an example of explicate study of qualitative quantity, thus morphometric could be re-assessed on the base of the concept of qualitative quantity. Morphometrics refers by definition to the quantitative analysis of form, a concept that encompasses size and shape, thus some qualitative characteristics are presented in this pure quantitative approach of morphometrics. The current lack or gap of qualitative approach in the study of morphometrics can be fulfilled with the qualitative quantity approach and research in the area. Morphometric study requires measurement of qualitative aspects of the shape /form/ and change. (Bookstein, 1991)

[251] Fred L. Bookstein, Morphometric Tools for Landmark Data: Geometry and Biology, Cambridge, England: Cambridge University Press, 1991.

Morph from Greek meaning of “form” and “metric” which means “measurement”. We are having here study concerned with the quality of shape, measuring the quality of the shape /form/. Morphometric as statistical science on the base of qualitative quantity concept could be implemented as visual statistics. According to Fred L. Bookstein – morphometric analyses are commonly performed on organisms and useful in analyzing their fossil records, the impact on mutants on shape, developmental changes in form, covariances between ecological factors and shape, as well for estimating quantitative – genetic parameters of shape. Morphometrics ca be used to quantify a trait if evolutionary significance, and by detecting changes in the shape, deduce something of their ontogenic function or evolutionary relationship. (Bookstein, 1991)

The qualitative quantity could be explored in the quantitative study of biological shape variation. In his “Biometrics, Biomathematics and the Morphometric synthesis”[252] (Bookstein, 1996), Fred L. Bookstein provides an excellent acount on the historical development of biometrics, biomathematics and morphometrics.

In 1980, the traditional analysis – the quantitative study of biological shape variation, changed from statistical application to the new approach combining techniques from the mathematical statistics with non-Euclidean geometry and computer graphics, all combined in a coherent new system of tools for the complete recognized quantitative analysis of landmar point together with the biometrical images in which they are seen. This new enhanced statistic was enhanced by the visual element – of biomedical images – thus visual statistics. This visual part corresponds with the qualitative element – the visual statistic and visual math are in the domain of qualitative quantity’ concept. Bookstein’s new approach and analysis in morphometric is qualitative quantity application. Bookstein is critical about the quantitative approach in the statistic methods of morphometric and emphasizes on the gap of measurment and qualitative approach in morphometric research and study :

“Over most of this century, techniques for quantitative geometric study of organic form have fallen under one of two incommensurate headings. In one style of analysis, deriving from the

[252] Fred L. Bookstein, “Biometrics, Biomathematics and the Morphometric synthesis …. The quantitative study of biological shape variation” Bulletin of Marhemancal Biology, Vol. 58, No. 2, pp. 313-365, 1996, Elsevier Science Inc., Society for Mathematical Biology.

biometrics of Karl Pearson and Sewall Wright, conventional multivariate techniques are applied to an undisciplined roster of quantifications of single forms. The only algebraic structures involved are those of multivariate statistics, limited mainly to partitions of sums-of-squares, diagonalizations of covariance matrices and solution of linear systems in which they are involved. No aspect of the geometric organization of the measures or their biological rationale is reflected in these statistical maneuvers. In particular, the geometry of the typical form plays no formal role in the analysis of variation." (Bookstein, 1996)

"In the other class of shape analyses, often associated with the name of D'Arcy Thompson, but actually dating from the discovery of perspective transformations half a millenium ago, changes of biological form are visualized directly as distortions of Cartesian coordinate systems that carry meaningful biological labels right along with the coordinate grid. Such analyses are inextricably graphical; generation after generation of devoted amateurs failed to produce any rigorous grammar for the quantitative apersus to which they sometimes lead." (Bookstein, 1996)

"These two analytic traditions were pursued separately right through the 1970s, yet in the middle 1980s, without any premonitory ferment, the barrier between them was very rapidly breached by an unprecedented combination of algebraic and geometric tactics.

The breakthrough began-as breakthroughs in quantitative science often do-when it was realized that previous attempts at a morphometric synthesis had been struggling toward the wrong goal. What constituted the appropriate subject matter for analysis of forms with labels was not, as I erroneously (if understandably) claimed in Bookstein (19781, the construction of a canonical coordinate system for D'Arcy Thompson's grids, which apply to only two forms at a time. Those grids were an artifact. Efforts at improving their quantification only distracted from the far more important task of constructing a canonical manifold for biomathematically salient aspects of shape across extended samples. In the tangent space of that manifold, any vector becomes a potential "shape process" that summarizes evidence from a sample. Conversely, any pairing of forms, as of individual specimens with their common average, becomes a descriptor that, after projection onto the tangent space, can be aggregated with other commensurate descriptors, examined for patterns and associated with explanations. Within this shape space, trends can be named and their statistical reifications can be assessed;

the reliable diagrams that obtain of actual effects on real shapes are often very conducive to biomathematical insights. To combine biomathematical with biometrical aspects of biological."[253] (Bookstein, 1996)

According to Bookstein, the necessity resolution is to combine biomathematical with biometrical aspects of biological shape studies. Biomathematics of shape change gives us the qualitative aspect – by the visualization of transformation grid. Bookstein: "To combine biomathematical with biometrical aspects of biological shape studies, it is first necessary to demarcate the boundary between them." (Bookstein, 1996)

In "Biometrics, Biomathematics and the Morphometric synthesis", 'section 3. Biomathematical Studies of Shape Transformation, 3.1. Historical background/, Bookstein wrote:

"What we borrow from the biomathematics of shape change is, of course, the visualization by transformation grid. Although this idea is usually associated with the famous treatise ***On Growthand Form*** by the British naturalist D'Arcy Thompson (19171, it is actually hundreds of years older than that. The first "transformation grids" reflect efforts of Renaissance artists to comprehend the variability of the human forms that they were just beginning to reproduce realistically. Figure 5, for instance, assembled from Albrecht Diirer's (1528) vier ***Biicher uon Menschlicher Proportion,*** explores diverse types of "transformation grid," both affine and localizable, in the effort to explore the limits of normal variation and the strategies of effective caricature. The semiotics is that of geometric perspective, but the information conveyed is wholly different: no longer the effect of a change of vantage-point, but a change of organism.

[253] See: Figure 5 and 6 in Fred L. Bookstein, "Biometrics, Biomathematics and the Morphometric synthesis …. The quantitative study of biological shape variation" Bulletin of Marhemancal Biology, Vol. 58, No. 2, pp. 313-365, 1996, Elsevier Science Inc., Society for Mathematical Biology.

Figure 5. One head with a standard grid and 11 transformations onto other German types. From Diirer (1528)
Figure 6. Cartesian transformation, Diodon to "Orthagoriscus." From Thompson, (1917).

In Thompson's own examples, including the famous exemplar reproduced here as Fig. 6, the Platonic thrust of homogeneity clearly dominates any concern for realism. Thompson's fond hope that these figures would reveal the origins of form in force (an assertion he meant literally) was never realized, and while several later generations of quantitative biologists were tempted by this graphical style, it proved never to lead to quantification in the global mode that Thompson had intended. As a consequence, this once-promising method underwent only "vicissitudes," not development, from its publication in 1917 to its supersession in the middle 1980s by the biometrical graphics of the synthesis. For a historical review, see Chapter 5 of Bookstein (1978).

This formal theme of shape transformation as the explicit object of biometric discussion was most clearly set forth in the famous Chapter XVII of Thompson (19171, “On the Theory of Transformations, or the Comparison of Related Forms.” Thompson’s goal is distinctly old-fashioned and much too Platonic to articulate with biometrics without severe modification:

[If] diverse and dissimilar [organisms] can be referred as a whole to identical functions of very different co-ordinate systems, this fact will of itself constitute a proof that variation has proceeded on definite and orderly lines, that a comprehensive ‘law of growth’ has pervaded the whole structure in its integrity, and that some more or less simple and recognisable system of forces has been in control. . . .” (Bookstein, 1996)

In my view – wrote Bookstein – the synthesis is a major methodological triumph, one to be realized by both statistics and mathematical biology, that has for the first time matched descriptive inferential technique, to a powerfull classical mode of qualitative biological intuition.” (Bookstein, 1996)

**Grid transformation is a method of converting coordinates from one datum to another by interpolation within a grid points. The transformation grid models the distortion between datums and contains corrections for latitude and longtitude.**

To my knowledge, Hegel’s dialectic of qualitative quantity was never before investigated through the D’Arcy Thompson’s shape (form) change by the grid transformations. Back in 1989s I was typing my paper “Quality of Quantity” in which I was exploring the relation between Hegel and D’Arcy Thomson’s ideas, on my old fashioned typewriter. Today computer methods are described for trend-surface analysis of D’Arcy Thompson transformation grids, illustrating data on shapes and forms. (see: P.H. A. Sneath, D’Arcy

Thompson's transformations, an interactive animation allows us to explore D'Arcy Thompson's transformations with new eye.) [254] [255]

D'Arcy Thompson considered how mathematical functions could be applied to pictures of one living organism to transform them into others. Among his most striking examples are the use of linear and non-linear functions to, for example, alter pictures of baboon skulls into skulls of other primates or humans, to transform the profiles of various fishes into one another, as well as showing how the corresponding bones (shoulder blades for example) in different species were related.

The picture below shows a pair of pictures from *On Growth and Form* of species of fish and the way that a transformation of the plane can move one into the other.

It is my understanding that D'Arcy Thompson is the first to apply the concept of qualitative quantity considering how mathematical /qualitative/ functions could be applied to picture /quality/ of the living organism to transform them into others. Among his striking examples are the use of linear and non-linear functions to alter qualities of shape /form/ of pictures like baboon skulls into skulls of other primates or humans, to transform the profiles of various fishes into one another, as well as showing how the corresponding bones /shoulder blades in different species were related.

The continuous notion of qualitative quantity is presented in the ability to vary the parameters continuously. The two pictures below show another of D'Arcy Thompson's pair of fishes and how, although the map he evidently used is not a quadratic map, the use of a quadratic map can achieve much the same effect.

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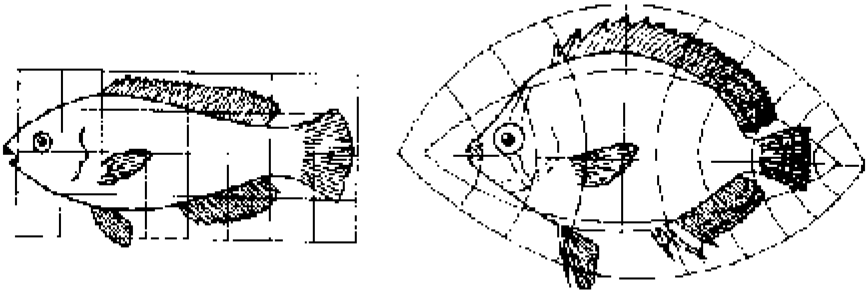
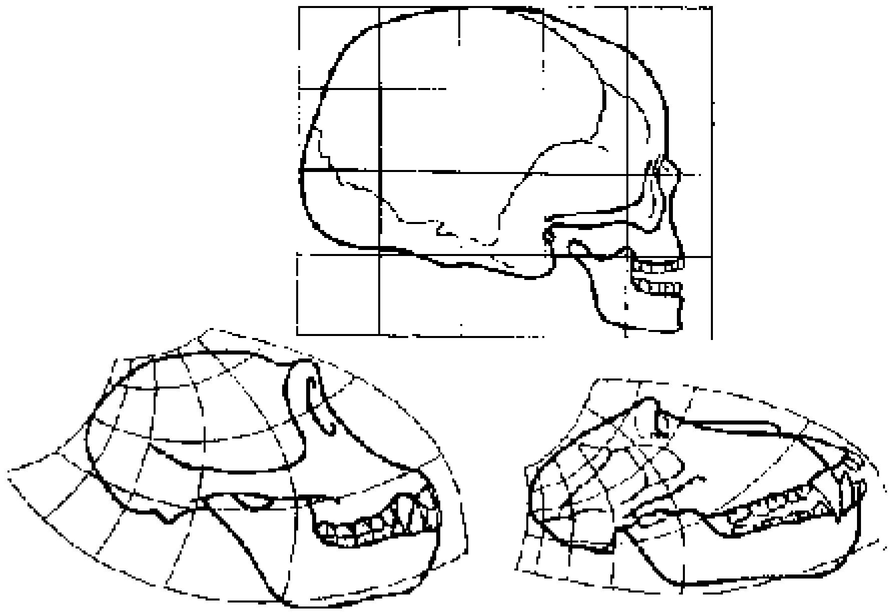
[254] P.H. A. Sneath, Trend-surface analysis of transformation grids, Journal of Zoology, Volume 151, Issue 1, pages 65–122, January 1967 – "Computer methods are described for trend-surface analysis of D'Arcy Thompson transformation grids, illustrated by data on skulls and jaws of hominoids. In comparing two diagrams the following steps are required. First, corresponding points on each diagram are marked, and their co-ordinates are recorded. Second, the diagrams are scaled and fitted to give the best possible fit; this gives measures of size and shape difference. Third, the displacements of each point relative to its partner on the other diagram are subjected to trend-surface analysis. The displacements are analysed in terms of linear, quadratic, cubic and higher order trends. Fourth, the differences based on the trends alone can now be estimated. The results on the illustrative examples are discussed, together with the difficulties in applying such methods. The nature of taxonomic (phenetic) affinity is also discussed, with suggestionsfor measuring different components of this concept. The techniques show promise for a wide variety of taxonomic and morphological applications.

[255] See: Using a computer to visualize change in biological organisms, School of Mathematics and Statistics, University of St. Anrews, Scotland - http://www.gap-system.org/~history/Miscellaneous/darcy.html

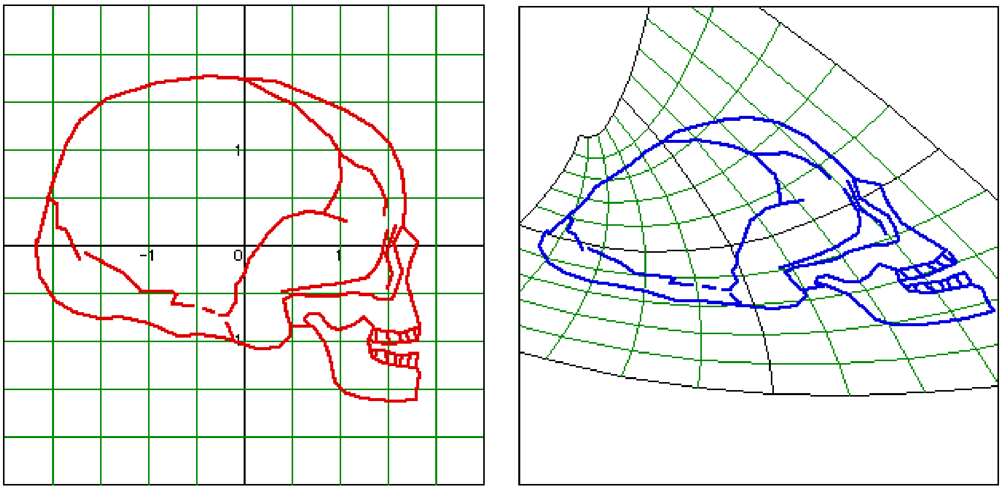


*Scarus* sp. *Pomacanthus.*

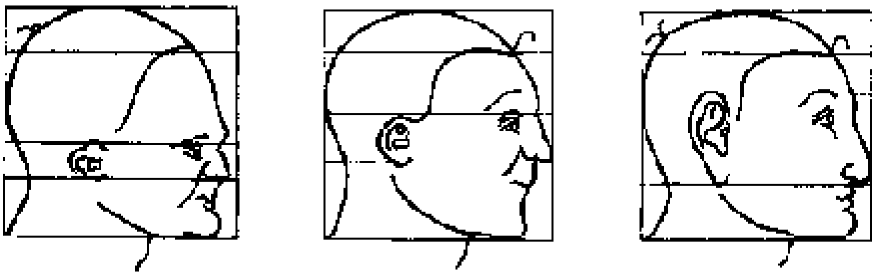
Some other pictures from *On Growth and Form* and the corresponding images from School of Mathematics and Statistics, University of St. Andrews, Scotland - http://www.gap-system.org/~history/Miscellaneous/darcy.htmlour project are shown below.



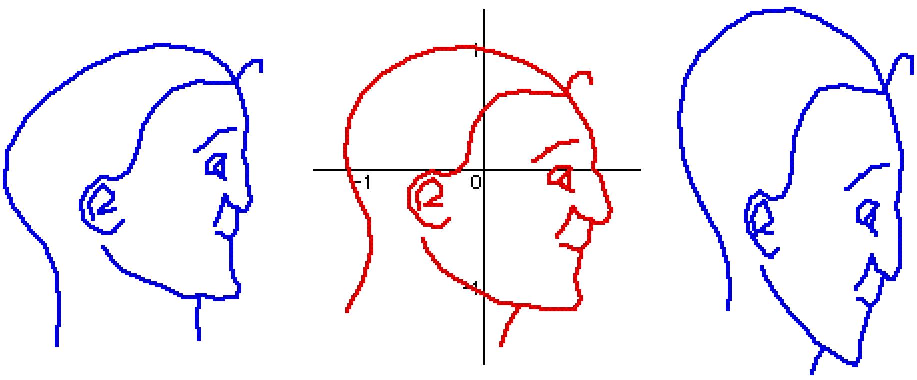
Skulls of a human, a chimpanzee and a baboon
and transformations between them



One set of images used by D'Arcy Thompson was taken from the artist (and mathematician) Albrecht Dürer's treatise on proportion: *De Symetria Partium in Rectis Formis Humanorum Corporum Libri...* published in Nuremberg after his death in 1528.



D'Arcy Thompson (after Albrecht Dürer)




Dürer's head and some of the
other things one can do to it

In mathematical biophysics Robert Rosen explores the derivation of D'Arcy Thompson' theory of transformations from the theory of optimal design.[256] (Rosen, 1962)

Rosen established that "Cohn's theory of Optimal Forms can be construed as a comparative theory, and that when this is done the celebrated theory of transformations of D'Arcy Thompson follows as a consequence. The implications of this type of theoretical foundation for the Thompson theory with regard to problems of comparative morphology are discussed, and some suggestions for the further implementation of the theory are described." (Rosen, 1962)

Robert Rosen introduced also one intriguing interpretation of Thompson's concept of biological transformation.[257] (Rosen, 1962)

In his paper "Dynamical similarity and the theory of biological transformations" Rosen established that "The D'Arcy Thompson concept of biological transformations is developed in a form analogous to such physical concepts as the Law of Corresponding States in thermodynamics, and the Principles of Similitude found in engineering. According to Robert Rosen "such concepts depend on a distinction between fundamental and derived quantities, in which the values assigned to the fundamental quantities set the natural scales for the derived ones. Robert Rosen: - "we see that critical phenomena, such as phase transitions, arise as an immediate consequence of this distinction." In a biological context, Robert Rosen explores "the implications of Thompson's hypothesis that closely related organisms are phenotypically similar, assuming that the organisms we see are the result of selection processes operating on phenotypes." (Rosen, 1962)

In my view, Robert Rosen's remarks about phenotype and phenotypology could be interplayed as phenomeno-type in the context of ***transition of phenomenology and phenomenotypology to phenomenotopology.*** **[258]**

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[256] Robert Rosen, The derivation of D'Arcy Thompson' theory of transformations from the theory of optimal design, Buletin of Mathematical Biophysics, Volume 24, Number 3, 279-290, 1962

[257] Robert Rosen, Dynamical similarity and the theory of biological transformations, Buletin of Mathematical Biophysics, Volume 40, Number 5, 549-579, 19…

[258] The term "Phenomeno-topology" first appear in Alain Badiou's "Deleuze: the Clamor of Being" /translated by Louise Burchill, 2000, The University of Mianesota Press/. Writing about Deleuze's "topology of the

The issue of Cultural Phenomenology of Qualitative Quantity is the transition of PhenomenoTypology to PhenomenoTopology.

Robert Rosen claims that "According to our analysis, the Van Der Waals [259] (Rosen, 1989) equation satisfies D'Arcy Thompson's Principle of Transfromation." [260] (Rosen, 1989)

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outside", Alain Badiou claims that: "Deleuze devotes innumerable pages to this stage of his ontological identification, multiplaying the cases and refining the investigations – to such a degree that some have believed him to do noting other than replace phenomenology by a phenomeno-topology." The aspects of the qualitative quantity in Deleuze's "Renewed Concept of the One" /title of the chapter from Alain Badiou's "Deleuze: the Clamor of Being"/, could be found in the Badiou's assertion that "Deleuze is indeed he who announces that the distribution of Being according the One and Multiple must be renounced." Alain Badiou claims that "…as always Deleuze, going beyond a static /quantitative/ opposition always turns out to involve the qualitative raising up of the one its terms."

**Deleuze's view on the interrelation of qualitative and quantitative** is demonstrated in his "Nietzsche and philosophy". (Originaly published in France in 1962),

In the chapter "3. Quality and Quantity", Deleuze discussed qualitative quantity as the "difference in quantity" on the interpretation in Nietzsche: "Forces have quantity, but they also have the quality which corresponds to their *difference in quantity*: the qualities of force are called "active7' and "reactive". We can see that the problem of measuring forces will be delicate because it brings the art of qualitative interpretations into play. The problem is as follows:1) Nietzsche always believed that forces were quantitative and had to be defined quantitatively. "Our knowledge, he says, has become scientific to the extent that it is able to employ number and measurement. The attempt should be made to see whether a scientific order of values could be constructed simply on a numerical and quantitative scale of force. All other 'values' are prejudices, naiveties and misunderstandings. They are everywhere reducible to this numerical and quantitative scale" (VP I1 352lWP 710).

2) However Nietzsche was no less certain that a purely quantitative determination of forces remained abstract, incomplete and ambiguous. The art of measuring forces raises the whole question of interpreting and evaluating qualities. " 'Mechanistic interpretation: desires nothing but quantities; but force is to be found in quality. Mechanistic theory can therefore only *describe* processes, not explain them" (VP I1 46/WP 660 - for an almost identical text cf. I1 187). "Might all quantities not be signs of quality? . . . The reduction of all qualities to quantities is nonsense" (VP I1 343/WP 564). Is there a contradiction between these two kinds of texts? If a force is inseparable from its quantity it is no more separable from the other forces which it relates to. *Quantity itself is therefore inseparable from difference in quantity.* Difference in quantity is the essence of force and h e r e l a t m f o r c e to force. To dream of two equal forces, even if they are said to be of opposite senses is a coarse and approximate -- dream, a statistical dream in which the living is submerged but which chemistry dispels Q~achti me that Nietzsche criticises the concept of quantity we must take it to mean that quantity as an abstract concept always and essentially tends towards an identification, an equalisation of the unity that forms it and an annulment of difference in this unity.

Nietzsche's reproach to every purely quantitative determination of forces is that it annuls, equalises or compensates for differences in quantity. On the other hand, each time he criticises quality we should take it to mean that qualities are nothing but the corresponding, difference in quantity between two forces whose relationship is pre-supposed. In short, Nietzsche is never interested in the irreducibility of quantity to quality; or rather he is only interested in it secondarily and as a symptom. What interests him primarily, from the standpoint of quantity itself, is the fact that differences in quantity cannot be reduced to equality. Quality is distinct from quantity but only because it is that aspect of quantity that cannot be equalised, that cannot be equalised out in the difference between quantities. Difference in quantity is therefore, in one sense, the irreducible element *of* quantity and in another sense the element which is irreducible *to* quantity itself. Quality is nothing but difference in quantity and corresponds to it each time forces enter into relation. "We cannot help feeling that I mere quwtitative differences are something fundamentally distinct from quantity, namely that they are *qualities* which

This principle asserts, in my own paraphrase, that closely related species are similar. Since "closely related" pertains to distances between genomes, and "similar" pertains to phenotypes or behaviours, this assertion is highly non-trivial. In the case of Van Der Waals equation, the assertion holds without qualification. But this is a very special situation, as we shall now see; in general, the Principle of D'Arcy." The words used by Robert Rosen – "closely related and similar" – "without quantification" directs to the similitude and similarity as domain and exhibit form of the qualitative quantity. (Rosen, 1989)

can no longer be reduced to one another" (VP I1 108lWP 565). The remaining anthro-pomorphism in this text should be corrected by the Nietzschean principle that there is a subjectivity of the universe which is no longer anthropomorphic but cosmic (VP I1 15). "To want to reduce all qualities to quantities is madness.. By affirming chance we affirm the relation of *all* forces. And, of course, we affirm all of chance all at once in the thought of the eternal return. But all forces do not enter into relations all at once on their own account. Their respective power is, in fact, fulfilled by relating to a small number of forces. Chance is the opposite of *acontinuum* (on the **L** *continuum* cf. VP I1 356). The encounters of forces of various quantities are therefore the concrete parts of chance, the affirmative parts of chance and, as such, alien to every law; the limbs of Dionysus. But, in this encounter, each force receives the quality which corresponds to its quantity, that is to say the attachment which actually fulfills its power. Nietzsche can thus say, in an obscure passage, that the universe presupposes "an absolute genesis of arbitrary qualities", but that the genesis of qualities itself presupposes a (relative) genesis of quantities (VP I1 334). The fact that the two geneses are inseparable means that we can not abstractly calculate forces. In each case we have to concretely evaluate their respective quality and the nuance of this quality.

**- Gilles Deleuze, Nietzsche and philosophy**

The concept of qualitative quantity is embedded in the concept of **phenotype.** Phenotype is an organism's observable characteristics or traits: such as its morphology, development, biochemical or physiological properties, behaviour, and products of behavior (such as a bird's nest). Phenotypes result from the expression of an organism's genes as well as the influence of environmental factors and the interactions between the two. Thus qualitative quantity is applicable in study of morphology, development, biochemical and physiological properties, behaviour, and products of behaviour such as burd nest, a cathedral termite mound. The Qualitative quantity as a trait is applicable in the study of characteristic or property of object because trait is indeed a characteristic or property of some objects. It is aplicable to the psychological study of personality due to the known trait theory in psychology. Qualitative quantity could be created as environment and developed as behavioural phenotype. In biology, trait involve genes and characteristics of organisms. Phenotypic trait in biology is a distinct variant of a phenotypic character of an organism that may be inherited, environmentally determined or be a combination of the two. For example, eye color is a *character* or abstraction of an attribute, while blue, brown and hazel are *traits*. In psychological study of personality. Trait is a Quality but extended quality, which could be reffered as qualitative quantity.

The interplay of qualitative and quantitative is obvious in the interaction of genotype and phenotype and genotype-phenotype distinction. This genotype-phenotype distinction. was proposed by Wilhelm Johansen in 1911 to make clear the difference between an organism's herefity and what that heredity produces. This distinction is similar to that proposed by August Weisemann, who distinguished between germ plasm (heredity) and somatic cells (the body). The concept of Qualitative quantity is aplicable to the Genotype-Phenotype concept.

The interaction between genotype and phenotype has often been conceptualized by the following relationship:
**genotype (G) + environment (E) → phenotype (P)**
The qualitiative quantity could be recognized in one more nuanced version of the relationship between genotype and phenotype:
**genotype (G) + environment (E) + genotype & environment interactions (GE) → phenotype (P)**
The qualitative quantity is explicit in the relationship "**genotype & environment interactions (GE) → phenotype (P)" thus leading to the phenotype.**

[259] Robert Rosen, Similitude, similarity, and scaling, Landscape Ecology vol. 3 nos. 3/4 pp 207-216 (1989), SPB Academic Publishing bv, The Hague.

In his famous book “Extended Phenotype”, Richard Dawkins wrote about D’Arcy Thompson the following:

“Lorenz’s (1937) discovery that a behavior pattern can be treated like an anatomical organ was not a discovery in the original sense. No experimental results were adduced in its support. It was simply a new way of seeing facts that were already commonplace, yet it dominates modern ethology /Tinbergen 1963/, and it seems to us today so obvious that it is hard to understand that it ever needed “discovering”. Similarly, D’Arcy Thompson’s (1917) celebrated chapter “On the theory of transformations…” is widly regarded as a work of impotance although it does not advance or test a hypothesis. In a sense it is obviously necessarily true that any animal form can be turned into a related form by a mathematical transformation, although it is not obvious that the transformation will be a simple one. In actually doing it for a number of specific examples, D’Arcy Thompson ivited a “so what?” reaction from anyone fastidious enough to insist that science proceeds only by the falsifying of specific hypotheses. If we reed D’Arcy Thompson’s chapter and then ask ourselves what we now know that we did not know before, the answer may well be not much. But our imagination is fired. We go back and look at animals in a new way; and we think about theoretical problems, in this case those, not so presumptuous as to compare the present modest work with the masterpiece of a great biologist. I use the example simply to demonstrate that it is possible for a theoretical book to be worth reading even if it does not advance testable hypostheses but seeks, instead, to change the way we see.” [261](Dawkins, 1989)

Not only the relevance of D’Arcy Thompson for the modern science, but the relevance of Hegel’s logic of qualitative quantity, is recognized in the study of Harvey Goldstein and Francis E. Johnston, “A Method for studying shape change in children” [262] (Goldstein H., and Johnston, F.E., 1978) This method is given for measuring shape change with age, utilizing two

[260] Johanes Diderik van der Waals /1837-1923/ was a Dutch theoretical physicist and termodynamicist famous for his work on an equation of state for gases and liquids. His name is primarily associated with the Van Der Waall equation of state that describes the behaviour of gases and their condensation to the liquid phase. As I noted in my study “Quality of the Quantity” /1989/, the qualitative quantity’s approach applied to the process of phase transition of the fluid crystals or the behavior of the gases and their condensation to the liquid phase.

[261] Dawkins, Richard (1989). The Extended Phenotype. Oxford: Oxford University Press

dimensional body outlines. Problems of standardization for overall size are discussed and it is shown how the method can be used to draw D'Arcy Thompson-type transformation grids. In this study the authors states that: “One of the first serious attempts to study the shapes of biological organisms was made by D'Arcy Thompson (1917). He considered two-dimensional diagrammatic representations and studied ways in which diagrams based on different organisms were related--his well known "transformation grids". Here shape is defined in terms of the geometrical relationships between corresponding features of two diagrams. An alternative development, mainly developed by Huxley (1932), and known as allometry, defines "shape" in terms of relationships among a set of lengths of body segments. Huxley's ideas were often applied to relationships observed in growing individuals, whereas Thompson's ideas found most application in the comparison of different species. The original ideas of "bivariate" allometry, where only two lengths were related have undergone considerable mathematical development, have been generalized to the case of several lengths, and found applications in several areas (see, e.g., Sprent, 1972). Thompson's ideas on the other hand, seem not to have aroused so much interest, and it is the purpose of this paper to apply a development of his ideas to the study of shape change in children. This development is originally due to the work of Sneath (1967), and in the next section his methods are summarized.” (Goldstein H., and Johnston, F.E., 1978)

In my review of the D’Arcy Thompson heritage in the present science and reconsideration of Hegel’s topological notion of qualitative quantity, I must underline the remarkable work of Norman MacLeod[263] in Paleo-Mathematics. Norman MacLeod’s “Paleo-Math 101, Shape Models II: The Thin Plate Spline”[264] (MacLeod, 2010) enhanced the relevance of D’Arcy Thompson and illustrates Hegel’s notion of qualitative quantity.

In “Paleo-Math 101, Shape Models II: The Thin Plate Spline”, Norman MacLeod demonstrates the mathematically sophisticated way to do the same thing that, at least on a

[262] Harvey Goldstein and Francis E. Johnston, A Method for studying shape change in children, Annals of Human Biology, 1978, Vol. 5, No. 1 , Pages 33-39

[263] *www.nhm.ac.uk/hosted_sites/**paleo**net/**MacLeod**/**MacLeod**CV.pdf*

[264] Norman MacLeod, Paleo-Math 101, Shape Models II: The Thin Plate Spline
http://www.palass.org/modules.php?name=palaeo_math&page=26

superficial level, makes connection with the 'deformation grid' approach to shape modelling developed by D'Arcy Thompson in his classic treatise *On Growth and Form* (1917).

MacLeod provides comprehensive review on what Thompsonian transformation grids are and why they were developed:

"In his original /*On Growth and Form*/ (1917), and in the later, expanded (1942) edition, Thompson's goal was to "correlate with mathematical statement and physical law certain of the simpler outward phenomena of organic growth and structure or form while all the while regarding the fabric of the organism, ex hypothesi, as a material and mechanical configuration" (Thompson 1917, p. 17). In other words, Thompson sought to "see how, in some cases at least, the forms of living things, and of parts of living things, can be explained by physical considerations and to realize that in general no organic forms exist save such as are in conformity with physical and mathematical laws" (*ibid*, p. 15). Thompson was, of course, aware of evolutionary theory and agreed that natural selection operated to sort mechanically efficient from inefficient forms in the manner Darwin had suggested. But he bridled at the idea that every aspect of a form is now, and always has been, under direct adaptive scrutiny, preferring to believe that some aspects of form owe their origin to the physical forces with which they must contend. Thompson saw organic form as a 'diagram of forces' from which inferences can be made regarding the nature of the forces that act upon it now or that have acted upon it in the past. Using this force metaphor, Thompson saw the mathematical comparison of forms as a way of deducing how these fields of forces changed during both ontogenetic and evolutionary history." [265] (MacLeod, 2010)

**Norman MacLeod's text reveals the "topological D'Arcy Thompson":**

**"**Thompson's proposed method of force-field analysis was to take two simple line drawings of species' bodies or some corresponding structural element therefrom (e.g., copepod, ungulate cannon bone, leaf). For convenience we'll label one form as the 'reference' and the other as the 'target'. In order to better visualize the nature of the geometric transformation Thompson superimposed a rectilinear grid on the reference form. He then worked out simple sets of

[265] Norman MacLeod, Paleo-Math 101, Shape Models II: The Thin Plate Spline
http://www.palass.org/modules.php?name=palaeo_math&page=26

mathematical transformations that would map point location coordinates of the reference into **topologically** corresponding locations on the target. Applying those same mathematical transformations to the coordinate locations of the grid vertices **Thompson obtained a striking image-based summary of the implied geometric transformation (Fig. 1).** In this view Thompson perhaps reflects the same level of discomfort with hyperadaptationist arguments criticized more recently by Gould and Lewontin (1979), among others." [266] (MacLeod, 2010)

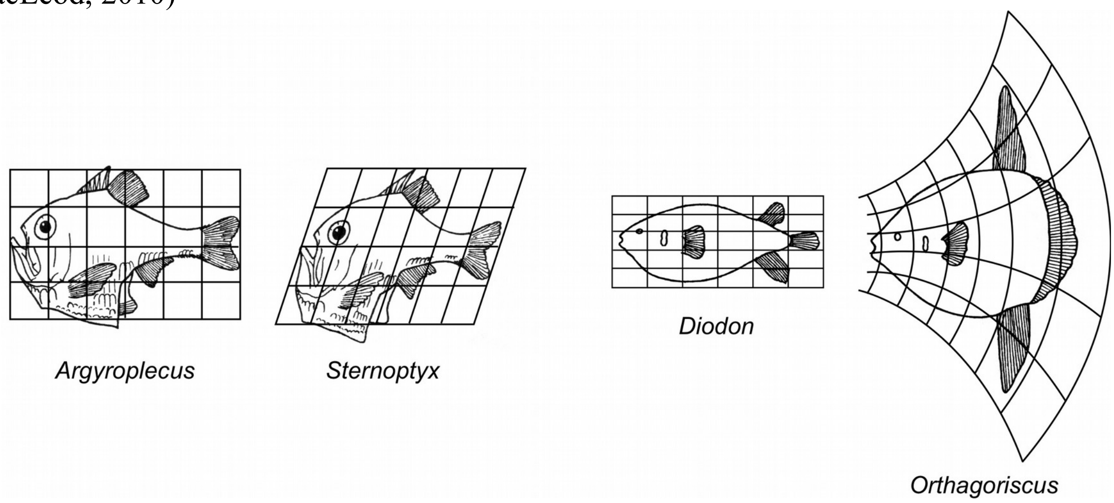


Figure 1./168/ Example Thompsonian transformation grids specifying uniform (upper) and non-uniform (lower) transformation functions. For each comparison the reference form is located on the left and the target form on the right. Redrawn from Thompson 1917.

MacLeod establishes: "It is clear from Thompson's many statements throughout the last chapter of *On Growth and Form* that he regarded the mathematical transformation as pertaining to, and being constrained by, all mathematical points comprising the line drawing and that he respected the principle that biological homology pertained to structures, but not necessarily individual point locations on structures. Rather, it was the configuration of the entire ensemble of mathematical points — represented diagrammatically by the superimposed grid — that he looked to in judging whether he had devised biologically reasonable formulae for a particular form transformation. Similarly, it is clear the only purpose served by the mathematical grid was to passively express the overall geometry of the transformation in the manner of a deformed, map-like coordinate system." [267] (MacLeod, 2010)

[266] Norman MacLeod, Paleo-Math 101, Shape Models II: The Thin Plate Spline http://www.palass.org/modules.php?name=palaeo_math&page=26

[267] Norman MacLeod, Paleo-Math 101, Shape Models II: The Thin Plate Spline

Reffering to the figure 1, Norman MacLeod discusses the fact that "Thompson used his approach to provide examples of both linear (uniform) and non-linear (non-uniform) transformation modes. Like the later 'relative growth' studies of Otto Snell, Julian Huxley and Georges Teissier (see below), the thing that impressed morphologists about Thompson's transformation grids was the fact that seeming complex form changes appeared to be able to be described accurately by simple mathematical transformations applied consistently to all point locations over a form. This suggested to many at the time that the underlying principles and/or determinants of morphological change might be simple when expressed in, or studied using, the language of mathematics." [268] (MacLeod, 2010)

"While Thompson's transformation-grid approach resulted in the creation of compelling diagrams — so much so both his original set of drawings and many subsequent variations of them have been reproduced in countless books on biology and evolution despite the fact that the physicalforce theory these drawings represent is almost never discussed in those same texts — his geometric approach to the analysis of form never caught on during his lifetime. Thompson himself provided some guidance regarding how to operationalize his transformation grids, which he thought of as a visual tool akin to a modern-day spatial morphing algorithm. **Those algorithms subdivide an image into a set of points and then smoothly map a subset of these between a reference and target form with their difference displacements informing the displacement of intermediate points via simple linear interpolation.** For example, it is this linear interpolation approach to transformation grid analysis that Thompson used to create his morphlike model of the complex geometric transition between *Hyracotherium* and modern-day *Equus* (Fig. 2)." [269] (MacLeod, 2010)

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[268] Norman MacLeod, Paleo-Math 101, Shape Models II: The Thin Plate Spline
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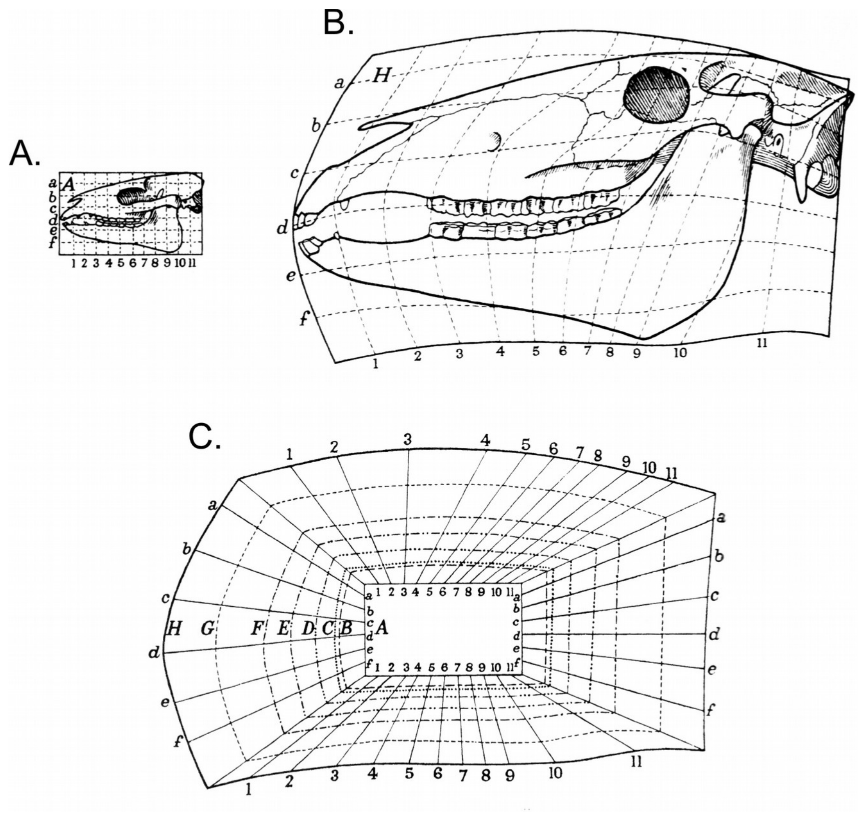

Figure 2 /172/. Thompson’s transformation grid analysis for the transition from the Eocene *Hyracotherium* skull form (A) to the modern *Equus* skull form (B). Note the representation of hypothetical intermediate stages of the transformation via linear interpolation (C). From Thompson 1917.

“Although Thompson’s original, interpolation-based approach to the realization of transformation grids was fine if all you wanted to do was map one form into another, it was not well suited to the summarization of geometric information across a larger sample of data. Arguably Huxley (1924) and Teissier (1929, see also Snell 1892 and Huxley 1932) were more successful in developing an analytic approach to the general problem of form variation than was Thompson. But the logistic regression equations used by students of relative growth — or allometry in modern parlance — were not used to create graphic models of form change with anything like the visual impact of Thompson’s grids.

Curiously, this failure to capitalize on the modelling capabilities of regression-based methods when treating morphological data remained in place for a half-century during which time Huxley and Teissier’s bivariate regression-based approach to form analysis was extended to the multivariate case (via PCA, see Jolicouer and Mosimann 1960) and holistic modelling approaches were developed for other aspects of morphological analysis (see Olson and Miller 1958). As we have seen in the last column, **all the mathematical machinery for implementing at least some aspects of useful geometric shape modelling was in place by the 1960s. Yet, no new developments in this area took hold until 1980s despite a few attempts to formulate an explicitly Thompsonian modelling approach in the form of morphological trend surfaces (Sneath 1967) and biorthogonal grids (Bookstein 1978).** [270] (MacLeod, 2010)

In retrospect there appear to be two reasons for this. The first was that the largest school of morphometrics (multivariate morphometrics) tended to look to the communities of statisticians and psychometricians for methodological guidance, neither of which were particularly interested in creating morphological models. The second was that, ever since the 1930s, the tradition in bivariate and multivariate morphological studies was to analyze pairs or sets of distances between landmark locations rather than configurations of Cartesian coordinate locations scattered over a sample of forms. Once the power of shape coordinates had been established by Bookstein (1986) and the outlines of shape theory had begun to emerge (see Kendall 1984), the stage was set to renew the search for an analytic method that could combine the intuitive appeal of Thompson's transformation grids with the equally popular, and far more powerful, tools of multivariate morphometrics. The key insight that allowed this new approach to shape modelling to be realized was specification of a new spatial metaphor for shape similarity.

In traditional multivariate analysis the similarity between two objects is quantified by calculating the distance between them across all variables." [271] (MacLeod, 2010)

I should highlight here that this approach is pure quantitative approach, lacking "the ability to track varying patterrns of size /shape/ similarity and difference in the different regions of the forms". And Norman MacLeod emphasizes also the qualitative element in this analysis:

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[270] Norman MacLeod, Paleo-Math 101, Shape Models II: The Thin Plate Spline
http://www.palass.org/modules.php?name=palaeo_math&page=26

[271] Norman MacLeod, Paleo-Math 101, Shape Models II: The Thin Plate Spline
http://www.palass.org/modules.php?name=palaeo_math&page=26

This is fine for a quick-and-dirty summary of form differences, but lacks the ability to track varying patterns of size/shape similarity and difference in different regions of the forms. In the days when morphometricians characterized forms using sets of linear distances between landmarks, this distance-based metaphor seemed both natural and practical. After all, distances are simply magnitudes. There is no information about geometry in a list of distance values. Since geometry can't be reconstructed precisely from a table of distance values there wasn't any point in worrying about shape models. But with the move to characterizing forms using the coordinate values of the landmarks themselves — and especially the transformation of landmark coordinate values to *Procrustes* shape coordinate values via standardization for position, scale, and rotation — it became possible to represent the form and shape similarities or differences between any two objects precisely in a manner that retained the fundamental geometry of the landmark configurations." [272] (MacLeod, 2010)

The qualitative element in analysis is exactly in the variety of patterns of size /shape/ ..in the "**similarity and difference in the different regions of the forms".**

The notion of qualitative quantity is evident in **Norman MacLeod's examples of geometric Deformations** / Figure 5. from Alternative modes of uniform shape deformation and and Figure 6 - the so-called 'square to kite' deformation from Norman MacLeod, Paleo-Math 101, Shape Models II: The Thin Plate Spline".[273] (MacLeod, 2010)

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[272] Norman MacLeod, Paleo-Math 101, Shape Models II: The Thin Plate Spline
http://www.palass.org/modules.php?name=palaeo_math&page=26

[273] **Thin plate splines (TPS)** were introduced to geometric design by Duchon (Duchon, 1976). The name *thin plate spline* refers to a physical analogy involving the bending of a thin sheet of metal. The acronim "TPS" is a brief derivation for the closed form solutions for *smoothing Thin Plate Spline*. Details about these splines can be found in (Wahba, 1990).
Thin plate or layer: lamelli-, lamell- from Latin: thin plate or layer; **lamella** (s)**, lamellae** (pl) 1. A thin plate, scale, membrane, or layer, as of bone, tissue, or cell walls. **Lamelliform:** 1. Shaped like a thin plate or scale; shaped like a lamella. 2. Thin and flat; platelike, scalelike; lamellar.
**Elastic maps** provide a tool for nonlinear dimensionality reduction. By their construction, they are system of elastic springs embedded in the data space. This system approximates a low-dimensional manifold. The elastic coefficients of this system allow the switch from completely unstructured k-means clustering (zero elasticity) to the estimators located closely to linear PCA manifolds (for high bending and low stretching modules). With some intermediate values of the elasticity coefficients, this system effectively approximates non-linear principal manifolds. This approach is based on a mechanical analogy between principal manifolds, that are passing through "the middle" of data distribution, and elastic membranes and plates. The method was developed by A.N. Gorban, A.Y. Zinovyev and A.A. Pitenko in 1996–1998.

According to MacLeod, there are two broad classes of possible geometric deformations. These go by various names. Uniform deformations (also called affine or linear) includes all modes of deformation in which lines that are parallel prior to the deformation remain parallel after the deformation. There are six types of uniform deformations (Fig. 5). (MacLeod, 2010)

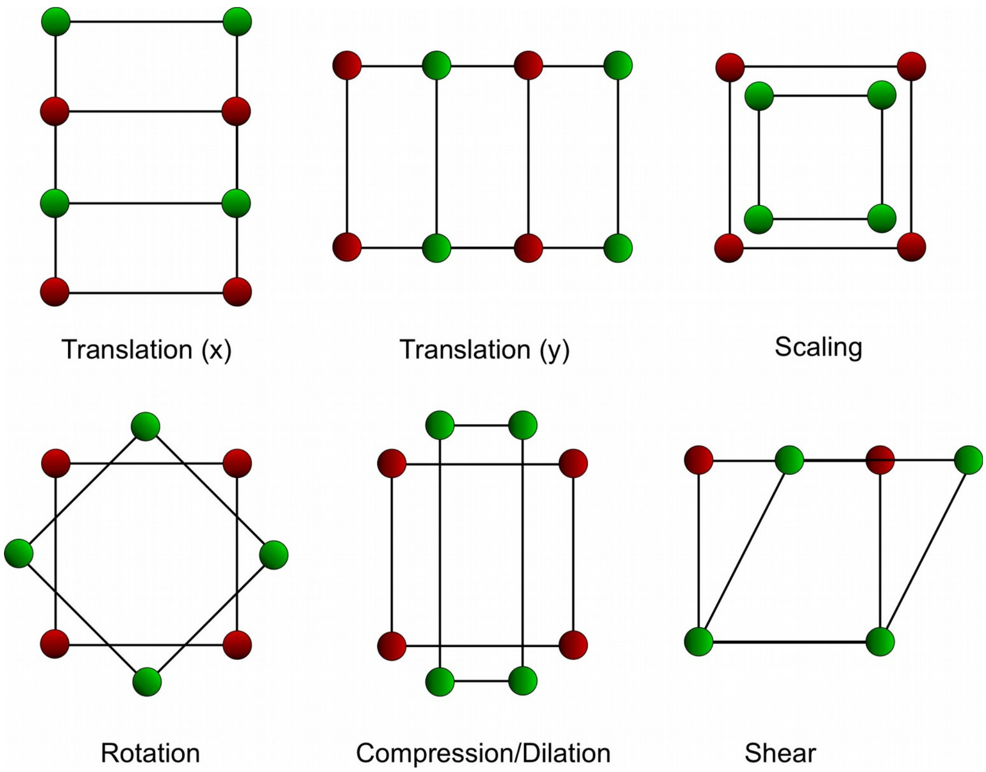



Figure 5. Alternative modes of uniform shape deform (MacLeod, 2010)

And “Among these you’ll recognise the deformation modes that are corrected during Procrustes superposition. Nevertheless, the compression/dilation and shear modes can be used, or combined, to describe aspects of genuine shape change. As for the ‘other’ category, it’s usually referred to as a non-uniform deformation in the morphometric literature, but can also be termed a non-affine or non-linear deformation. These are deformations in which lines that are parallel prior to the deformation are not parallel after the deformation. Examples are numerous, but the simplest is the so-called ‘square to kite’ deformation (Fig. 6).” (MacLeod, 2010)

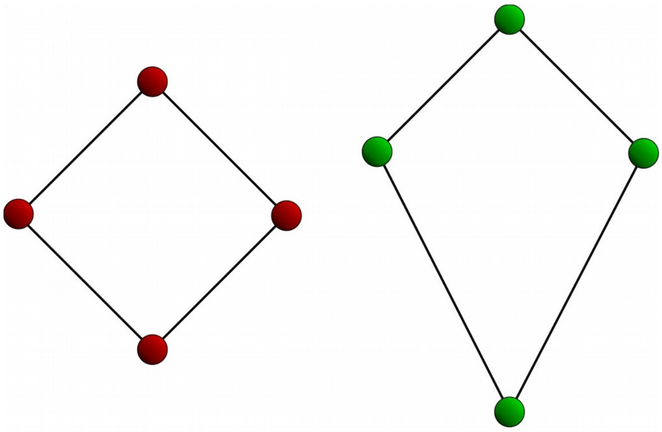



Figure 6. One simple example of a non-uniform shape deformation

For MacLeod, most ‘real-world’ geometric deformations are combinations of uniform and non-uniform deformation modes like in the so called Acaste-Calymene deformation.

The goal of **Norman MacLeod in “Paleo-Math 101, Shape Models II: The Thin Plate Spline” is** “to represent shape transformation as a thin plate spline in mind /see the note underline -me/, and with an appreciation of the fact that this spline is (likely) going to be composed of both uniform and nonuniform deformation modes, we’re now in a position to begin a (very generalized) discussion of TPS mathematics. Since we’re going to be minimizing the hypothetical bending energy in the specification of our shape transformation surface, we’re going to need to calculate an index of bending energy at each landmark location. This first step toward this is achieved by the following equation.” (MacLeod, 2010)

$$U\left(r_{ij}\right) = r_{ij}^2 \ln r_{ij}^2$$

(MacLeod, 2010:177)

The notion of qualitative quantity we could experience by way of an example, given by MacLeod for the *Acaste-Calymene* TPS surface in which further decomposition goes into uniform and non-uniform deformation modes, as shown in Figure 8. (MacLeod, 2010)

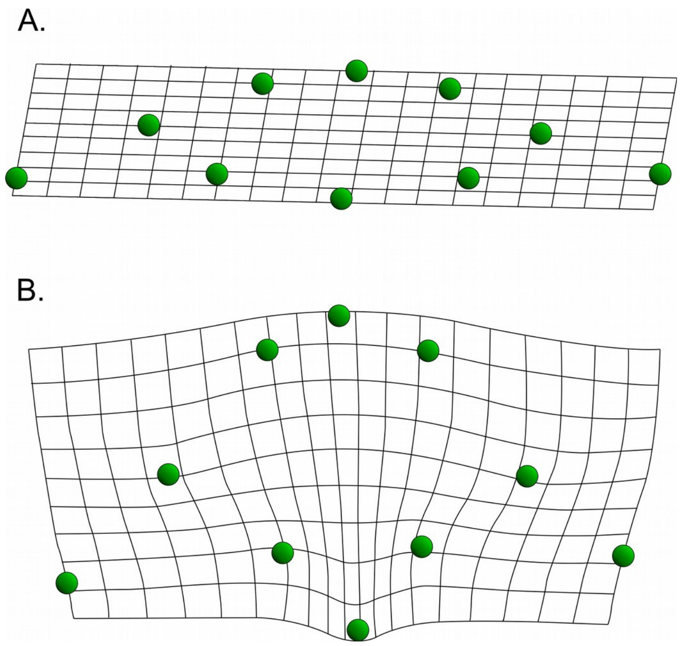



Figure 8. /178/ Deformational modes of the *Acaste-Calymeme* geometric transformation. (A) Uniform (affine) TPS surface. (B) Non-uniform (non-affine) TPS surface.

For MacLeod – "Accordingly, the deformation shown in Figure 7D can be described as a combination of a uniform deformation that combines aspects of a clockwise shear, strong anterio-posterior compression, and clockwise rotation (Fig. 8A). To this is added a pronounced non-uniform deformation centred in the glabella involving a strong element of asymmetrical latero-posterior compression with movement of the three anterior landmarks strongly forward relative to the remaining landmarks. This relative movement results in elongation of the glabella and anterior of the cranidium, lateral migration of the eyes, and latero-posterior migration of the intersection between the crandidium's the posterior lateral projection and the posterior lateral margin of the glabella (Fig, 8B, see MacLeod 2009, Fig. 5 for landmark definitions). (MacLeod, 2010)

Thin plate splines for the entire trilobite dataset for which these ten landmarks can be located are shown in Figure 9. In these analyses the sample mean shape was used as the reference shape. Also provided is the value of the total bending energy specified by each spline surface. This number is analogous to the total shape variance and can be used to identify the shapes that deviate more (or less) from the reference (= mean) shape than others.

I hope this brief explanation and demonstration of thin plate splines has demystified the topic for you, at least a bit. Thin plate splines are a very attractive way of graphically depicting shape changes and, because of that, they are also very seductive. They should be used more widely than they are, but they need to be used with caution. (MacLeod, 2010)

Because these splines are depicted as surfaces that encompass the landmarks themselves, the areas between the landmarks, and even areas outside the region covered by the landmark set, there is tendency to make more of the details of the spline's configuration than is actually warranted. It should be remembered that, except for the areas immediately surrounding the landmark locations, all other aspects of the spline are artificial interpolations. While the geometry of the spine between the landmarks can identify regions of potential interest (see Bookstein's 2002 method of TPS creases), interpretations involving these inter-landmark regions should be made with caution. Ideally once inter-landmark regions of interest have been identified, landmarks should (if at all possible) be placed at or near their location and the analysis repeated." (MacLeod, 2010)

The notion of the Quality of numbers /qualitative quality is evident in the Thin Plate Surface (MacLeod, 2010)  TPS - Figure 9 (MacLeod, 2010)  – MacLeod remarks: "Total TPS surfaces for the comparisons between the trilobite sample mean shape (red) and the landmark shape configuration for 18 genera from the trilobite dataset. Numbers /quantity/ in parentheses beside each genus name are the total bending energies associated with that shape configuration relative to the sample mean shape. This value is analogous to the shape variance" . The value which is analogous to the shape varience is the quality as well as the shape varience itself.

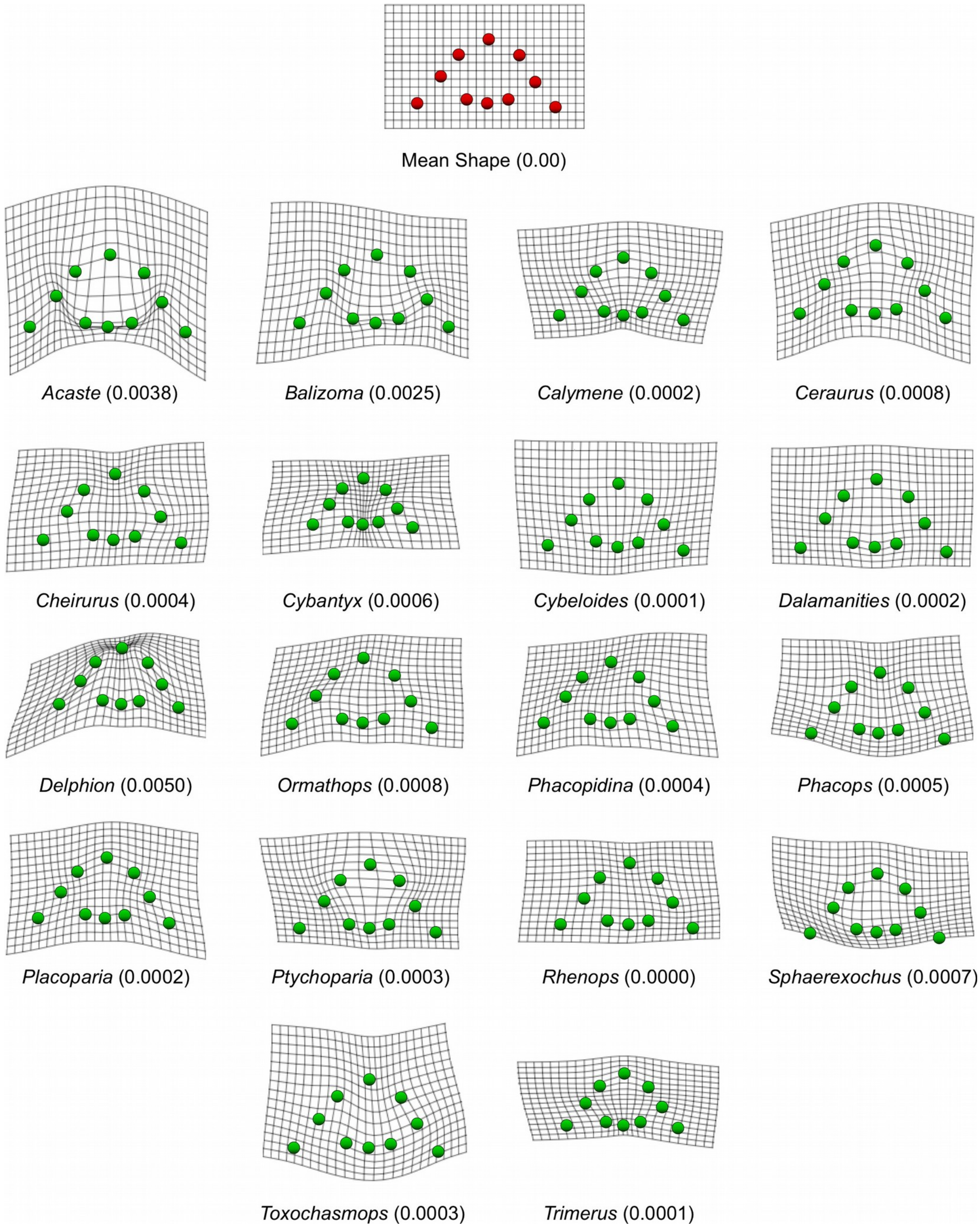



Figure 9. /181/ Total TPS surfaces for the comparisons between the trilobite sample mean shape (red) and the landmark shape configuration for 18 genera from the trilobite dataset. Numbers in parentheses beside each genus name are the total bending energies associated with that shape configuration relative to the sample mean shape. This value is analogous to the shape variance. (MacLeod, 2010)

MacLeod concludes:

"Is the thin plate spline the long-sought realization of the Thompsonian transformation grid concept? In some ways it is and in some ways it isn't. I suspect Thompson himself would have absolutely loved thin plate splines. D'Arcy Thompson was a great believer in the constraints materials and physical processes place on morphological arrangements. The idea that the TPS algorithm involves a metaphorical concept of bending energy which is required to be minimized by the resulting geometry would have spoken to one of his most deeply held beliefs about the organic world. However, no data or morphological patterns have come to light in the 93 years that have elapsed since On Growth and Form's publication to lend support the idea that evolutionary processes operate in such a way as to minimize physical parameters such as bending energy. To be sure, organic design cannot exceed the performance limits imposed by the materials used to execute the design. This represents an absolute limitation. But evolutionary history abounds with examples of structures that are inefficient from a purely mechanical point of view. The reason for this this is that mechanical design is only one of the parameters evolutionary processes seek to optimize."[274] (MacLeod, 2010)

The topological notion of "betweenness" typical to the category qualitative quantity which is "between" quality and quantity, could be seen in Fred L. Bookstein, When One Form is Between Two Others: An Application of Biorthogonal Analysis.[275] (Bookstein, 1980) In this essay Bookstein builds again on D'Arcy Thompson, presenting a method for measuring the degree to which one biological outline form lies in between two others. To my understanding this approach is about measuring "qualitative quantity". As Bookstein explains – "The procedure does not measure forms separately, but rather compares pairs of tensors expressing D'Arcy Thompson's "Cartesian transformations" according to the biorthogonal formalism of Bookstein. In analogy with conventional methods, betweenness is computed as a similarity

[274] Norman MacLeod, Paleo-Math 101, Shape Models II: The Thin Plate Spline :: There are computer programs for implementing a TPS representation of shape difference. The industry standard remains Jim Rohlf's tpsSplin and tpsRelw packages (http:// life.bio.sunysb.edu/morph/soft-tps.html). Øvind Hammer's PAST package (http://folk.uio.no/ ohammer/past/) also calculates thin plate splines though his description of the algorithms it employs to do so (Hammer and Harper 2006) appears to differ in many ways from the canonical descriptions provided by Bookstein (1991), Rohlf (1993) and Zelditch et al. (2004). Other packages that can be used to perform TPS analyses include Dave Sheets' IMP software (http:// www3.canisius.edu/~sheets/morphsoft.html), Paul O'Higgins' Morphometrika (http:// sites.google.com/site/hymsfme/downloadmorphologica) and Jon Krieger's Morpho-Tools online morphometrics data analysis tools site (http://www.morpho-tools.net/).

[275] Fred L. Bookstein, When One Form is Between Two Others: An Application of Biorthogonal Analysis, Amer. Zool. (1980) 20 (4): 627-641.

score, the cosine of a non- Euclidean angle between the tensors." Bookstein established that "the new quantities, size-betweenness and shapebetweenness, enable comparisons of form series against *a priori* orderings intra- and interspecifically." (Bookstein, 1980)

After Fred L. Bookstein's "When One Form is Between Two Others", one can see how qualitative quantity is category and expression of beetweeness – Quality and Quantity are related to the growth and forms and their changes. It is well known from the classic dialectics how quantitative addition lead to the qualitative changes. This clishe is well established in ontogenesis explenation and biorthogonal analysis and affect the approach/s and methods of study in morphometrics in quantitative style and quantitative research. There is a lack widly recognized in science of Morphometrics about the qualitative elements and methods. Qualitative quantity is a concept that would bridge this gap due to the suitable character of this category to explain the "term" and the "concept" of "betweenness".

And here is Bookstein for D'Arcy Thompson and **Geometric Morphometrics:**

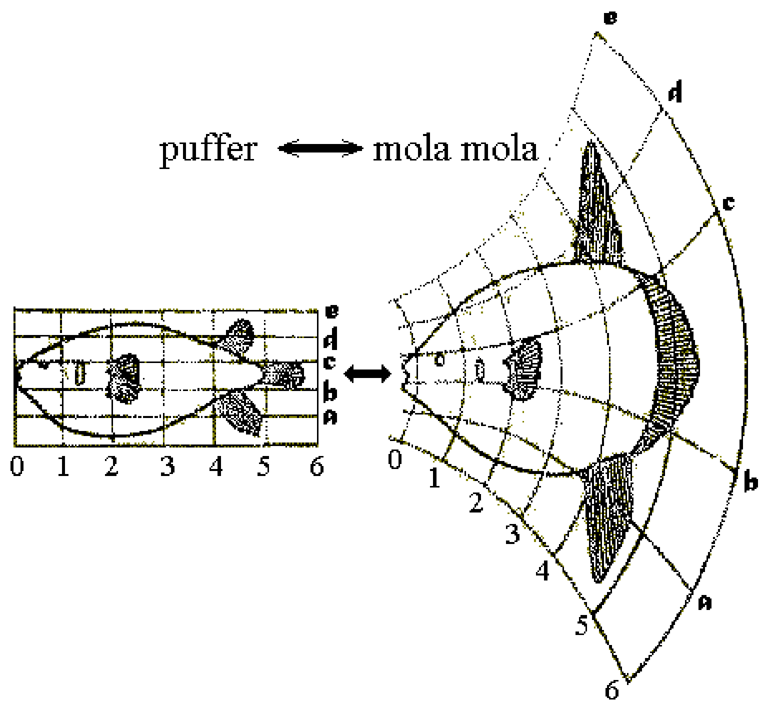
"Geometric morphometrics inspire, partially at least, in the work of D'Arcy W. Thompson (1942) who approached the study of biological shape change as distortions occurring in a cartesian coordinate system which have been previously selected on the basis of its biological homology. Shape is a definite entity, a configuration of points that keep geometric relationships among them and cannot be split into isolated items (like length or height). Confronted with a biological shape, the morphometrician will attempt to describe it in terms of transformation from an original reference shape. Although the approach proposed by Thompson was very appealing and promising it was not accompanied by any analytical procedure. It was the arrival of the computer age, several decades later, that makes it possible to develop application for morphometric analysis based on Thompson's ideas feasible (Bookstein 1993)." [276]

[276] See: Bookstein, F. L. 1984. Tensor biometrics for changes in cranial shape. Annals of Human Biology, 11(5): 413-437.; Bookstein, F. L. 1986. Size and shape spaces for landmark data in two dimensions. Statistical Science, 1: 181-242.; Bookstein, F. L. 1989. "Size and shape": a comment on semantics. Systematic Zoology, 38: 173-180.; Bookstein, F. L. 1991. Morphometric tools for landmark data: Geometry and Biology. New York: Cambrige University Press.; Bookstein, F. L. 1993. A brief history of the morphometric synthesis. In: L. F. Marcus, E.

The continuous change as an exhibit form of qualitative quantity corresponds with the evolution and in particular with the morphological evolution of the brains, also with the communication and behavioral drive. The topological notion of qualitative quantity is evident in D'Arcy Thompson's early attempt to quantitatively describe the mechanism of shape change, published by D'Arcy Thompson in his landmark book. The particular image that inspires me in my research on qualitative quantity is illustrated above - the "Puffer" to "Mola mola" transformation. Some 60 years later Fred Bookstein is the another scientist who made an effort to formalize the technique into a procedure he called 'bioorthogonal analysis' (Bookstein, 1980). In 1981, a technique was published for alligning and comparing homologous sets of landmark-coordinates /Siegel, 1981, 1982/. The technique was explained in relatively simple ways but the most important advance, to the average biologist, was that a computer algorithm to compute the allignments was also published. This date was also significant in being close to the beginning of the Personal Computer (PC) Era. This allowed biologists to follow the published algorithm of Siegel and start studying shape-change this way.

In the late '70s and early '80s the evolution of shape was embroiled in controversy about the validity of newly developed techniques, some developed *ad hoc* by non-biometricians. Some of the topics were also controversial. In particular, a study was published out of the Allan Wilson laboratory claiming a rapid morphological evolution of the shapes of the humanoid line of evolution compared to a measured slow evolution of the frogs (Cherry, Case and Wilson, 1978). A figure comparing human to chimpanzee illustrates the dramatic change in shape/proportions for a species pair whose DNA similarity is closer than almost all recorded sibling mamalian species.

The desire to explain the apparent rapid evolution of the hominoids led to a paper (Wyles, Kunkel & Wilson, 1983) which ascribes the rapid morphological evolution to large brains. A large brain is a trait associated with vocal communication in groups. Thus the hominoids, the song birds and marine mammals each communicate within their own species. They all need larger brains with which to carry on this communication and the attendant processing of information which other mammals are not endowed with and do not benefit from. The communication and learning of information from one-another enlargens the interface of the individual with the environment. This increases the rate of evolution (including morphological evolution) and has been given the name "behavioral drive". In effect, the evolution of a larger brain and communication skills leads to an autocatalytic increased rate of evolution.




D'Arcy Thompson's classic fish transformation

In March 2008, Tuns Press and Riverside Architectural Press, Faculty of Architecture and Planning of Dalhousie University, Halifax, Canada, published the book “On Growth and Form - Organic Architecture and Beyond”, edited by Philip Beesley & Sarah Bonnemaison. The book is such a celebration of D’Arcy Thompson. Sarah Bonnemaison and Philip Beesley addressed the issue - Why Revisit D'Arcy Wentworth Thompson's *On Growth and Form*? [277] (Beesley P., and Bonnemaison, S., 2008)

The editors, Sarah Bonnemaison and Philip Beesley answered the question with the following:

"D'Arcy Wentworth Thompson demonstrated new working methods for understanding the influence of physical forces in the environment, and the architectural projects in this book owe much to Thompson's research. They explore structural systems that use tension and 'tensegrity', in which forces animate the entire structure. Digital design tools now allow such complex interactions to be quantified and dynamically modeled, and digital prototyping and manufacturing play important roles in their realization. Instead of relying on centralized systems that resist environmental changes, new generations of buildings can accommodate shifting forces, distributing loads to better withstand undesirable deformation. Such buildings involve new methods of construction using chains of components and distributed structures." (Beesley P., and Bonnemaison, S., 2008)

According to Sarah Bonnemaison and Philip Beesley:

"Recent research confirms Thompson's empirical observations of biological form which showed that cell shapes are dictated by three-dimensional skeletons that mirror large-scale architectural space-frames. New developments in materials compatible with physiology, and miniature fabrication methods similar to those used for manufacturing computer chips have contributed to further this development in lightweight structural frameworks. Analytical tools

[277] "On Growth and Form - Organic Architecture and Beyond", edited by Philip Beesley & Sarah Bonnemaison, 2008, Tuns Press and Riverside Architectural Press, Faculty of Architecture and Planning of Dalhousie University, Halifax, Canada. Titles and Authors: Why Revisit D'Arcy Wentworth Thompson's *On Growth and Form*? - *Sarah Bonnemaison and Philip Beesley;* Geometries of Creation: Architecture and the Revision of Nature - *Ryszard Sliwka;*Old and New Organicism in Architecture: The Metamorphoses of an Aesthetic Idea - *Dųrte Kuhlmann;* Functional versus Purposive in the Organic Forms of Louis Sullivan and Frank Lloyd Wright - *Kevin Nute;* The Forces of Matter - *Hadas A. Steiner;* The Skin of the "Sky Bubble" at Expo '67 - *Sarah Bonnemaison;* The Geodesic Dome as a Metaphor for Expanding Consciousness - *Christine Macy;* Drawing Indeterminate Architecture and the Distorted Net - *Nat Chard;* Naturalization, in Circles: Architecture, Science, Architecture - *Reinhold Martin;* "A Diagram of Forces": Form as Formation in Nature and Design -*Ann Richards;* Tensegrity Complexity - *Thomas Seebohm;* Phenomeno-Logical Garden: A Work in Morpho-Logical Process - *Manuel Bбez;* Process in Nature and Process in Architecture: Inquiry into a Process of Unfolding - *Hajo Neis;* Synthesis of Form, Structure and Material – Design for a Form-optimized Lightweight Membrane Construction - *Edgar Stach.*

that support visualization in space and time have led to miniaturization of established technologies such as magnetic resonance imaging (MRI) and positron emission tomography (an imaging by sequential sectional cuts, known as PET), which permit the analysis of molecules and cells in living animals. The two-way street of evolutionary development involves molecular exchanges that can be detected with these tools. This ability to probe allows for the measurement of mechanical properties alongside observations of spatial and chemical dynamics. Adaptation to the environment through intimate linkages of natural forms and functions is now being described in mathematical detail. Molecular biology now asks critical questions about shape and structure at the scale of atoms, cells and organisms. A convergence of dynamic 'network' thinking from information technology has blurred the boundary between environment and organism. In turn, the natural world is being revealed in molecular detail as a dynamic ecology of interconnectedness. Similarly, computer-aided design is capturing the geometric relationships that form the foundation of architecture, building upon now-established practices of form-finding and finite element analysis (which breaks down a continuous structure into many simple, linked elements in order to find optimal thicknesses and arrangements of supporting elements). New developments in parametric modeling permit control of design through models that can coordinate and update themselves. These systems can automatically update the entire model or drawing set based on changes as small as a joint or as large as the entire floor plan, offering flexible design of deeply nested relationships. In much the same way that mutations in nature generate biodiversity, individual variation in architectural components can be achieved economically. Parametric design practice employs 'dependency' networks akin to the complex process diagrams used to express relationships in natural systems, offering increasingly fine-tuned approaches to building component design. Using these tools, Architectural disciplines are poised to work with increasing effectiveness in responsive, interactive systems." (Beesley P., and Bonnemaison, S., 2008)

Whether through design practice or a critical perspective on design, the essays in this book ask what might we learn by revisiting Thompson's way of seeing the world, and apply the answers to their own work. The array of design work reveals an ongoing interest in the pursuit of organicism, and the critical interpretations of history provide alternatives to a traditional historiography of modern architecture that celebrates the aesthetics of the machine, by bringing to light some of the questions raised by the thorny relationship between nature and artifice that make our world. In their own way, these essays contribute to this important discussion." (Beesley P., and Bonnemaison, S., 2008)

From the article "Invasion of the Shapeshifters" by the mathematician and writer Barry A. Cipra, published in *SIAM News*, Volume 40, Number 2, March 2007,[278] (Cipra, B.A., 2007) we learn about how D'Arcy Thompson influenced resent research in merger of topology and biology. Cipra introduces Monica Hurdal, a biomedical mathematician at Florida State University and Michael Miller, a biomedical engineer at Johns Hopkins University. In Boston, at 2006's SIAM Annual Meeting, Miller gave an invited presentation on computational anatomy and Hurdal spoke on "conformal cortical cartography" in a minisymposium on brain imaging. The goal of this mini symposium was to help in the diagnosis of neurological disorders, such as Alzheimer's disease and schizophrenia, and to understand how the brain works. In the presentation of computational anatomy, Miller explains, the realization of an idea dating back to D'Arcy Thompson's 1917 classic, *On Growth and Form*.

"Thompson proposed the use of coordinate transformations for comparing the shapes of related creatures. For example, the silver hatchetfish, *Argyropelecus olfersi*, can be transformed into its cousin *Sternoptyx diaphana*, the transparent hatchetfish, with a simple linear shear (Figure 1). (There is also a contraction: *diaphana* is smaller than *olfersi*.) Other comparisons use curvilinear transformations much like the pictures aerodynamic engineers draw when designing airfoils. A second, more recent and more mathematically advanced inspiration is Ulf Grenander's theory of deformable templates. Grenander, an applied mathematician at Brown University, is one of the founding fathers of mathematical pattern recognition, in which researchers aim to understand how it's possible to tell that two things are the same when everything about them is different. A deformable template is an idealization of a two- or three-dimensional object (the theory—and even some of its applications—extend to higher dimensions, but it's best to think in terms of visualizable pictures), which can be stretched, bent, twisted, and otherwise mangled in continuous fashion, to assume a range of "target images" corresponding to the object as it occurs in nature. You can think of it as an airbrushed photo printed on the proverbial rubber sheet of topology." (Cipra, B.A., 2007)

[278] "Invasion of the Shapeshifters" by the mathematician and writer Barry A. Cipra, published in *SIAM News*, Volume 40, Number 2, March 2007

Following Barry A. Cipra, we learn that:

"For anatomical studies, the template should represent the "normal" heart, brain, or other organ, whatever "normal" means. The basic idea is that abnormalities are present in a target image if matching the template to it requires a large deformation—whatever "large" means. One way to measure the size of a deformation is by defining a function on infinitesimal deformations that assigns a "cost" value at each point, depending on the amount of stretch or shear in its immediate neighborhood. (Rigid translations and rotations are usually cost-free, corresponding as they presumably do to simply lining up the pictures correctly.) As the template is continuously deformed onto the target image, each point racks up a total cost along the path it takes from template to target. The "cost" of the continuous deformation is the sum (actually the integral) of the costs for all the points. The distance between template and target is the smallest such cost among all such continuous deformations.

A continuous deformation that realizes this minimum distance can be thought of as a geodesic in the space of diffeomorphic connections between the template and target images. In fact, this gives, in theory at least, a way to specify a template: If one somehow has a probability distribution on the space of images according to their prevalence in nature, or perhaps according to their desirability, the template can be chosen as the image whose average distance to all other images is the least—a template, in other words, is a kind of infinite-dimensional centroid in a highly warped world." (Cipra, B.A., 2007)

Knowledge of physiology is crucial in analyzing the infinitesimal deformations, because tissue typically stretches more easily in some directions—along fibers, for example—than in others. This can be modeled mathematically with vector fields. Ideally, the imaging technology will measure the appropriate vectors. Diffusion tensor magnetic resonance imaging (DT-MRI) is a technology that does this. Miller and colleagues at the Center for Imaging Science at Johns Hopkins have developed what they call a "large deformation diffeomorphic metric mapping" (LDDMM) framework for data sets of this type, and applied it in numerical experiments on DT-MRI images of the heart and brain.

In a study of normal canine hearts, Miller and CIS colleagues Yan Cao and Laurent Younes, with Raimond Winslow of the Johns Hopkins Center for Cardiovascular Bioinformatics and Modeling, used the LDDMM software (see http://cis.jhu.edu/software/lddmm/) to match one image, chosen as the template, to the rest. What Thompson did painstakingly by eye and hand the computer now does automatically. By solving a variational problem (for which the researchers prove a general existence theorem), the algorithm zeroes in on the deformation that gives the best match between the vector fields of template and target (see Figure 2). (Cipra, B.A., 2007)

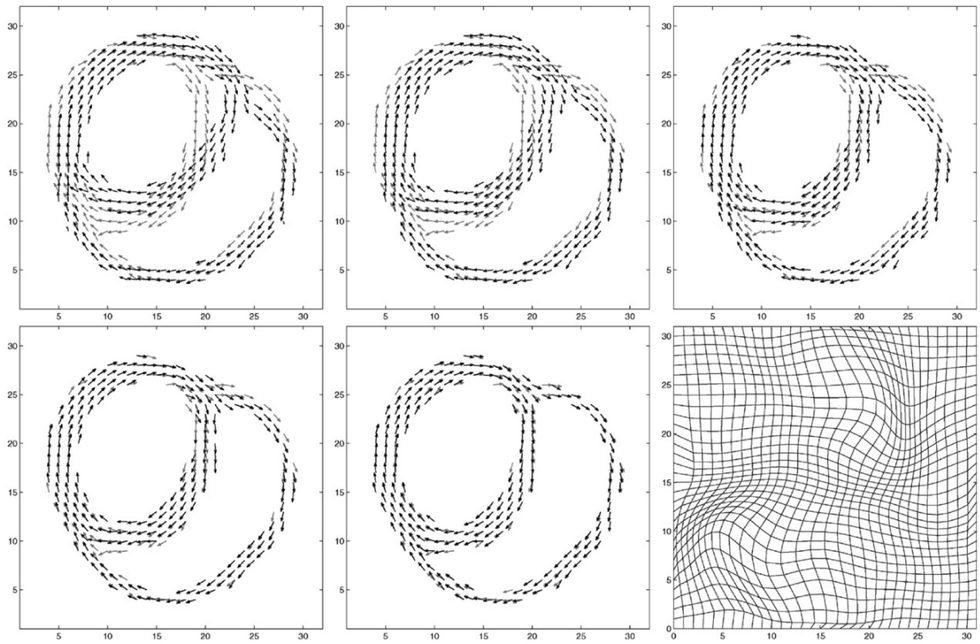


**Figure 2** /193/**.** Deformable dog hearts. Five steps along the way as a template canine heart matches with a target, and the grid that corresponds to the final result. (From Yan Cao, Michael I. Miller, Raimond L. Winslow, and Laurent Younes, "Large Deformation Diffeomorphic Metric Mapping of Vector Fields," IEEE Transactions on Medical Imaging, Vol. 24, September 2005.) (Cipra, B.A., 2007)

Structuralism and topology are indisputably linked with D'Arcy W. Thompson. He advocated structuralism as an alternative to survival of the fittnest in governing the form of species. Structuralism supports a non-Darwinian evolutionism. /194/. In his classic "On Growth and Form", D'Arcy Thompson analyzed biological form in terms of physical forces. With his famous set of diagrams, Thompson showed family ressemblences between species of fish by deforming grids through smooth coordinate transformation, suggesting that topology is basic to the overall plan of an organism.

D'Arcy W. Thompson's structuralism influenced thinkers as the antropologists Claude Levi-Sreauss[279] and Edmund Leach[280], and the topological phylosophy of Jacques Lacan, Jacques Derrida, Gilles Deleuze.

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279 In his survey of structural anthropology Michale Oppitz (1975) – See: Benjamin Baumann, On Centrism and Dualism - A Critical Reassessment of Structural Anthropology's Contribution to the Study of Southeast Asian Societies, Südostasien Working Papers No. 40, Berlin 2010 - gives a principal definition of the concept of structures, which he perceives as the totality of elements related in such a way that the modification of one element or one relation brings about a modification of the other elements or relations. This definition is based on the concept of totality, which includes the idea of interdependence between its constitutive elements, implying at the same time that the whole is more than the sum of its parts.

According to Oppitz, Levi-Strauss' conception of transformation was derived from the work of the British biologist D'Arcy Wentworth Thompson. Inspired by the Thompson's ideas, Levi-Strauss coupled the concept of transformation with that of structure, with this relationship becoming an essential aspect of his structuralism. This is, in fact, acknowledged by Levi-Strauss himself, who states that Wentworth Thompson's interpretation of the visible differences between species as transformations was an illumination for him that deeply affected his conception of structure. (See: cf. LÉVI-STRAUSS 1994: 427 [1991: 113] – in - **LÉVI-STRAUSS, Claude** 1994 [1991]. 'Anthropology, Race, and Politics: A Conversation with Didier Eribon', in: BOROFSKY (ed.), *Assessing Cultural Anthropology.* New York: McGraw-Hill: pp. 420-429/.

In fact Levi-Strauss confesed - "A very close relationship exists between the concept of transformation and that of structure, which occupies such a large place in our work" – See: LÉVI-STRAUSS 1976a: 18)." LÉVI-STRAUSS, Claude 1967a. 'The Story of Asdiwal', in: LEACH (ed.), *The Structural Study of Myth and Totemism.* London: Tavistock: pp. 1-49:18

280 Edmund Leach's concept of structure converges with the structuralism of Claude Lévi-Strauss. Leach was influenced by Lévi-Strauss and he introduced Claude Lévi-Strauss into British social anthropology. Leach's advocacy was based initially only on *The Elementary Structures of Kinship* (1949), as Lévi-Strauss's fuller presentations of structuralist theory had not yet been published. Leach's idea of structure is a transformation, but a recognizable one, concerned primarily with rules of social interaction, as was Lévi-Strauss at the time. The possibility that the rules of a given culture might conform to some underlying logical pattern of which the members of that culture—or, indeed, the social anthropologist—are unaware, is an important step away from the natural-science model of Radcliffe-Brown.

Leach proposes the topological concept of mathematical function as a ratio of two terms, themselves relational (e.g., maternal versus paternal filiation) as the basis for the comparative analysis of social structure. In his book "Rethinking Anthropology" /1961/ Leach described topology as a geometry of elastic rubber sheeting precisely because the shape and size of things or the distance between them is less significant than what holds them together; that is, the *ways* in which they are connected, the nature of their relatedness, so to speak (Leach, 1961)- See: Leach, E. R. [1961] *Rethinking Anthropology*. London: The Athlone Press.

Structural studies are, in the social sciences, the indirect outcome of these approaches that can offer tantalising and infinitely explorable links between the quantitative and the qualitative in research and shed light on the relevance of mathematics also to social sciences. Levi-Strauss forays into e.g. topology can be further explored. Another call for multi-disciplinarity. 'Considered mathematically society is not an assemblage of things but an assemblage of variables.' (Edmund Leach) : 'the geometry of elastic rubber sheeting." i.e. one can change the manifest shape of the original geometrical figure out of all recognition and still there is a sense in which it is the same figure all the time. 'The constancy of pattern is not manifest as an objective empirical fact but it is there as a mathematical generalization." Leach : since topology is a non-metrical form of mathematics it deserves especial attention from social scientists.. For Leach – topology allows us to cease to be interested in particular

From Lévi-Strauss' anthropology we can extract a series of conceptual oppositions which are relevant to the problem of biological form, and one of this opposition is a 'rational' system of transformations (a structure) *versus* a purely 'empirical' (temporal or spatial) order[281] (Kull, K., 2010)

This opposition of structuralism and Darwinism is inheried by Levi-Strauss by D'Arcy Thompson. And as Kalevi Kull claims in "Structuralism and semiotic of organic form" – "This opposition of structuralism and Darwinism is not simply a debating device. In the first place, it is of historical significance for a number of earlier writers ([William] Bateson, Driesch, D'Arcy Thompson) who employed structuralist concepts did so in the course of formulating critical evaluations of Darwinism.[282] [283] (Kull, K., 2010)

Atuhiro Sibatani in his report on the so called Osaka group [284] concluded Osaka group's consideration about Structuralism as a Scientific Movement. Sibatani assertes that "Some of the ideas described Above /"On Structuralist Biology"/ (Sibatani, A., 1987), may seem to be still groundless or premature at the best. However, Dave

relationships and look at the regularities of pattern among neighbouring relationships. Thus "the same structural pattern may turn up in any kind of society". Critique of Radcliffe-Brown, and support of Levi-Strauss.

281 Kalevi Kull, "Structuralism and semiotic of organic form", Presentation, 2010, Seminar SEMIOTICS OF ORGANIC FORM in the Department of Semiotics, Tiigi St. 78-119, University of Tartu, Tartu, Estonia

282 Kalevi Kull, "Structuralism and semiotic of organic form", Presentation, 2010, Seminar SEMIOTICS OF ORGANIC FORM in the Department of Semiotics, Tiigi St. 78-119, University of Tartu, Tartu, Estonia

283 *The Osaka Group and Other Recent Movements in Structuralist Biology.* Contributions by Webster and Goodwin during the years of 1982-1984 to structuralism in biology have made a great impact on certain biologists in different countries who are interested in evolution. See: Atuhiro Sibatani, On Structuralist Biology, Riv. Biol. - Biol. Forum 80 (1987), pp. 558-564

284 Osaka group: After contributions by Webster and Goodwin during the years of 1982-1984 to structuralism in biology have made a great impact on certain biologists in different countries who are interested in evolution. Goodwin and Webster were inspired by structuralism (*sensu lato*) as developed mainly by Lévi-Strauss, Piaget and Chomsky. They maintain that biological form has not emerged as a historical process under the influence of environment upon organisms subject to inherently structureless variation. According to them, the ahistorical nature of biological evolution is based on autonomous or intrinsic laws which apply on both animate and inanimate subjects. A few scientists published papers declaring 'structuralism' as their guiding principle: Kauffman (1985, 1987), Lambert and Hughes (1984), Sibatani (1985), Van der Hammen (1985). They exerted, in their turn, further influences upon other scientists. As a part of this process, played a major role in disseminating ideas borne by this newly, emerging trend in biology. Since many of these scientists worked in remote countries as minorities in their respective circles, and without personally knowing one another, people thought it useful to bring them together for interactions. It was the international workshop on structuralism in biology at Osaka, Japan, during 7-11 December 1986. The 20 odd scientists from overseas were joined by about 30 residents of Japan, both groups including some who did not necessarily identify themselves as structuralists.

Lambert at the workshop emphasized the crucial importance in science of the way of seeing things and the language to signify what is seen. Structuralism is a will to see things in the way it intends to see, prompted by its autonomous, internally determined objectives of the movement. Of course, eventually, the whole movement must be tested by the outcome – good correspondences with the biological reality and satisfactions generated after successfully understanding it. Until a sufficient time has past, it would be futile to argue against structuralism with the conventional views that depend upon strength of available evidence. The impressive fact is that, knowing that it is a minority view, structuralism in biology has still attracted quite a sizeable number of scientists, working mostly independently. Thus structuralism is a scientific movement, rather than a mere scientific methodology or metaphysics, with the object of making visible those things which may otherwise remain invisible." (Kull, K., 2010)

Structuralism and Semiotics are the field of qualitative quantity due to it exhibit form of "structural stability" and topological notion of gradualness. Qualitative quantity deals with both morphology /shape/ and physiology /process/, with the development as reversible changes of shapes and processes and evolution as irreversible changes of shapes and processes. Qualitative quantity and Semiosis are linked. And if "biological structuralism has been a forerunner for biosemiotics, likewise the structuralism in humanities has been a forerunner for semiotics." (Kull, K., 2010)

The philosophy of Qualitative quantity is the core of the topological philosophies and sustainability /living balance/ due to the claim that *living balance (**sustainability**) is themaintenance of **qualitative** (incommensurable) **diversity.***

**Semiosis is, in fact, *the* instrument which assures the maintenance of the steady state of any living entity.**

Qualitative quantity is indispensable to the topological descriptions of living processes and "Algebraic/topological descriptions of living processes are indispensable to the understanding of both biological and cognitive functions. [...] fundamental algebraic description of living/cognitive processes [...] exposes [their] inherent ambiguity. Since ambiguity is forbidden to computation, no computational description can lend insight to inherently ambiguous processes. The impredicativity of these models is not a flaw, but is, rather, their strength. It enables us to reason with ambiguous mathematical representations of ambiguous natural processes. The noncomputability of these structures means computerized simulacra of them are uninformative of their key properties. This leads to the question of how we should reason about them." (Louie, A.H., and Kercel S.W., 2007).[285]

## 1.4. Hegel and Topological Qualitative quantity Measure in Sorites Paradox [286] and Vagueness

Hegel presents the 'vagueness' and the sorites paradoxes (the Heap and the Bald), in the first chapter entitled "Specific Quantity" ("Qualitative quantity"), in the second section following the discussion of Specific quantum (section one), (the third part of the Logic of Being – Measure). With the Heap and the Bald, Hegel illustrates the "Specific quantum".

Hegel states the following under section a. the specific quantum:

1.Measure is the simple self-reference of quantum, its own determinateness in itself; quantum is thus qualitative. At first, as an immediate measure it is an immediate quantum and hence some specific quantum; equally immediate is the quality that belongs to it; it is some specific quality or other. – Thus quantum, as this no longer indifferent limit but as selfreferring externality, is itself quality and, although distinguished from it, (**21.330)** it does not extend past it, just as quality does not extend past quantum.

Quantum is thus the determinateness that has returned into simple selfequality – which is at one with determinate existence just as determinate existence is at one with it.

[285] A. H. Louie & Stephen W. Kercel (2007). Topology and Life Redux: Robert Rosen's Relational Diagrams of Living Systems. *Axiomathes* 17: 109–136

[286] For Mathematical Induction Sorites and general topological characterization of the paradox.See Sorites Paradox In Atanford Encyclopedia of Philosophy, 1997, referring to Weber, Z. and Colyvan, M., 2010. 'A topological sorites', The Journal of Philosophy, 107: 311–325 concerning their proposal for a fully general form of the paradox, a general topological characterization of the paradox.

If a proposition is to be made of the determination just obtained, it could be expressed thus: “Whatever is, has a measure.” Every existence has a magnitude, and this magnitude belongs to the very nature of a something; it constitutes its determinate nature and its in-itself. The something is not indifferent to this magnitude, as if, were the latter to alter, it would remain the same; rather the alteration of the magnitude alters its quality. As measure this disappearance as if before one’s eyes: since the quantum is posited as the external limit which is by nature alterable, the *alteration* (of quantum only) then follows by itself. But in fact nothing is thereby explained, for the alteration is at the same time essentially the transition of one quality into another, or the more abstract transition of one existence into a non-existence, and therein lies another determination than just gradualness, which is only a decrease or increase, and the one-sided holding fast to magnitude.”[287]

Hegel states that “The ancients had already taken notice of this coincidence, that an alteration which appears to be only quantitative suddenly changes into a qualitative one, and they used popular examples to illustrate the inconsistencies that arise when such a coincidence is not understood. Two such examples go under the familiar names of “the bald” and “the heap.” They are *elenchi*, that is, according to Aristotle’s explanation, two ways in which (**21.332)** one is compelled to say the opposite of what one has previously asserted.( See Aristotle, *Sophistical Refutations*, 164b 27–165a 2.).

The question was put: does the plucking of one hair from someone’s head or from a horse’s tail produce baldness, or does a heap cease to be a heap if one grain is removed? The expected answer can safely be conceded, for the removal amounts to a merely quantitative difference, and an insignificant one at that. And so one hair is removed, one grain, and this is repeated with only one hair and one grain being removed each time the answer is conceded. At last the qualitative alteration is revealed: the head or the tail is bald; the heap has vanished. In conceding the answer, it was not only the repetition that was each time forgotten, but also that the individually insignificant quantities (like the individually insignificant disbursements from a patrimony) *add up*, and the sum constitutes the qualitative whole, so that at the end this whole has vanished: the head is bald, the purse is empty.

[287] Georg Wilhelm Friedrich Hegel, The Science of Logic, edited and translated by George Di Giovanni, 2010, Cambridge University Press, p.291 (21.330)

The embarrassment, the contradiction, produced by the result, is not anything sophistic in the usual sense of the word, as if the contradiction were a pretense. The mistake is committed by the assumed interlocutor (that is, our ordinary consciousness), and that is of assuming a quantity to be only an indifferent limit, that is, of taking it in the narrowly defined sense of a quantity. But this assumption is confounded by the truth to which it is brought, namely **that quantity is a moment of measure** and **is linked to quality;** refuted is the one-sided stubborn adherence to the abstract determinateness of quantum. – Also those elenchi are, therefore, not anything frivolous or pedantic but basically correct: they attest to a mind which has an interest in the phenomena that come with thinking.

Quantum, when it is taken as indifferent limit, is the side from which an existence is unsuspectedly attacked and laid low. It is the *cunning* of the concept that it would seize on an existence from this side where its quality does not seem to come into play – and it does it so well that the aggrandizement of a State or of a patrimony, etc., which will bring about the misfortune of the State or the owner, even appears at first to be their good fortune."[288]

For Hegel these elenchi of "the bald" and "the heap" are basically correct and attest to a mind the transformation of Qualitative quantity (measure), the measure that could be regarded as the measure of 'soros', where gradualness is presented with the change 'little by little'.

In logic, the sorites paradox is the name given to a class of paradoxical arguments, also known as 'little-by-little arguments', which arise as a result of the indeterminacy surrounding limits of application of the predicates involved. The Sorites Paradox demonstrates the vagues of qualitative-quantitative relation.

Within the Qualitative quantity measure, transformation between qualitative and quantitative is topological, emphasizing on such topological concepts as 'boundary' and 'limit'. For example, the concept of a heap appears to lack sharp boundaries and, as a consequence of the

[288] Georg Wilhelm Friedrich Hegel, The Science of Logic, edited and translated by George Di Giovanni, 2010, Cambridge University Press, p.291 (21.330)

subsequent indeterminacy surrounding the extension of the predicate ‘is a heap’, no one grain of wheat can be identified as making the difference between being a heap and not being a heap. Given then that one grain of wheat does not make a heap, it would seem to follow that two do not, thus three do not, and so on. The single quantitative addition one by next seems as disconnected by their quality, the quality of the quantitative and this ‘disconnection’ is perceived as real, indeed there is connection yet inaparent. In the end it would appear that no amount of wheat can make a heap. We are faced with paradox since from apparently true premises by seemingly uncontroversial reasoning we arrive at an apparently false conclusion.

The paradigmatic cases of the sorites paradox—heaps of sand and bald heads—are cases where the changes in question are small but discrete. Weber and Colyvan, in their original paper *A topological sorites* proposes a topological formulation of sorites and the problematic concept of vagueness. Weber and Colyvan asserts that in the case both of the heap and of baldness, there is a natural ordering, in terms of the number of grains of sand and the number of hairs. The authors calls such versions of the sorites paradox discrete and numerical.

In part three of their paper Weber and Colyvan assert that:

“For millennia, geometers attempted to prove Euclid’s parallel postulate. In the late eighteenth century came awareness that there are models of the first four Euclidean axioms that do not respect the parallel postulate. By the nineteenth century, in his landmark paper on the foundations of geometry, Riemann was able to diagnose why there are such models: The first four postulates, he saw, codify topological properties of the space, while the fifth is a specifically metric property.”[289] (Weber, Z. and Colyvan, M., 2010)

The lesson from Euclid is that there is a distinct science of space that does not deal in metric, quantitative notions, but only in qualitative notions like closeness.

<u>“It will be useful to describe the standard concepts of point-set topology.</u>

The basic primitive (though intuitively familiar) notion is that of open set. Let X be a set. A topology is a collection of open subsets of X, closed under union and finite intersection, and

[289] Weber, Z. and Colyvan, M., 2010. ‘A topological sorites’, *The Journal of Philosophy*, 107: 311–325.

including X and the empty set ;. Let A be a member of the topology on X. A point x is interior to A, and A is a neighborhood 10 of x, iff there is an open set U where x 2 U # A. A set A is open iff all its points are interior, that is, A is a neighborhood of all x 2 A.

The interior of A is its largest open subset, the union of its open subsets, A°. The closure of A is its smallest closed superset, the intersection of closed supersets, A−. The interior, the set, and the closure sit like this:

A+ # A # A−: A set A is open if A is contained in its interior, A # A°, and A is closed if A contains its closure, A− # A. Therefore a set is both open and closed if A° 5 A−.

Definition 1 A space X is connected iff the only sets in the topology of X that are both open and closed are X and . . "(Weber, Z. and Colyvan, M., 2010).

The following consequence could serve equally well as the definition of connectedness."[290] (Weber, Z. and Colyvan, M., 2010)

For Weber and Colyvan, "The space of threatened species, for example, has several degrees of freedom: species numbers, area and quality of habitat, rate of decline, and others. While each of these is arguably numerical, it is not at all clear how to combine them in a meaningful fashion. Any proposed metric in this space, it seems, will be problematic.

Our topological characterization leaves open the question of whether the space really is connected; it leaves the question sharpened. Perhaps the concern can be put differently—as a concern about maintaining a neutral dialectic. We have proven that whenever we have a sorites series, the underlying space must be disconnected. This should be music to the ears of epistemicists about vagueness, for they take the lesson learned from the sorites paradox to be that all such spaces are (surprisingly) disconnected." [291] (Weber, Z. and Colyvan, M., 2010)

[290] Weber, Z. and Colyvan, M., 2010. 'A topological sorites', *The Journal of Philosophy*, 107: 311–325.p.318

[291] Weber, Z. and Colyvan, M., 2010. 'A topological sorites', *The Journal of Philosophy*, 107: 311–325.

Weber and Colyvan's discussion of a generalized sorites via an appeal to classical point-set topology was not intended to suggest that classical topology should be used in solving the problem, as the authors assrets "We do not expect that we have said enough here to convince everyone that the sorites is essentially topological. An important concern is whether the generalization is too general. A topological characterization of vagueness is a generalization on the canonical discrete, numerical sorites in the same way that topology itself is a generalization on certain properties of the real numbers. One important consequence of the topological approach is that, since a topological sorites does not require assumptions about order, any proposed solutions to sorites that trade on the details of order are not going to be general solutions."[292] (Weber, Z. and Colyvan, M., 2010)

Weber and Colyvan's conclusion is that "a topological sorites is recognizably a generalization of the canonical sorites and that the topological characterization captures the essential ingredients— namely, connectedness and local and global constancy."[293] (Weber, Z. and Colyvan, M., 2010)

This phenomenon at the core of the paradox is exibit form of Hegel's Qualitative quantity and nowadays is recognised as the phenomenon of vagueness.

In analytic philosophy and linguistics, a concept may be considered **vague** if its extension is deemed lacking in clarity, if there is uncertainty about which objects belong to the concept or which exhibit characteristics that have this predicate (so-called "border-line cases"), or if the Sorites paradox applies to the concept or predicate.

Vagueness is philosophically important. Suppose one wants to come up with a definition of "right" in the moral sense. One wants a definition to cover actions that are clearly right and exclude actions that are clearly wrong, but what does one do with the borderline cases? Surely, there are such cases. Some philosophers say that one should try to come up with a definition that is itself unclear on just those cases. Others say that one has an interest in making his or her definitions more precise than ordinary language, or his or her ordinary

[292] Weber, Z. and Colyvan, M., 2010. 'A topological sorites', The Journal of Philosophy, 107: 311–325.
[293] Weber, Z. and Colyvan, M., 2010. 'A topological sorites', The Journal of Philosophy, 107: 311–325.

concepts, themselves allow; they recommend one advances precising definitions.The philosophical question of what the best theoretical treatment of vagueness is - which is closely related to the problem of the paradox of the heap, a.k.a. sorites paradox - has been the subject of much philosophical debate.

The phenomenon of vagueness bear a topological character. (Hill, Brian, 2006). In *The (topo)logic of vagueness,* Brian Hill proposed a '*topological*' theory of vagueness."[294] Hill assrets that "such a theory of vagueness may find inspiration, or perhaps even a formalisation, in the mathematical theory of *topology*, which deals with abstract notions of *space* and notably *closeness*."[295] (Hill, Brian, 2006)

Hill states that "the problem of vagueness concerns scales, whose most important aspects are their ability to *distinguish* among objects or points. But intuitively, what cannot be distinguished is *close* in a certain sense, and so there is a natural affinity between mathematical topology and the notion of scale. In fact, the notion of scale introduced above is a primitive version of what mathematicians call a *topology*: a topology is commonly defined as a set of so-called "open sets", satisfying certain conditions. Indeed, several recent advances in topology, and notably the introduction of the field of formal topology, where no sets of points are *supposed*, may prove useful. …The relationship with topology is currently intuitive and prospective; if it could be fruitfully developed, one would indeed obtain a '*topological*' theory of vagueness."[296] (Hill, Brian, 2006)

Hill concludes that "Vagueness has been recognised as an important topic in philosophy. A large number of everyday and philosophical terms are vague, and are none the less useful for it. They can be used to say important things in many interesting cases; however, there are situations where they are less useful, there are questions to which they do not permit answers. Instead of attempting to force a truth value on these cases (of whatever logical flavour), perhaps a theory of vagueness should be more concerned with understanding where vague

[294] Hill, Brian, 2006*, The (topo)logic of vagueness,* paper presented at the IHPST (May 2006), ENFA (June 2006), Mind-Aristotelian Society Meetings (July 2006) and The Prague International Colloquium (September 2006). The author would like to thank all audiences for comments., p.14

[295] Sutherland, W. A. (1975). Introduction to Metric and Topological Spaces. OUP, Oxford.

[296] Hill, Brian, 2006*, The (topo)logic of vagueness,* paper presented at the IHPST (May 2006), ENFA (June 2006), Mind-Aristotelian Society Meetings (July 2006) and The Prague International Colloquium (September 2006). The author would like to thank all audiences for comments., p.14

terms “work” and where they don’t, with which terms they can be used and with which ones they can’t. At worst, such a theory would introduce a different perspective on vagueness, and its relationship to precision.”[297] (Hill, Brian, 2006)

As a conclusion on the above discussion, my suggestion is that Hegel’s notion of Qualitative quantity/ Specific quantity/ Qualitative quantum is instrumental in constructing a topological theory of vagueness” and revealing topological ‘relationships between specific qualitative quanta’.

Vagueness is philosophically important and can be traced in philosophy of Kant[298] and Hegel[299].

The topological notion of Qualitative quantity/ Specific quantity/ Qualitative quantum (Hegel) and the ‘relationships between specific qualitative quanta’ [300], the measure of Qualitative quantity, where determinancy (of real measure) becomes a determinate determinacy, relations between measures, demonstrates itself with the “dichotomy” of Zeno’s paradox as Zeno’s paradox of measure. [301]

Zeno’s “dichtomy” paradox argues that a runner in a race will never finish: before reaching the finish, he must get to the half-way point $p_1$; but once he has got to $p_1$, he must still reach the point half-way between $p_1$ and the finish, say $p_2$; but once he has reached $p_2$ . . . So, one might conclude, he never really finishes the race.

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297 Hill, Brian, 2006, *The (topo)logic of vagueness,* paper presented at the IHPST (May 2006), ENFA (June 2006), Mind-Aristotelian Society Meetings (July 2006) and The Prague International Colloquium (September 2006). The author would like to thank all audiences for comments., p.14

298 Boniolo Giovanni and Valentini Silvio, 2008, Vagueness, Kant and Topology: A Study of Formal Epistemology, *Journal of Philosophical Logic*, Vol. 37, No. 2 (April 2008), pp. 141-168, Springer. – Boniolo and Valentini‘s approach a vagueness as characterized by two features. The first one is philosophical following a Kantian path emphasizing the knowing subject's conceptual apparatus. The second one is formal, faceing vagueness, and a philosophical view on it, proposing the use of topology and formal topology. The authors show that the Kantian and the topological features joined together allow an atypical, but promising, way of considering vagueness.

299 **See: Angelica Nuzzo’s Vagueness and meaning variance in Hegel's logic, in** Nuzzo, A. ed., 2010, Hegel and the Analytic Tradition, London: Continuum, 2010, pp.208

300 Winfield, Richard Dien, 2012, Hegel's Science of Logic: A Critical Rethinking in Thirty Lectures, Rowman & Littlefield Publishers, Inc., p.145

301 Brian Hill, Zeno’s Paradox of Measure . . .

The sorites paradox argues that a man with a full head of hair may count as bald: for a man with 0 hairs counts as bald; and if a man with 0 hairs counts as bald, a man with 1 hair counts as bald; and if a man with 1 hair counts as bald, a man with 2 hairs counts as bald; and . . . ; and if a man with 4999 hair counts as bald, a man with 5000 hairs counts as bald, so a man with 5000 counts as bald. [302]

As Brian Hill asserts, "there is a certain structural ressemblance between these arguments. Each involves a long series of apparently legitimate steps, which, taken together, yield an unacceptable conclusion. Despite this similarity, philosophers have tended to treat the two paradoxes separately and in different ways. One commonly accepts Zeno's infinite sequence as "compatible" with the race finishing in a finite number of seconds, whereas one often reacts to the sorites paradox by proposing a semantic, epistemic or pragmatic theory of vagueness which denies the sorites argument. Consequently, it poses a certain set of challenges to prospective theories of vagueness."[303] (Hill, 2006, )

Hill asserts that there is apparent *structural* similarity between the two paradoxes and on the base of this structural similarity one can transpose aspects of a common reply to Zeno's paradox onto the sorites case. More precisely, it shall be noted that compatibility of Zeno's argument with the race finishing in a finite time is not enough to defuse the paradox, for it is also necessary to assert that the number of seconds (not the number of stages of Zeno's sequence) is the *appropriate* measure for whether the runner has finished the race or not. If one tries to apply this aspect of the reply to Zeno's paradox in the sorites case, one ends up rejecting the argument on the grounds that the number of hairs is not appropriate for reasoning about baldness. This will have some interesting consequences for the conception of the *problem* which vagueness poses, and thus for what one should expect from a theory of vagueness."[304]

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[302] Hill, Brian, 2006, *The (topo)logic of vagueness,* paper presented at the IHPST (May 2006), ENFA (June 2006), Mind-Aristotelian Society Meetings (July 2006) and The Prague International Colloquium (September 2006). The author would like to thank all audiences for comments., p.1

[303] Hill, Brian, 2006, *The (topo)logic of vagueness,* paper presented at the IHPST (May 2006), ENFA (June 2006), Mind-Aristotelian Society Meetings (July 2006) and The Prague International Colloquium (September 2006). The author would like to thank all audiences for comments., p.1

[304] Hill, Brian, 2006, *The (topo)logic of vagueness,* paper presented at the IHPST (May 2006), ENFA (June 2006), Mind-Aristotelian Society Meetings (July 2006) and The Prague International Colloquium (September 2006). The author would like to thank all audiences for comments., p.1

Hill assrets that “such a theory of vagueness may find inspiration, or perhaps even a formalisation, in the mathematical theory of *topology*, which deals with abstract notions of *space* and notably *closeness*.”[305]

Hill states that “the problem of vagueness concerns scales, whose most important aspects are their ability to *distinguish* among objects or points. But intuitively, what cannot be distinguished is *close* in a certain sense, and so there is a natural affinity between between mathematical topology and the notion of scale. In fact, the notion of scale introduced above is a primitive version of what mathematicians call a *topology*: a topology is commonly defined as a set of so-called “open sets”, satisfying certain conditions. Indeed, several recent advances in topology, and notably the introduction of the field of formal topology,[306] where no sets of points are *supposed*, may prove useful. …The relationship with topology is currently intuitive and prospective; if it could be fruitfully developed, one would indeed obtain a ‘*topological*’ theory of vagueness.”[307]

Hill concludes that “Vagueness has been recognised as an important topic in philosophy. A large number of everyday and philosophical terms are vague, and are none the less useful for it. They can be used to say important things in many interesting cases; however, there are situations where they are less useful, there are questions to which they do not permit answers. Instead of attempting to force a truth value on these cases (of whatever logical flavour), perhaps a theory of vagueness should be more concerned with understanding where vague terms “work” and where they don’t, with which terms they can be used and with which ones they can’t. At worst, such a theory would introduce a different perspective on vagueness, and its relationship to precision.”[308]

Hegel’s notion of Qualitative quantity/ Specific quantity/ Qualitative quantum is instrumental in constructing a topological theory of vagueness” and revealing topological ‘relationships between specific qualitative quanta’.

[305] Sutherland, W. A. (1975). Introduction to Metric and Topological Spaces. OUP, Oxford.

[306] Sambin, G. (2003). Some points in formal topology. *Theoretical Computer Science*, 305:347–408.

[307] Hill, Brian, 2006*, The (topo)logic of vagueness,* paper presented at the IHPST (May 2006), ENFA (June 2006), Mind-Aristotelian Society Meetings (July 2006) and The Prague International Colloquium (September 2006). The author would like to thank all audiences for comments., p.14

[308] Hill, Brian, 2006*, The (topo)logic of vagueness,* paper presented at the IHPST (May 2006), ENFA (June 2006), Mind-Aristotelian Society Meetings (July 2006) and The Prague International Colloquium (September 2006). The author would like to thank all audiences for comments., p.14

Most of the Zeno's paradoxes (the dichotomy paradoxes) exhibit the notion of gradualness of 'qualitative quantity' transformation dealing with counterintuitive aspects of continuous space and time.

There are cases, concludes Caws, in which "cumulative imperceptible changes in *x* lead to the emergence of *y* , and there are cases in which they just lead to more *x* —and either *x* or *y* can be indifferently qualitative or quantitative predicates; everything depends on the particular case, and can only be learned by looking. Adding atom after atom to a lump of uranium 235 eventually produces an atomic explosion and an assortment of vaporized fission products; adding atom after atom to a lump of gold just produces a bigger lump of gold. Water when refrigerated changes into ice; iron when refrigerated gets colder but doesn't change into another form." (Caws, P. 1993:250)

Caws is radical in his conclusion that "the dialectical law of the passage of quantity into quality, like its companions, the law of the interpenetration of opposites and the law of the negation of the negation, is thus seen to be an entertaining but nonessential red herring." (Caws, P. 1993:250)

### 1.5. Topological Notions of Multiplicity in Hegel's Fourfold of Infinities

The thesis of Topological Notions of Multiplicity in Hegel's Fourfold of Infinities is strongly supported by the research of Andrew Haas, in particular his book "Hegel and the Problem of Multiplicity". (Haas, A. 2000)

If for Heidegger quantification is the emasculation of spirit, a misinterpretation of quantity ("the quantity took on quality of its own") due to the fact that Heidegger based his thesis on Nietzschean metaphor, opposing the Cartesian proper of mathesis universalis that seeks to become master and processor of nature via technologic innovation, for Hegel, the spirit can receive the mark of quantification, then it is because quantity is not just misinterpretation.

For Hegel, quality is the expression of essence. Quality, for Hegel implies the pure interchangeability of beings. Beings are radically and qualitatively incomparable and yet completely and quantitatively convertible due to the possibility of translation to quantitative determinateness. Quantity is the other of quality. The truth of qualitative determination lies on its quantitative side. Quality is repulsion and attraction sunk into equilibrium, mutually into one-sided identity, that posits itself, like being, as pure – but this time, as pure quantity. Quantity in Hegel's logic is not the reduction of all quality to numbers, since numbers for Hegel are not simply numbers. Quantity in Hegel is not the abstraction of all determinations in terms of amount of units, the death of things in mechanism, calculation and computational codes, but it is the movement of the moments of pure quantity, quantum, that is determinate quantity and quantitative ratio. (Haas, A. 2000: 113)

The particular quality of quantification, the reproduction and restoration is taken care of in the dialectic of qualitative-quantitative concept – and in Hegel, *multiplicity means the quantification of quality and the qualification of quantity, a multiplicity of the double-entendre, of the inevitable double-meaning.* (Haas, A. 2000: 113)

Quantitative 'multiplicity' is *first* posited as continuity. *Second,* 'multiplicity' breaks up as discreteness, the negation of continuity. *Pure quantitative multiplicity* is the continuous discreteness that contains the totality of its prior moments within itself. In this way quantitative purity has the possibility and necessity of self-production "without further determination": space and time as the perpetual self-creation of multiple units, where space is as being-out-of-itself in points, lines, and planes, and time is as coming-out-of-itself in past, present, and futures. Pure quantitative multiplicity is discrete continuity—multiplicity without quality. The many seems as quantity in-and-for-itself, the simple unity wherein each is what the other is, namely 'one among many.' The concept of pure quantity is the unity of continuing and discrete multiplicity, a quantitative multiplicity, which, in its self-negation, remains multiple.

Continuity and discreteness are simultaneously in relation to themselves via negation of the other. Discrete and continuous quantity, are determinate magnitudes of being. In other words, pure quantity as discrete continuity (and continuous discreteness) is multiple (and unified) only insofar as it is limited—and this concept of limitation, the limit that is and is not a limit, is called quantum.(Haas, A. 2000: 115)

For Hegel, the inability to read history as two-sided, as both continuous and discrete, to see beings as double, as both qualitative and quantitative, is the road to the destruction of spirit.

Andrew Hass, proposed and answered a question with great importance, the question of understanding the core and essence of Hegel's logic and dialectics in terms of homology and topology – *What does it mean to think multiplicity as two-sided, as subject to the process or movement of double-edged, de-limitation, to the logic of the concept and its relational borders, frontiers, horizons, thresholds? . . . and What does it mean to think multiplicity as both quality and quantity?* (Haas, A. 2000: 115)

My reply to this question is Topology, and the Topological notions of Hegel's multiplicity, the Topological notion of Hegel's fourfold of infinities, and the topological cobordism of Qualitative quantity.

In my understanding of the topological implementation of Qualitative quantity, I agree with Andrew Hass's claim that "Quality is in the core of quantitative infinity. Infinity is a quality of quantity (genetives objectivus and subjectivus) – and qualitative multiplicity is the limit of quantitative multiplicity. Yet with the concept of infinity, quantum is no longer simply immediate, rather, it determines itself (via negation of negation, the return of already superseded qualitative infinity) as good and bad, achived and unachieved qualitative and quantitative infinity". (Haas, A. 2000: 121)

The qualitative aspect of conceptual infinity appears most clearly in ratios. Haas concludes that, "with respect to the limit of the ratio, for example, Hegel invokes calculus in order to argue for the implicit quality of quantity, the qualitative relation of quantitative ratios; and of the qualitative character of quantitative infinity." And "in other words, the qualitative aspect of quantitative means – qualitative opposition, that is, having determinateness in in another, by means of non-being, having its being by virtue of nothing." (Haas, A. 2000: 125)

Hegel's fourfold of infinities is build on the fourfold of quality and quantity ratio – the notions and ratios of *quantitative quantity; quantitative quality; qualitative quantity; qualitative quality*. The fourfold of the qualitative and quantitative ratios, relate to the fourfold of the measure. All measures are ratios of two other measures. Measure is twofold, divided into two

—the external and the internal measure. This is the dialectical real of the real measure. Measure is the unity of quality and quantity, yet in *the center* of the series of measures is a *master signifier* that organizes everything, even while escaping measurement. This 'master signifier', the empty center *between* quality and quantity is like the *hole of the torus*, the nothing, the void, and Hegel names it *'substrate.'* The substrate is *discontinuous* within the series of measure *and continuous* at the same time. The substrate is what Hegel calls a *true infinity*. Substrate can be organized in a series of measures. Substrate is abstract measureless. What is important to see at this point in relation to measure is entailed in a duality between the nodal relation of quantity and quality, on the one side, and substrate, on the other side. The first side is measure as such—quantity and quality. The second side—the substrate—is something deeper than quantity and quality.

Concerning the four measures of quantitative quantity; quantitative quality; qualitative quantity; qualitative quality, the question proposed by Andrew Haas— How the multiplicity of measure is possible? (Haas, A. 2000: 140) —is in place. If all things are plural, states Haas, then must their measure not also be plural? For the science of logic, the multiplicity of all things is measured by the concept, and the concept of measure is the multiple measure of all things. Here measure is political, physical, chemical, biological, social, economic, musical, aesthetic, and so on. The measure is quality and quantity, the qualitative-quantitative and quantitative-qualitative concept (Haas, A. 2000: 139) – the Conceptual Measure of Qualitative quantity (concept).

### 1.6. Alain Badiou's Mediation on the fragile verbal foortbridge of Hegel's multiplicity

To my knowledge, in contemporary dialectics and philosophical research the significant notion of Hegel's category "qualitative quantity" remained inapparent in Heidegger's sense of his "Phenomenology of Inapparent". (Heidegger, M. 1973) The reason for this is probably Hegel's warning about "the intellectual difficulty" (*The Science of Logic*, § 777, *The Greater Logic*), and his accusing "the attempt to explain coming-to-be or ceasing-to-be on the basis of gradualness of the alteration" as "tedious like any tautology."[309] In contemporary philosophical research the creative power of tautology was definitely recognized.[310]

This "inapparent" nature and notion of Hegel's qualitative quantity based on and related to the "gradualness" of transformations (as an exhibit form of the qualitative quantity), are established and built by Hegel himself, and "blocked" by him with the self-imposed 'embargo' of "tautology" implemented in Hegel's claim that "the attempt to explain coming-to-be or ceasing-to-be on the basis of gradualness of the alteration" is "tedious like any tautology."

Probably this is the reason why Alain Badiou calls Hegel's notion of infinities—the domain of the qualitative quantity—a "fragile verbal foortbridge." (Badiou, A. 2006: 168-169)[311]

In 1988 with the first publication of *L'être et l'évènement* (Being and Event),[312] translated in English only in 2005, in "Mediation Fifteen on Hegel", Alain Badiou recognizes "qualitative quantity" as the core of the domain of "quantitative infinity", claiming that "Quantitative infinity is quantity qua quantity, the proliferator of proliferation, which is to say, quite simply, *the quality of quantity*, the quantitative such as discerned qualitatively from any other determination." (Badiou, A. 2006: 168-169)

309 For Hegelian and Heidegerian Tautologies, see: Tze-Wan Kwan, Hegelian and Heidegerian Tautologies, Analecta Husserlliana, The yearbook of Phenomenological research, Logos of Phenomenology and Phenomenology of Logos, Volume LXXXVIII, 2005, The World Institute for Advanced Phenomenological Research and Learning

310 In the works of Gregory Bateson, also see: Allen Thiher, The power of tautology: The roots of leterary theory, Associated University Press, 1997

311 After the distance of these twentyfour years, since my first exploration on Hermann Haken/ D'Arcy Thompson thesis what I could see is **the growing relevance of D'Arcy Wentworth Thompson,** especially in support of my research on Qualitative quantity. Under the arch of the "inapparent" nature and notion of Hegel's "Qualitative quantity" and the "tautology" coined by Hegel for any "attempt to explain coming-to-be or ceasing-to-be on the basis of gradualness of the alteration", stand the figure of the "father of the gradualness" - D'Arcy Wentworth Thompson, with his masterpiece "On Growth and Form", the book called by Stephen Jay Gould "the greatest piece of scientific writing of the twentieth century", the book regarded by G.E. Hutchison as "one of the very few books on a scientific matter written in this century which will, one may be confident, last as long as our too fragile culture." The year of 2014 marked 154 years since the birth of D'Arcy Thompson and I may see his masterpiece like walking all these years through the "fragile verbal footbridge" (expression used by Alain Badiou in his disagreement with Hegel) of Hegel's Logic, waking up the "Hegelian halutination" (again Alain Badiou). D'Arcy W. Thompson's "On Growth and Form" has had a profound influence on present day science, art, architecture, engineering and anthropology.

312 The title recalls Heidegger's "Being and Time", and Badiou explicitly agrees with Heidegger that philosophy can only be done on the basis of the ontological question. Establishing that mathematics is ontology and ontology is a situation, in ―Being and Event‖ Badiou offers thirty-seven interlaced 'meditations'. Meditation Fifteen /over Hegel/ of Badiou's ―Being and Event‖ is recognized as one of the most forbidding encounters of that book, pitching the Hegelian doctrine of the infinite against Badiou's own Cantorian conception. (See: Jonas Jervell Indregard, (2010). Badiou, Hegel, and the Infinite – part I: Introduction, 2010)

Badiou *agrees* with the concept of being as a qualitative essence of quantity, *but disagrees* with the naming of this as “infinity.” According to Badiou, the name “infinity” suits only qualitative infinity (in Hegel – “the bad infinity”):

“because it was drawn from the void, and the void was clearly the transfinite polarity of the process. In numerical proliferation there is no void because the exterior of the One *is* its interior, the pure law which causes the same-as-the-One to proliferate. The radical absence of the other, indifference, renders illegitimate here any declaration that the essence of finite number, its numericity, is infinite. In other word, Hegel fails to intervene on number. He fails because the nominal equivalence he proposes between the pure presence of passing – beyond in the void (the good qualitative infinity) and the **qualitative concept of quantity (the good quantitative infinity)** is a trick, an illusory scena of the speculative theatre. There is no symetry between the same and the other, between proliferation and identification. However heroic the effort, it is interrupted de facto by the exteriority itself of the pure multiple. Mathematics occurs here as discontinuity within dialectics. It is this lesson that Hegel wishes to mask by suturing under the same term – infinity - two disjoint – discursive others.” (Badiou, A. 2006: 168-169)

Badiou claim is radical. He states that “the good quantitative infinity is a properly Hegelian hallucination”. (Badiou, A. 2006: 168-169)

Regarding Badiou‘s disagreement with Hegel, I share the position of Jonas Jervell Indregard. (Indregard, J.J. 2010). Indregard claims that “in fact, Badiou *does not object* to Hegel‘s deduction of the good *qualitative* infinite. And in fact, Badiou recognizes that there is not, in Hegel, a simple dichotomy between qualitative and quantitative infinity, but rather a fourfold of infinities: the bad qualitative infinity, the good qualitative infinity, the bad quantitative infinity, and the good quantitative infinity.” (Indregard, J.J. 2010). According to Indregard, Badiou seems to argue that Hegel cannot properly ground the third infinity, the bad quantitative infinity: “One must recognize that the repetition of the One in number cannot arise from the interiority of the negative.” (Indregard, J.J. 2010). Badiou argues that, in any case, what Hegel puts forward as the fourth infinity, the good quantitative infinity, cannot properly be called "infinity" at all: "I have no quarrel with there being a qualitative essence of quantity, but why name it 'infinity'?" (Badiou, A. 2006:169)

### **1.7. Topological Fourfold of infinities within Hegel's philosophical development of the concepts of space, time, matter and aether**

Hegel's categories of quality and quantity, in particular qualitative quantity in relation with his concepts of space and time, is subject to which much attention has not been paid by Hegel's expositors and commentators and yet the importance of it cannot be denied. (Haldar, 1932).[313]

As Hiralal Haldar asserts, "Time and space are not among the categories of Hegel. They are not discussed under the names in his logic, but as regarded of though very meagerly and obscurely in his Philosophy of Nature."[314] (Haldar, 1932:520).[315] But there is nothing in the phenomena of nature of which the ground plan, the basis, is not to be found in the forms of thoughts or the categories. (Haldar, 1932:520). If space and time belong to nature, if they are the necessary aspect under which the objects of perception are always presented, they must have their logical grounds. What are these categories? This is the important question which need consideration. (Haldar, 1932:520).

Hegel's philosophy of space and time is presented in his *Philosophy of Nature*[316] (Miller tr. Hegel's Philosophy of Nature 1970), and in *Wissenschaft der Logik* (the Greater Logic), and in his early treatment of geometry, his *Geometrische Studien,* [317] This early work of Hegel contains fragents of what remains of a more extensive work on geometry which Hegel wrote

[313] Hiralal Haldar, 1920, Space and Time in Hegel's Philosophy, The Monist., Volume 42, Issue 4, October 1932. Devoted to the Philosophy of Science. Pages 520-532.
[314] Hiralal Haldar, 1920, Space and Time in Hegel's Philosophy, The Monist., Volume 42, Issue 4, October 1932. Devoted to the Philosophy of Science. Pages 520-532.
[315] Hiralal Haldar, 1920, Space and Time in Hegel's Philosophy, The Monist., Volume 42, Issue 4, October 1932. Devoted to the Philosophy of Science. Pages 520-532.
[316] Hegel's Philosophy of Nature, translation of Hegel's *Naturphilosophie* by A. V. Miller, Clarendon Press, Oxford, 1970.

[317] *Geometrische Studien (GS),* by G. W. F. Hegel, pp. 288-300 of: *Dokumente zu Hegels Entwicklung*, herausgegeben von Johannes Hoffmeister, Frommann, Stuttgart, 1936. (This early work of Hegel contains fragents of what remains of a more extensive work on geometry which Hegel wrote in his Jena years. A detailed discussion of it is given in the present writer's paper: *Hegel's Early Geometry*, Hegel Studien 39/40, 2004-2005, 61-124. A translation of *GS*, together with an Introduction (both by the present writer) is given in: *G. W. F. Hegel: Geometrical Studies - translated with Introduction and Notes,* Bulletin of the Hegel Society of Great Britain 57/58, 2008, 118-153 (*ITGS).*))

in his Jena years. Hegel arrived at the University of Jena in the year 1801, and left the university and the town in the year 1807. During those few years in Jena, Hegel worked on the manuscripts and drafts of his later 'system' which was published in a considerably revised form only after Hegel had left the town. For this reason Gruner and Bartelmann call Hegel's time in Jena his 'proto-systematic' phase. [318] (Gruner and Bartelmann, 2015: 44).
Hegel's nature-philosophical speculations bears continuous relevance for our 21st century. As Manfred Gies wrote *Hegel, in his thoughts about the notions of 'space', 'time', and 'matter', had reached insights which touch precisely those points at which the discipline of physics possibly reaches its own methodological barriers*[319] (Gies, 1983: 51).

In the logic of ***Wissenschaft der Logik*** (the Greater Logic), the moments of the concept of *quantity* are *continuity* and *discreteness,* and Hegel argues that these two moments equally imply the other, so that neither can be considered in complete isolation, but "slides over" to the other. (This, for Hegel, is the explanation of Kant's second cosmological antinomy.) (Paterson 2006:4)[320]

In ***Philosophy of Nature***, Hegel's account of space and time is initial and completely abstract. Hegel introduces a *self-externality of space*, a very limited form of space, in particular, not yet developed enough for geometrical thought. (Paterson 2006:4) Here Hegel presents *only the continuity level*. Hegel names this a "*perfect continuity"*. (Miller tr. Hegel's Philosophy of Nature 1970:29). For Hegel, the space is ``pure self-externality'' and what is external to itself is always pointing away to what it is not, a defiance of any fixity, and this is (at least initially) our idea of continuity. (Paterson 2006:4)

[318] Gruner, Stefan; Bartelmann, Matthias, 2015, The notion of 'Aether': Hegel versus contemporary physics, Cosmos & History; 2015, Vol. 11 Issue 1, p 41-68 - Hegel's proto-systematic manuscripts of that era –which were, by the way, never released by himself during his own lifetime– contained several metaphysical reflections on his nature-philosophical concept of 'aether', particularly in the years 1802-1806. Typical for Hegel's approach to philosophizing there was no essential difference between 'aether' (as a theoretical notion) and *aether* (as an ontic entity), because Hegel's meta-philosophy (on which all his philosophy was based) postulated an ultimate identity of idea and being. In Hegel's later –systematic– publications (after Jena) on the philosophy of nature, the German *word* <<Äther>> (as a lexical object) does not play a prominent role in his texts any more, such that it seems plausible to classify the *notion* of 'aether' as one of Hegel's particularly proto-systematic concepts.
[319] M. Gies: *Einführung in Hegels Naturphilosophie*. Lecture Notes Vol. 3339-9-01-S1, Fern-University of Hagen, 1983
[320] Paterson, Alan L. T., (2006), A modern Hegelian Philosophy of Special Relativity, 32 pages, 2006. https://sites.google.com/site/apat1erson/ (Paterson 2006)

Intuitively, a continuous function has no ``jumps" in its graph, for running into one would "stop"' it and not point anywhere outside itself. Since we have only pure continuity at this stage, the science of space, geometry, cannot start: for there are no points – a point is an *interruption* of the continuity of space (Philosophy of Nature, p.29). (Paterson 2006:4/5)

Points, we may say, form the *discrete* side of space. More generally, for geometry to start, we need to have *points*, *lines* and *planes*. This comes about through the logic of the concept as described in Wissenschaft der Logik (the Greater Logic). (Paterson 2006:4/5)

As Paterson states "There, the two moments of quantity, *continuity* and *discreteness*, do not conceptually ``live" in isolation but the presence of each implies the presence of the other. We can see that even in the modern mathematical definition of continuity: continuity in mathematics is continuity at every point, and with the mention of the point, discreteness enters. More precisely, and as explained in Wissenschaft der Logik (the Greater Logic: p.200f ), each form of magnitude, continuity and discreteness, contains that other and that ``*the distinction between them consists only in this, that in one of the moments the determinateness is posited, and in the other, it is only implicit*''. Further, in terms of the unit (one), the two moments ``*possess the absolute possibility that one may be posited in them at any point*''. In other words, . . . continuity is continuity at a point (as a one) while conversely, discreteness taken as concept is the continuity of its ones. Ultimately, this is based on Being-for-self (*Fürsichsein)* in the relation of the one and the many, in which one gives rise to continuity, and many to discreteness, but each of the one and the many refers to the other, though in any particular situation, we may choose not to make this reference-to-each implicit." (Paterson 2006:4/5)

For Paterson, "with discreteness made explicit in space as a many of points, (a potential infinite), Hegel's theory – a development of the Pythagorean view of point, line, plane and space – then gives the basis for Euclidean geometry. We can briefly motivate this development as follows. We start off, as discused above, with the (immediate) concept of space as *continuity*, a ``flowing''. This, made more precise – again as we have seen - makes explicit the moment of *discreteness* through the positing of the *point* at which the continuity is located. But the point is, as Euclid says, ``*that which has no part*''. How can you have a spatial object that has no extension? To clarify this, on Hegelian grounds, the point is *contradictory* in character simply because it is discreteness in isolation and *so is implicitly continuity as well*. So our problem with the point and its lack of extension is due to us holding in our minds the point in separation on its own, suppressing its intrinsic conceptual shift bringing out its continuity. The continuity of space was its self-externality. This self-externality is determined further in the continuity of the discrete moment of space, viz. the point. The *continuity* of the point is its existing ``*outside itself*''[321], which is, for Hegel, realized as the *line*, paradigmatically the *straight* line." (Paterson 2006:5)

Hegel points out[322] that curved lines presuppose two (and, we may say, even three) dimensions and therefore come after the determination of the surface. Surely Hegel's account of (a closed) surface as determining three dimensional space by its bounding such a region is open to the same kind of objection, viz. that for a surface to bound such a region it already presupposes three dimensional space so that one cannot derive the latter from the former.

Hegel's account in *Philosophy of Nature* of how the surface gives rise to three dimensional space differs from that in his *Geometrical Studies.*[323] In the latter (influenced by Aristotle), the point, line and surface are *limits* but the three dimensional body is not limit.) (Paterson 2006:5)

Paterson asserts that "this is realized in classical physics in the motion of a point mass which traces out a curve (a line)." (Paterson 2006:5) For Paterson, "**Hegel then argues dialectically that space is mediated through its *first* negation as this straight line, its *second* negation being the plane. Obtaining the plane from the ``line through point'' can be thought of physically by obtaining the plane as swept out by a line rotating about a point on it (like a propellor blade in motion).** However, the plane in two dimensions corresponds to the straight line in one dimension. More generally, and corresponding to a curved line, the negation of the line is the (curved) *surface.* We can think of this as swept out by a rotating curved line. For example, a circle rotated about a diameter gives a (hollow) sphere. (Paterson 2006:5)

[321] Miller tr. Hegel's Philosophy of Nature 1970:31

[322] in Miller tr. Hegel's Philosophy of Nature 1970:32

[323]

For Paterson, “On philosophical grounds (**for Hegel) the surface is then the negation of the negation of space and so is space mediated.** But the surface is two dimensional and so is not, as we would expect, full three dimensional space. Hegel points out, however, that if we take a *closed* surface, e.g. a (hollow) *sphere*, it is the limit of its interior, i.e. of a full spatial region. And it is in this respect, that the surface is the ``*restoration of the spatial totality*''.

In this way, then, the point, the line and the plane (surface) are set up as the primary concepts required for Euclidean geometry. Of course, many other concepts, axioms and postulates are also required and Hegel discusses these in some detail in *Wissenschaft der Logik* (the Greater Logic and *Geometrical Studies* though not so much in *Philosophy of Nature*. (Paterson 2006:6)

**Time**, Hegel says, is ``*the negative unity of self-externality*''[324]. Now on the other hand, space, for Hegel, is ``*the abstract universality of Nature's self-externality*''[325]; it follows that time is the ``*negative unity of space*'' (at least at the universal level). But we have already seen the negativity of space in the point-line-surface-space (three dimensional) development - for example, in space as a ``*perpetual becoming other*''. So in what respect does time as the negative unity of space differ from the negativity of the geometric development described above? This is explained by Hegel in terms reminiscent of the transition from *Quality* to *Being-for-self* (*Fürsichsein*) in *Wissenschaft der Logik* (the Greater Logic), although here, the transition is from *Quantity* (with its indifference) to Being-for-self.

In this latter transition (which is *time*), Hegel points out that the negativity of space ``*is equally for itself*''. In contrast, the point-line-surface-space negativity is *not* for itself, but rather the expression of the negativity working through spatiality ``behind the scenes'' as it were.

Another way to express the difference between the negative unity of space (as geometrical) and time is that in the former[326], negativity takes the form of space falling apart ``*into indifferent subsistence*'' while still being space as a whole, but in the case of time, negativity

[324] Miller tr. Hegel's Philosophy of Nature 1970:34

[325] Miller tr. Hegel's Philosophy of Nature 1970:28)

[326] Miller tr. Hegel's Philosophy of Nature 1970:34

is sublated in *existence*, existence that is no longer indifferent but is difference that has ``*stepped out of space*''. So we may say, space falls apart into the point which sublates itself consecutively in line, surface and then space reconstituted as its original totality, space, as it were, parting and reforming, the divisions articulating the negativity being just as easily erased, i.e. they are *indifferent*, and can just as much be explicitly present as blanketed out, i.e. their fundamental incompatibility is not allowed to stand out permanently. This process of spatial negativity, however, stands out on its own in *time* as *existing*, so that spatial differences, which in the negative unity of space on its own, are *internal* and indifferent, are now determined as *external*, out there, not emerging only to disappear again. The point[327], that was only a ``*possibility*'' for geometry, and is posited only as ``*the being-forself*'', ``*the negation of space, a negation that is posited in space*'' is now, through time, existing. Put yet another way, the point in geometry is only *ideal* and, but in time, it becomes *real*. For Hegel, ``*the point ... developed for itself is time*''[328]. (Paterson 2006:6:7)

The role of time, in Hegel, is to make space *real* in Euclidean geometry Itself. The geometrical figure, though ideal, becomes real, assumes drawn form, through the constancy of the figure *in time*. (Paterson 2006:7)

In Hegel's dialectic of space, the point's sublation in the line achieves *reality* in the *drawing* of a line through a point *in time*, and the line's sublation in the plane achieves *reality* through the sweeping out of a plane by the rotation of a line about a point on the line *in time*. (Paterson 2006:7)

Paterson asserts that time as giving reality to space is easy to illustrate in our subjective experience with the example, when we ``lose track of time'', e.g. in listening to music, spatial difference is smothered in constancy, receding into implicitness. This is the first form of negativity. When we ``wake up'' and look around the room, spatial difference is made explicit and real through the temporal process – looking around, moving our eyes. While this example of the two kinds of negativity comes from our experience in the world, Hegel, I think, would have argued that the two kinds are observer-independent, intrinsic to the concepts of space and time. (Paterson 2006:7)

[327] Miller tr. Hegel's Philosophy of Nature 1970:29

[328] Miller tr. Hegel's Philosophy of Nature 1970:37

For Paterson, “since time is the *Fürsichsein* of space, the positing of the negation of space as for itself, external, the moments of this negativity are now explicitly posited.

In Hegel’s *Philosophy of Nature*, time is a *becoming* – the becoming present in space – and according to the logic [329](*Wissenschaft der Logik*), becoming is the passing of being into nothing and its reverse.

In this context, the passing of being into nothing is the *past* and of nothing into being is the *future.* The *Now* is the ``*immediate vanishing of these differences into singularity*'', is the *present*. The three moments of time, namely, past, present and future, *are* because of the loss of indifference of spacein the transition from space to time. The three moments not realized in existence, posited out there in front of us for inspection: the closest that one can come to the awareness of the moments is in reflection on one’s own experience: past, present and future are, for Hegel, ``*only in subjective imagination, in remembrance and fear and hope*''. (These themes are developed existentially much later by Heidegger in *Being and Time*.) (Paterson 2006:7)

Space first expresses its negativity in point, line and plane, and time expresses its negativity in the *Now*, the present as the transition from past to future.

The three moments of space give rise to the three dimensions and relate to the science of geometry. Yet, this is not the case for time. (Paterson 2006:7)

Paterson asserts that “the reason for this is that time does not have the ``*indifference of self-externality*'' that space had. It is this indifference which allows spatial elements to exist in harmony side by side in geometric ``*configurations*'' such as triangles and cubes, with their points, lines and planes. Since time is the positing of the negativity of space, its determinations are no longer those of indifference, but its constancy lies in is its *shifting* which only *is* when it *is not*. There are no temporal triangles. For a constant determination for time, one has to fall back on the most abstract, paralyzation of thought, and this is the *One*. This, Hegel says, is the ``*uttermost externality of thought*'' completely difference-less, which

[329] Miller tr. Hegel’s Philosophy of Nature 1970:37

is manipulated purely externally in time as *arithmetic,* the practical mathematics of number.[330] (Paterson 2006:7)

Hegel understands Kant's formulation of geometry as derived from space as the form of pure intuition and arithmetic as the form of inner intuition.

The study of time in Physics involves also a continuum and the Calculus, and the study of number in mathematics goes far beyond arithmetical computation, involving profound theorems such as the *prime number theorem.* (Paterson 2006:8)

Regrettably, Hegel does not discuss these here, and, as developed in greater detail in *Wissenschaft der Logik*, one is left with the impression that for Hegel, number is a purely mechanical affair (though the Concept ``explains'' the various kinds of arithmetic process, addition, multiplication, powers and the like) though surely he must have been aware that arithmetic computation is the only the start of the mathematics of number.

From the concepts of space, time and place Hegel derive the concept of matter. Hegel's idea of matter emerging from space and time has apparently influenced Hermann Weyl (Weyl 1919)[331].

As we have seen, space is self-externality in its ``*asunderness and differenceless continuity*'' (*Philosophy of Nature,* p.40) whose negativity becomes for itself in time. So time arises as the explicit negativity of space, but conversely, ``*time is the immediate collapse into indifference*'' and hence into space. Time is the negativity *of space* and ``*the positive, i.e. the being of the differences of time, is space*''. Time is the spatial process posited and is only realized in space. Conversely, space in its intrinsic difference process is realized only when its truth is posited explicitly in time. In this way, as Hegel puts it, the *negativity* of space is time and the *positivity* of time is space. (The thinking involved here is a special case of the logic of *positive* and *negative* in the *Essence* of *WL*: there (*SL*, pp.424-427) the positive and the negative are ``*the sides of the opposition that have become self-subsistent*'' and ``*their positedness, or the reference-to-other in a unity which they are not themselves, is taken back*

[330] Miller tr. Hegel's Philosophy of Nature 1970:38

[331] Hermann Weyl, Raum, Zeit,Materie, 1919 (web) (Weyl 1919)

*into each*''. Hegel also stresses there that the positive and negative ``*are at the same time in and for themselves positive and negative*'' [my translation]. (Paterson 2006:8)

Space is the positive and time is the negative (the negativity of former), and space has three dimensions while time has none but rather is the transition of past and future through the now. On the other hand, in the logic of *Wissenschaft der Logik* (***Wissenschaft der Logik***, p.426) Hegel says that ``*although one of the determinatenesses of positive and negative belongs to each side, they can be changed round, and each side is of such a kind that it can be taken equally well as positive as negative*''. But space and time (which, in the case we are examining, are the positive and negative) do not have this symmetry for Hegel. (In relativity theory, as we will see, there is also a symmetry between space and time (and as in the Hegelian view, a *difference*).)

The explanation for the apparent contradiction in Hegel's view of space and time - that while space and time are ``posited unequally" yet the two sides can be ``changed round" - is to be found in the *selfexternality* of Nature at the beginning, i.e. the *initial* presupposition or determination of Nature is what determines one as the positive and the other as the negative, the becoming, thus introducing the asymmetry between the two. (Paterson 2006:8)

Continuing with Hegel's treatment of space and time in *Philosophy of Nature*, their unity is that *each is only in reference to the other*, is in fact the *truth* of both. This truth, posited in space, realizes the point as *Place.*[332]

The full threedimensionality of space is present in Place and the latter is the point existing in its truth as universal: space is no longer a falling apart into discrete points but rather cohesively presented as the *complete* portion of space as temporally dependent.

With Place, the point, instead of being the *abstract* point of geometry, is now ``*concrete*''. The transition of time is present in Place through the indeterminacy of its boundary.

[332] *Place* was much discussed by Aristotle in terms of *boundary* and *motion* in Book IV of his *Physics*, and the influence of Aristotle's account on Hegel's treatment of Place is apparent.

For Hegel Place is the existence of the point *as a universal*, in its truth. In Place, both spatial and temporal determinations apply so that space and time are no longer separated.

According to Paterson, we have in Place, to use relativistic language, a ``space-time'' *event* (interpreted non-technically. Place is the unity of *Here and Now*. From Place, Hegel procedes to obtain the concept of *Matter* through *motion*. The argument goes as follows. We have said that Place is the truth of space and time. But, of course, in normal language, place does not have temporal connotations. If I visit you at your *place*, place here is thought of as time independent – it is the same place, yesterday, today or tomorrow. The temporal moment in Place has been negated in its *spatiality*, which Hegel defines as ``*indifferent, spatiality*''. (*Philosophy of Nature*, p.41)

Place shifts with the making of the temporal *Now* explicit in *Place*. Here we could recall the previous discussion about 'point' above, where the point established itself in its self-externality as the *line*. This establishment of the point as the line was purely abstract and Geometrical, not primarily temporal in nature. At the present stage Hegel presents a more concrete determination of Place, it is determined as *this* place (the *Now*). Place here is *self-external, a spatial* Place. It is *another* place. In this transition, time is posited, made explicit. This positing of time in place is the *Motion*. According to Paterson, with this positing of time in place, Hegel rejects the Pythagorean account of line as the *motion* of a point. For Hegel motion comes later in the concrete setting of Place. For Paterson, Hegel's understanding of motion is just the making explicit of the shifting of the boundaries of Place as discussed above. Hegel's treatment of the unity of Place in movement from one place to another, or ``*the immediately identical and existent unity of both*'' is the matter. The conclusion that follows is that space and time are intrinsic to matter, and matter is the expression of concrete existence of space and time in their unity.

Hegel explains mechanics in which matter has mass moving with a certain velocity by the spatio-temporal content of matter: velocity (distance=space over time) is intrinsic to matter just as much as is its mass, since matter is the concrete existence of space and time.

Matter, says Hegel in a *Zusatz* (*Philosophy of Nature*, p.44) ``*is the relation of space and time as a peaceful identity*''. At this stage in *PON*, Hegel's discussion moves on to force and gravity.

The reality of space-time becomes the being of time in space and the being of space in time, and the "real union" of the two is "the real infinity of the ether", of absolute matter, which he terms motion. (**Cantillo, 2013:29**)

Reviewing Hegel's criticism of Newton's system of worlds and examining critically the many aspects of it that seems to anticipate the approach to mathematical physics, associated today with Einstein, Henry Paolucci notes that Hegel's criticism on Newton was well informed:

„Through hundreds of well documented pages of both of his Science of Logic and Philosophy of Nature, Hegel develops the meaning of Newton's fluxional calculus, his concepts of space, time, mass, inertial, centripetal and centrifugal forces, his law of motion, his gravitational world system, and finally his theory of light and colors. Particularly under the heading of ‚Quality' and ‚Measure' in the Logic and ‚Mechanics' in the Philosophy of Nature, Newton's doctrine provides mach of the empirical datum upon which the Hegelian philosophical dialectics operates." (Paolucci, 2001: 55)[333]

**In the Lesser Science of Logic**, Hegel does not explicitly discuss the Newtonian celestial mechanics. He does, however, discuss the logical presuppositions upon which any analysis of the law of phenomena into laws of forces (such as Newton framed out to be based, if it is to make philosophical sense. And this he does of considerable lenght under the subheadings „Repulion and Attraction", „Quantity", „Magnitude", „Quantum", „Number", „Degree", „Quantitative Ratio", „Measure", „Thing", „Properties", „Matter", „Form", „Phenomena", „Forces" – all of them terms we now familiarly encounter in contemporary treatises on post Plankian physical theory. (Paolucci, 2001: 55)

[333] Henry Paolucci, Hegel and the Celestial Mechanics of Newton and Einstein, Volume 64 of the series Boston Studies in the Philosophy of Science, pp 55-85

**In the Great Science of Logic**, Hegel provides what amounts to a running comentary on the the terse paragraph of the Logic of the Enciclopedia, takes us to the philosophical core of Hegel's criticism of the Newton's celestial mechanics. No one pretending to access the adequacy of that criticism can afford to ignore, or skim over, what Hegel has to say about „Quantity", „Quantum", and „Quantitative Ratio" in his book – lenght discussion of „Magnitude", or about „Specific Quantity", „Real Measure", and the „Measureless" (as transition to Essence) under the heading of „Measure" . ..Under the ‚Magnitude' and ‚Measure', Hegel examines, among other things, the basic notion of the Newtonian-Leibnizian calculus of the mathematical infinite and infinitesimals upon which that calculus is build. (Paolucci, 2001: 63)

As Henry Paolucci notes, „Hegel considers of lenght the dificulties of assimilating analytical calculus to analytical geometry and applying both to the analysis of accelerated rectilinear and non-linear curvilinear motion, and anticipating things to come in our own time". Hegel speculates on the possibility of developing *a mathematics of qualitative quanta* which would be *a science of measures* competent to deal with the qualities as well as the quantities of existent things, remarks Paolucci, (as for instance, Einsteinian and Plankian physics now deals with qualitatively determined quanta, or measures, of spaces, time, light and a host of electro-magnetic phenomena). (Paolucci, 2001: 77)

Paolucci emphasizes on Hegel's anticipation of the development of science done by Reimann and Einstein, „Anticipating the need for Reimann-Einsten advance, Hegel observes: „There is no science of time corresponding to the science of space, to geometry." (Paolucci, 2001: 77) Hegel observes that a science of measures, competent to deal with time in its negativity, as the moving point of space, „would be the most difficult of all sciences." (Paolucci, 2001: 77). For Hegel, a thing in „place" is understood to be a thing in Space-time, here and now or there and then. .....Anticipating the language of Erns Mach, einstein and Plank, Hegel defines space and time as positive and negative determinations of Motion, which is Mass. (Paolucci, 2001: 77)

Within Hegel's concepts of logic, categories of quality and quantity, and the concepts of space and time are embedded in his vision for philosophical science as a circle of sciences articulating (what he called) the Concept (Begriff). In the usual order, one starts with logic, which is 'pure' science, where qualitative and quantitative are developed, the logic then is realized in the externality of space and time in the philosophy of nature. This in turn leads to life and the philosophy of spirit, culminating in ‘absolute spirit’ in its forms of art, religion and philosophy. Logic itself, as Hegel says, begins with the qualitative, since it starts with being as ``abstractly first and immediate’’. It is this immediacy that gives to the start of logic that spontaneously given wholeness that is characteristic of the qualitative. This is reflected even in normal, non-Hegelian logic, where predicates (properties) are really the qualitative in the Hegelian sense. (Paterson 2006:4)

For Alan Paterson, the problem of the transition from logic to nature is also present in normal logic, in the relation between the intensive or qualitative (such as a property) and the extensive or quantitative (in the extension of a property). (Paterson 2006:4) If in the Wissenschaft der Logik Hegel starts with the qualitative, it is with the quantitative that Hegel resumes his philosophy of nature. In the logic, quantity is the disruption of quality in Fürsichsein, beingfor-self, with its combined moments of attraction and repulsion expressing the relation of the one and the many. In Wissenschaft der Logik 6, Hegel understands space, time in terms of the dynamics of the one and the many: these are ``expansions, pluralities that are a coming-out-ofself’’, ``a perennial self-production of their unity’’. Space is ``absolute self-externality’’, which is self-identical in its ``perpetual becoming other’’ while time is ``an absolute coming-out-ofitself’’, that generates the one in the form of ``a point of time, the now’’.

With philosophy, the concept becomes ‘pure’ again and brings us back to the starting place of logic. Hegel’s circle of the transition from logic to nature, beginning from logic with qualitative and quantitative and moving to the nature where space and time have their proper treatment, has always been controversial. Yet, reality and thought are omnipresent throughout Hegelian philosophy. As Alan Paterson states “Any meaningful thought involves reality – it is true that there is probably not in existence a ``golden mountain’’ but there is in existence ``gold’’ and a ``mountain’’. In the logic, despite its ``purity’’, the world is implicit, and in nature the thought of the logic that structures it is implicit. One cannot treat thought and being separately, the one is only with the other. This is consistent with themes in present day philosophy of science, where observation is seen to be theory laden.” (Paterson 2006:3/4)

Indeed, we can take ``pure'' logic to express the structure of the world implicitly, while the world (or nature) is just what logic is ultimately about.

The transition from logic to nature in Hegel's thought is then just the Being world that was implicit all along and through which the thought categories of the logic achieved their meaningfulness, the rock which assured the validity of that logic, emerging explicitly.

Indeed, one can trace in the logical development of Wissenschaft der Logik a greater and greater sense of reality, from the abstraction of pure Being, through number and measure, then to the logic of judgement and syllogism, then to objects of science, then to life, then to the initial emergence of space in the logic of Euclidean geometry in cognition and finally in the Idea where nature at last emerges explicitly and pure logic recedes into implicitness in the philosophy of nature. (Paterson 2006:3)

In the explicitness of nature, the logical structure lies under the surface (which is ultimately why one needs experiment to tease that structure out into the open). This distance between the implicit logic and nature 'out there'' appears most strikingly at the start of the Philosophy of Nature, with space and time, which for Hegel are ``selfexternality in its complete abstraction''.[334] In Philosophy of Nature (p.28), self-externality assumes its positive form as space and its negative form as time.

From § 10 of the *Philosophical Encyclopaedia*, which Hegel composed in Nürnberg (from 1808), in the context of his work as a *Gymnasium* teacher, it clearly emerges that "the three main parts" in which "science as a whole" may be divided – "logic", the "science of nature" and the "science of the spirit" – reflect three modes of being of the "essence", namely: "the logical element" (or "pure concept" and "abstract idea"), which is the "eternally simple essence [considered] in itself"; "nature", which is "this essence externalised" (or "the reality of the idea" in the form of "exterior being-there"); and "the spirit", which is the "return of the essence within itself from its externalisation" (or "the reality of the idea" in the form of "self-consciousness").[335] (Cantillo, 2013:31)

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[334] *Hegel's Philosophy of Nature*, translation of Hegel's *Naturphilosophie* by A. V. Miller, Clarendon Press, Oxford, 1970., p.28

[335] Hegel (1952), *Briefe von und an Hegel,* Bd. 1, hrsg. v. J. Hoffmeister, Hamburg, Meiner., Hegel 1952, 59-60

Giuseppe Cantillo emphasizes on Hegel's use of the term and notion of manifold in in his essay *Differenzschrift* (The Difference Between Fichte's and Schelling's System of Philosophy, 1801). Cantillo points out that "Hegel already explicitly noted this need for systematicity and cohesiveness at the beginning of his Jena period, in his *Differenzschrift.* Here he argues that because the relation of the limited to the absolute is manifold "philosophizing must aim to posit this manifold as internally connected, and there necessarily arises the need to produce *a totality of knowledge, a system of science*".[336] The system is established neither through an analytical method, nor through a synthetical one; rather, it is founded on the "development of reason itself"; it is a "self-production of reason", in which "the Absolute shaps itself into an objective totality, which is a whole in itself held fast and complete, […] an organisation of propositions and intuitions"[337] [338]. (Cantillo, 2013:32)

For Hegel, space emerges as the positive unity of mass, i.e. of inert matter, which nonetheless has within itself the memory – so to speak – of its own original unity with motion. This lends matter an inner animation in the form of the negative unity of the "absolute point":

For Hegel "mass is absolute unity equal to itself, in which negative unity subsists, the absolute point."[339] The insertion of the "absolute point", identified in the Dissertatio as the application of time to space, generates a multiplicity of points and limits, meaning that the mass, space/matter, divides itself into countless indivisible parts, or atoms. An infinite quantity (Menge) of quanta (Quanti) thereby emerges, which is to say of parts stemming from the negation of the universal mass. Each of them, as itself mass, contains its own centre (Mittelpunkt) within itself. While mutually distinct, quanta are all essentially the same: "one is a quantum as much another". [340] In their self-equality and non-differentiation, they are inert,

[336] G. W. F. Hegel, *The Difference between the Fichtean and Schellingian Systems of Philosophy*, translated by J. P. Surber, Ridgeview Publishers, 1978

[337] G. W. F. Hegel, *The Difference between the Fichtean and Schellingian Systems of Philosophy*, translated by J. P. Surber, Ridgeview Publishers, 1978

[338] Hegel, 1970, *Differenz des Fichte'schen und Schelling'schen Systems* [1801], in Hegel*, Werke in zwanzig Bänden*, Bd.2: *Jenaer Schriften* (1801-1807), hrsg. v. E. Moldenhauer u. K.M. Michel, Frankturt/M, Suhrkamp.

[339] Hegel (1975), *Gesammelte Werke*, hrsg. in Verbindung mit der Deutschen Forschungsgemeinschaft v. d. Reinisch-Westfälischen Akademie der Wissenschaften, Bd. 6: *Jenaer Systementwürfe I*, hrsg. v. K. Düsing u. H. Kimmerle, Hamburg, Meiner., page 11

[340] Giuseppe Cantillo, 2013, The concept of space in Hegel: The Early Jena Years, Lexicon Philosophicum, 1, 2013, http://lexicon.cnr.it/, p.46

yet this inertia affects their very mutual independence and distinction; as the removal of their independence, it takes the form of gravity, which presents itself as the fall of a body with a rectilinear motion that finds its beginning and end, its starting point and point of arrival, in its opposite, namely rest.

Hence the philosophy of nature distinguishes itself from the common way of envisaging nature, for it presents the latter as consisting "of wholes and parts in different quantities, and in a causal relation, as well as a quantity of 'these' [single, determined things]". (Cantillo, 2013:45)

What Hegel says about the relationship between 'wholes and parts' and the 'different quantities' as well as a 'quantity of these' single determined things, is truly topological in character. Topological notions of whole and parts relation, manifold, and different quantities, are set by Hegel in relation with space and time. The categories such as qualitative and quantitative are related with such concepts as space and time, within the manifold of multiplicity. Something more, in his interpretation Hegel emphasizes on the plasticity and elasticity of the ether, the spiritual substance of the absolute. As Cantillo asserts, "The spiritual substance first finds objective expression as ether or absolute, infinite and indeterminate matter. In its absolute restlessness, this ether, or absolute matter, is like the spirit, which is equal to itself in its being other than itself: it is the capacity to take up any form – absolute plasticity and elasticity."(Cantillo, 2013:45) In his discussion on the plasticity and elasticity of spirit, or ether, Cantillo relates to Giordano Bruno's argument that "ether […] knows no specific quality, but receives all those lent to it by nearby bodies." (Cantillo, 2013:46) As Cantillo asserts, "Ether, as the highest part of the sky, as light expanding in an absolute way, is absolutely infinite. Its moments are only "ideal moments, which in their mutual opposition refer to one another"; they "are absolutely restless in this relation" and remove one another in their separateness. These moments of the ether, "which immediately discloses itself as genuinely infinite", are space and time. This "infiniteness is motion"; or rather: "as totality, it is a system of spheres and motions [heavenly bodies]".(Cantillo, 2013:47) Space and time present themselves as ideal moments of the ether, which is to say of nature as directly posited by the spirit as its first manifestation. These are ideal moments because the reality of the ether is both its self-equality, its sameness to itself, and its infiniteness, its infinite motion. Its self-equality and being at rest represent the moment of space, considered separately; yet insofar as this manifests itself, insofar as it objectively fulfils its being in itself, it represents the opposite of self-equality and being at rest: it is the moment of infinity and motion (from the equal to its opposite, the unequal and different) – hence, time.

Just as infinite as space, time, as an ideal moment, in becoming other than itself spills into its opposite, into self-equality and rest, thus becoming space. Ether, as the whole, possesses this motion within itself, so that from space it becomes time and from time space, since the whole is the unity of the self-equal and the different ."(Cantillo, 2013:47)

Hegel's ether immediately posits itself as space and this idea is similar to how ether is understood by Bruno and Kepler, according to whom all space is filled with ether. Even Newton believed that ether fills space, at least within the solar system.

For Cantillo, "Yet this notion is here turned on its head, so to speak, since according to Hegel it is ether, as the totality of all moments, that constitutes the reality of space and time: "the reality of space and time […], as separate terms, is the expression of the totality of moments"; the "essence" of each of these remains their mutual "relation". (Cantillo, 2013:47)

For Hegel, space as absolute space, by its very concept, is self-equality which has difference, the infinite, outside itself, and hence also the negative, limit, outside itself. Within absolute space, this limit, this counterpart to the infinite, or time, is the point: "Just as time moves outside itself and becomes space, so space must move into itself and remove itself through the point". The point introduces a limitation within the indeterminate continuity of space. This, however, does not make absolute space determined space, but rather introduces "the concept of a general dimension within it". Yet, because of this indeterminate limitation, in the very act of positing itself in space, this dimension is likewise "removed" – not eliminated, but simultaneously negated and preserved. This means that "a second dimension in general" is placed "as a [determined] limiting which relates to the indeterminate limiting of space". This new dimension presents itself as "the first being- limited of space, as surface", which is "the relation between two dimensions", "the union of the negative and space". This means that the surface is the being-other or opposite of space, placed in relation to space itself, which therefore appears to be divided, yet not negated, but on the contrary reconfirmed in its indifference and universality:  It, the surface, is indeed the limiting of space, yet it is not the free limit itself, like a negative, but rather the union of the negative and space, the synthesis of both. In other words, it is the opposite of space placed in relation to space itself, as the negation of space, so that this is only divided – there are two spaces, but in such a way that space is indifferent in this negation and remains equal to itself, and its negation is nothing at all.

In the text from 1804-05 Hegel draws attention to the fact that while the dimensions of space are the moments of the realisation of space, they might also be other than three (“the determination of number – Hegel writes – immediately comes about as an extrinsic determination, which has the form of contingency”).

Büttner has examined Hegel's aether doctrine from the perspective of a Luhmann-kind-of system philosophy.[341] From such a system-philosophical perspective, Hegel's 'aether' (which, as we should remember, was associated with spiritual qualities) is said to play the role of an 'observer' which 'observes' the self-unfolding of the self-conscious spirit during the course of its history. Moreover Büttner also compared in [2] Hegel's notion of 'aether' with Plato's concept of *χώρα* (chora) in the book of Timaeus.

It is an important aspect of modern physics, not just of quantum physics, that *the observer determines part of what can be observed*. This realisation began with Einstein's theory of special relativity which gave up the Newtonian concept of absolute space and time. Two observers, for example, do not generally perceive the same time differences between events; the time differences they measure depend on their relative state of motion. A brief claim of relevance in favour of Hegel's philosophy of nature with regard to contemporary physics was made by Gies[342].

[341] S. Büttner: Von der Chora zum Äther: Rezeption und Transformation des platonischen Chorakonzepts in Hegels Jenenser Naturphilosophie, pp. 107-127 in K. Vieweg (ed.): Hegels Jenaer Naturphilosophie. Jena-Sophia Studien und Editionen zum Deutschen Idealismus und zur Frühromantik, Sect. II, Vol. 1, Wilhelm Fink Publ., 1998.

[342] M. Gies: *Einführung in Hegels Naturphilosophie*. Lecture Notes Vol. 3339-9-01-S1, Fern-University of Hagen, 1983.

There are comments on the similarity of Hegel's physics to aspects of general relativity[343](Wandschneider 1982) and comprehensive work on Hegel's philosophy of special relativity (Paterson 2006).[344] Topology is Analysis Situs, the science of space. Topological account could be found in Hegel's assertion that space preceding time.[345] (Paterson 2006:3)

As Paterson asserts in his *A modern Hegelian Philosophy of Special Relativity* (2006): "The roles of space and time in Hegel's account are asymmetric – space preceding time. – whereas in special relativity, we must think of them as inseparably together, a unity of space-time." (Paterson 2006:3) Paterson argues that "in fact looked at more closely, space and time are distinguished in special relativity and that the Hegelian account, suitably developed, makes philosophical sense of space and time as it appears in relativity." (Paterson 2006:3)

**Paterson shows how the Hegelian approach can be sutiably developed and gives an adequate philosophical account of the spatio-temporal basis for special relativity.**

**William Lawvere, in the series of works, also emphasize on the presence of spacetime concept in Hegel.** For William Lawvere, in modern terminology Hegel's discussion in Philosophy of Nature would be read as addressing spacetime event, namely in the disappearance and regeneration of space in time and of time in space as motion– a becoming, which, however, is itself just as much immediately the identically existing unity of both, or matter.

One of the important concept and phenomena related with the manifold of spacetime is the Elasticity or Plasticity. Elasticity is commonly used as an illustration via analogy of the nature of space in physics, specifically that of manifolds and spacetimes. William Lawvere emphasized on e**lasticity and analogy with a quality of space, pointing out that** Hegel goes on to speak of cohesion being refined to elasticity in the Paragraph 297 of Philosophy of Nature, where Zusatz/Elasticity is the whole of cohesion. According to Paragraph 298 from the Philosophy of Nature this elasticity is related to the unity of opposites that constitute

343 Dieter Wandschneider, Raum, Zeit, Relativität, 1982 web (Wandschneider 1982)

344 Paterson, Alan L. T., (2006), A modern Hegelian Philosophy of Special Relativity, 32 pages, 2006. https://sites.google.com/site/apat1erson/ (Paterson 2006)

345 Paterson, Alan L. T., (2006), A modern Hegelian Philosophy of Special Relativity, 32 pages, 2006. https://sites.google.com/site/apat1erson/ (Paterson 2006)

Zeno's paradox of motion, hence to the modern concept of differentiation via a limit of a sequence. For Lawvere, in terms of categorical logic this is precisely what is encoded in the infinitesimal shape modality and infinitesimal flat modality of “differential cohesion”. Sticking to imagery from solid state physics, these modalities are reminiscent of concepts in infinitesimal strain theory. Notice that this applies to **structures built from relatively stiff elastic materials.** According to Lawvere, In order to think of not just topology but Riemannian geometry in the above context of elasticity, the rigidity mentioned further above seems advisable. A rigidly elastic body is to be expected to produce sound when struck. This is a common imagery in Riemannian geometry, as in “hearing the shape of a drum”.--- Physical oscillations (of elastic bodies) are described via a wave equation and hence by the spectrum of a Laplace operator on a Riemannian manifold. The phrase hearing the shape of a drum in this context refers to the issue of recovering a Riemannian manifold from just the spectrum of the Laplace operator on it (“spectral geometry”). In plain Riemannian geometry there are counter-examples to the possibility of recovering Riemannian manifolds from just the spectrum of their Laplace operator. But from a little bit more of information, enhancing the Laplace operator to a spectral triple it becomes possible in general. This theorem due to Alain Connes is the motivation for the formulation of all of metric geometry via spectral triples, a program which is known as Connes' noncommutative geometry. References for elasticity in the solid state physics are provided by Lev Landau (Landau 1970), Andrei Sakharov (Sakharov 1967), Charles Misner (Misner 1973). [346]

William Lawvere asserts that the concept of cohesion/elasticity/plasticity is one of the most important concepts that is presented in Hegel's Philosophy of Nature (§296), that refers to the modern science. Lawvere discusses ‘cohesion’ with the following remarks:

In Hegel's Encyclopedia of the Philosophical Sciences (1817) there is discussion (in the section Physik – Die Kohäsion) of the cohesion and elasticity of some substance which Hegel says is the unity of space and time (PN§261): “This disappearance and regeneration of space in time and of time in space is motion – a becoming, which, however, is itself just as much immediately the identically existing unity of both, or matter.” (PN§261)

[346] Lev Landau, Evgeny Lifshitz, Theory of Elasticity, part VII of Course of Theoretical Physics, 1959, 1970; Andrei Sakharov, Vacvuum fluctuations in curved space and the theory of gravitation, Doklady Akad. Nauk S.S.S.R. 177 70-71 (1967) - An analogy for topology/differential geometry, the “rubber-sheet analogy of gravity”, summarized by Charles Misner with the saying that gravity is “an elasticity of space that arises from particle physics”; Charles Misner, Kip Thorne, John Wheeler, Gravitation, 1973.

In Hegel, space and time transmute into each other, forming a unity. The appropriate modern word for this is clearly spacetime. Hegel's perspective fits well with modern general relativity. (Wandschneider 1982:82)[347]

For Lawvere, "elasticity is the existence of the dialectic of these moments themselves" points to a unity of opposites. The cohesion and elasticity here is not so much that of actual physical bodies, but more of mathematical space itself (matter is the union of space and time anyway PN§261). Viewed this way speaking of a "sound of space" here is precisely what modern mathematics does, in the discussion of hearing the shape of a drum, where it refers to the spectrum of Laplace operators on Riemannian manifolds. Now Riemannian geometry is an example of a Cartan geometry, and Cartan geometry is one of the hallmark concepts formalized in differential cohesion. At the same time, followin PN§298 we think of differential cohesion as the formalization of Hegel's elasticity.

William Lawvere establishes proposals for formalization of Hegel's objective logic in categorical logic. (Lawvere 1991), (Lawvere 1992) (Lawvere 1994) (Lawvere 1995) (Lawvere 1997)[348]

For Hegel, Nature is, in itself a living whole. The movement of its idea through its sequence of stages is more precisely this: the idea posits itself as that which it is in itself; or, what is the same thing, it goes into itself out of that immediacy and externality which is death in order to

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[347] Dieter Wandschneider, Raum, Zeit, Relativität, 1982 web (Wandschneider 1982);

[348] William Lawvere, Some Thoughts on the Future of Category Theory in A. Carboni, M. Pedicchio, G. Rosolini, Category Theory , Proceedings of the International Conference held in Como, Lecture Notes in Mathematics 1488, Springer (1991) (Lawvere 1991); William Lawvere, Categories of space and quantity in J. Echeverria et al (eds.), The Space of mathematics , de Gruyter, Berlin, New York, pages 14-30, 1992. (Lawvere 1992);William Lawvere, Tools for the advancement of objective logic: closed categories and toposes, in J. Macnamara and Gonzalo Reyes (Eds.), The Logical Foundations of Cognition, Oxford University Press 1993 (Proceedings of the Febr. 1991 Vancouver Conference "Logic and Cognition"), pages 43-56, 1994. (Lawvere 1994);William Lawvere, A new branch of mathematics, "The Ausdehnungslehre of 1844," and other works. Open Court (1995), Translated by Lloyd C. Kannenberg, with foreword by Albert C. Lewis, Historia Mathematica Volume 32, Issue 1, February 2005, Pages 99–106 (Lawvere 1995)F. William Lawvere, 1996, Unity and identity of opposites in calculus and physics, Applied Categorical Structures, June 1996, Volume 4, Issue 2, pp 167-174 (Lawvere 1996);William Lawvere Toposes of laws of motion , transcript of a talk in Montreal, Sept. 1997 (pdf) (Lawvere 1997)

go into itself; yet further, it suspends this determinacy of the idea, in which it is only life, and becomes spirit, which is its truth.(PN§195).

In paragraph 202a from the Philosophy of Nature, Hegel asserts that "Other mathematical determinations, such as infinity and its relationships, the infinitesimal, factors, powers, and so on, have their true concepts in philosophy itself. It is awkward to want to take and derive these from mathematics, where they are employed in a nonconceptual, often meaningless way; rather, they must await their justification and significance from philosophy."

In the Philosophy of Nature Hegel made an important discussion of Measure, claiming that "The truly philosophical science of mathematics as theory of magnitude would be the science of measures, but this already presupposes the real particularity of things, which is only at hand in concrete nature." (PN§202b)

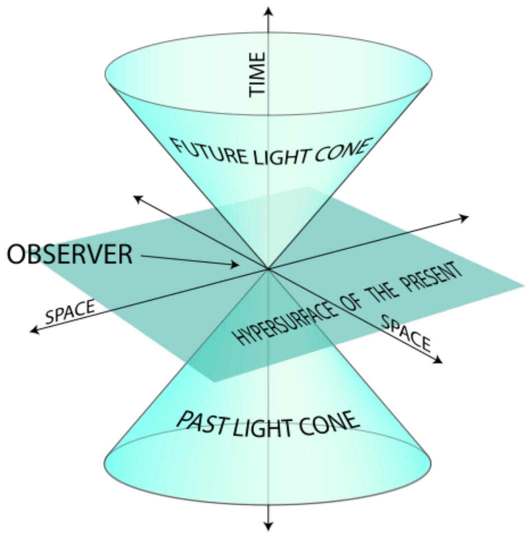
Four-dimensional Euclidean spacetime

In 1905, and later published in 1906, Henri Poincare showed that by taking time to be an imaginary fourth spacetime coordinate ($\sqrt{-1}\, c\, t$), a Lorentz transformation can be regarded as a rotation of coordinates in a four-dimensional Euclidean space with three real coordinates representing space, and one imaginary coordinate, representing time, as the fourth dimension. Since the space is then a pseudo-Euclidean space, the rotation is a representation of a hyperbolic rotation, although Poincaré did not give this interpretation, his purpose being only to explain the Lorentz transformation in terms of the familiar Euclidean rotation. [349]

This idea was elaborated by Hermann Minkowski, who used it to restate the Maxwell equations in four dimensions, showing directly their invariance under the Lorentz transformation. Minkowski further reformulated in four dimensions the then-recent theory of special relativity of Einstein. From this Minkowski concluded that time and space should be treated equally, and so arose his concept of events taking place in a unified four-dimensional spacetime continuum.

[349] Henry Poincare, 1905-1906, On the Dynamics of the Electron, pp.129-179

In mathematical physics, **Minkowski spacetime** is a combination of Euclidean space and time into a four-dimensional manifold where the spacetime interval between any two events is independent of the inertial frame of reference in which they are recorded.




Subdivision of Minkowski spacetime with respect to an event in four disjoint sets. The light cone, the absolute future, the absolute past, and elsewhere. [350]

Minkowski space is closely associated with Einstein's theory of special relativity, and is the most common mathematical structure on which special relativity is formulated.

S. Wickramasekara demonstrates how "a topology can be defined in the four dimensional space-time of special relativity so as to obtain a topological semigroup for time. The Minkowski 4-vector character of space-time elements as well as the key properties of special relativity are still the same as in the standard theory. However, the new topological structure allows the possibility of an intrinsic asymmetry in the time evolution of physical systems." [351](Wickramasekara, 2003).

[350] Sard, R. D. (1970). Relativistic Mechanics - Special Relativity and Classical Particle Dynamics. New York: W. A. Benjamin.
[351] *S. Wickramasekara, 2003, A Note on the Topology of Space-time in Special Relativity, Class. Quantum Grav., 18 (2001) 5353*

**Following the above discussion, we can conclude that,**

**t**here is fourfold of infinities in Hegel: 1) the bad qualitative infinity; 2) the good qualitative infinity; 3) the bad quantitative infinity; 4) the good quantitative infinity and there is a thesis that the fourfold of infinities in Hegel can be linked with the Ancient Greek concepts of Space and Time – the fourfold constructed of Chronos and Kairos, Chora and Topos.

(In "Mediation Fifteen – Hegel, The Arcana of Quantity," (Badiou, A. 2006:169) Badiou accepts the fourth element of Hegel‘s fourfold: "the good quantitative infinity" (qualitative quantity), called by Badiou, after Hegel, "the quality of quantity".)

In his study "Riemann's Conceptual Mathematics and the Idea of Space" (2009), Arkady Plotnitsky assetred that:

"One might argue that the ancient Greeks had *philosophical* topology, as is suggested by Plato's concept of *khora* in *Timaeus*, which may even be seen as already questioning the very concept of spatiality. But they did not have a mathematical discipline of topology; their only mathematical (exact and quantifiable) science of space was geometry. Anticipated by Leibniz's conception of "analysis situs" (the term used by Riemann and for a while after him), topological ideas were gradually developed by Riemann and others, especially Henri Poincaré, whose work was uniquely responsible for establishing topology as a mathematical discipline." (Plotnitsky, A. 2009: 105-130)

Examining the "The Spaces of the Baroque (with Leibniz, Riemann, and Deleuze)" (Tilottama Rajan, T. and Plotnitsky, A. 2012), Plotnitsky links the space /topos/ in the Baroque with Plato‘s khora /Timaeus/.

Following the direction given by Plotnitky for the topology of Baroque - topos and chora, we could see the notion of Hegel‘s qualitative quantity into the the fourfold of infinities in Hegel:

1/. the bad qualitative infinity;
2/. the good qualitative infinity;
3/. the bad quantitative infinity;

4/. the good quantitative infinity.

This Hegel's forufold of infinities is related with the fourfold interplay of the two pair of Ancient Greek categories of time and space. The fourfold is constructed of Chronos and Kairos, Chora and Topos as follow: "Chronochora", "Chronotopos", "Kairochora" and "Kairotopos".

There are two notions of Time and two notion of Space in the Ancient Greek concepts of time and space. The two temporal notion of Time are Chronos and Kairos. Chronos is the quantitative Time and Kairos is the qualitative Time. And there are two spatial notion of Space – Chora and Topos. Chora is the quantitative abstract notion of space and Topos is the qualitative notion of space. The abstract space is Chora and the concrete place is Topos. (Rämö, H. 1999: 309-328)

Aristotle defined Chronos quantitatively as the "number of motion with respect to the before and the after", which is a classical expression of the concept of (chronos) time as change, measure, and serial order. The definition of Chronos is focused on an exact quantification of time.

Following Aristotele analysis there are temporal and spatial pairs of *chronos*/*kairos* and *chora*/*topos*, relationships. Kairos is the time that gives value thus quality. Kairos is qualitatively defined. (Rämö, H. 1999: 309-328) The result would produce the following:

- quantitative quantity - /in the domain of Chronochora – Abstract Space and Abstract Time/;
- quantitative quality - /in the domain of Chronotopos –Meaningful Place and Abstract Time/;
- qualitative quantity - /in the domain of Kairochora – Abstract Space and Meaningful Time/;
- qualitative quality - /in the domain of Kairotopos – Meaningful Place and Meaningful Time/.

The temporal and spatial pairs of *chronos*/*kairos* and *chora*/*topos* relationships are proposed by Hans Rämö (Rämö, H. 1999: 309-328), after José Luis Ramirez [352] [353], a leading authority

[352] Ramírez, J.L. (1995) *Skapande Mening: En begreppsgenealogisk undersökning om rationalitet, vetenskap och planering* [Creative Meaning: A Contribution to a Human-Scientific Theory of Action]. Stockholm: NORDPLAN.

[353] José Luis Ramirez, professor of this subject at the Swedish University of Agricultural Sciences,who contributed to the development of rhetorical studies at Södertörn University College between 1997 and 2004. He

on Aristotle in Sweden. Rämö suggests fourfold model of time-space as configuration combining the four elements is the following neologisms: "Chronochora", "Chronotopos", "Kairochora" and "Kairotopos". Rämö relates these neologisms to the predominant Aristotelian modes of human activity Episteme, Techne and Phronesis, and creates the matrix of time and space configurations /Time-Space Manifold/.

Some particular proposition for the presence of Topological in Hegel is developed by Robert Groome, from P.L.A.C.E., Santa Monica[354]. Groome is the author of the original study “Formalization of Hegel’s Phenomenology of the Spirit”. (Groome, R. 2010) [355] In his study Groome proposed and developed the thesis of ‘topological’ Hegel, establishing that Hegel’s The Phenomenology of the Spirit [Phänomenologie des Geistes] first appeared in 1807 under the title System of Science. According to Groome, “Here, Knowledge [Wissen/Savoir] or Science [Wissenshaft] *has been called by Hegel* a System or Manifold (Mannigfaltigkeit), which has both a topological and philosophical sense.” Countering “the standard procedures that view the notion of a Manifold philosophically, if not metaphorically”, Groome’s aim is to read and construct its place topologically, both globally and locally, in a Theory. In proceeding in such a literal way, Groome’s aim is both to introduce a Hegelian theory that is not another philosophical commentary on Hegel's philosophy, but first and foremost an isolation of its structure. Groome’s conclusion is that the Phenomenology of the Spirit and Hegel's conception of phantasy are constructed in a topological structure, while showing the correspondence with Lacan's topological project. In the second part of his study Groome reconstructs the intuitive diagrams presented in the first part of his study in a more adequate topology, introducing a theory of categories, topoi, and sheaves in the formalization of Hegel's “Phenomenology of the Spirit”.

Excplicit references to the topology in Hegel, could be found in Alain Badiou’s work “The Rational Kernel of the Hegelian Dialectic”, (Badiou, A., Bellassen, J., Mossot , L. 2011) [356]

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maintains an interest in rhetorical studies in Spain.
http://www.retorikforlaget.se/index2.php?page=shop.product_details&flypage=flypage_images.tpl&product_id=411&category_id=18&option=com_virtuemart&Itemid=45

[354] http://lacanlosangelespsychoanalysis.com/

[355] Robert Groome, “Formalization of Hegel’s Phenomenology of the Spirit” http://www.lacanlosangelespsychoanalysis.com/classes/course/info.php?id=21

[356] Alain Badiou, Joël Bellassen and Louis Mossot , “The Rational Kernel of the Hegelian Dialectic”, See: (b) On the interior and the exterior. - Hegelian topology.

where Badiou states that “Hegel sustains, against Kant, *a new topology of knowledge*” and “the historical destiny of this topology is its inevitable division. We can in effect conceive it in a purely structural fashion: exterior and interior are to be discernable *at each point*, but indiscernible in the supposedly given all. We will say that there is a local subject of interior/exterior separation; however this demands an unconditional global unity of a law. This unity does not have any other evidence than its punctual effect, which is separation. The truth of the one is only insofar as it cannot be said *in whole* since the whole exists at each point as the act of a partition, of a two. This path is followed by Lacan (but already by Mallarme) in the usage that he makes of non-orientable surfaces like the mobius strip. In its global torsion, the ribbon does not admit to the distinction between interior and exterior. At each point, there is an ‘inverse’, thus an outside. As such, the all rematerializes as the scission interior/exterior, we need to cut the ribbon. We know that for Lacan this cut [*coupure*] is what precedes the subject: the subject is the act situated between the one of the all and its effect of the orientation of inside/outside. ‘A’ subject is the undoing of torsion.” (Badiou, A., Bellassen, J., Mossot, L. 2011) [357]

The **mathematical concept of mani*fold*** brings together geometry and **topology** and this concept is critical for the philosophical thinking in the tradition of Continental philosophy, as well as the roots of this tradition in Ancient Greek philosophy and geometry.

**The concept of manifold is present in Husserl as *Mannigfaltigkeit*** (variously translated as “manifold” or “multiplicity”). Husserl’s inspiration for elaborating the concept of *Mannigfaltigkeit* came from the father of the term maniforld - Bernhard Riemann. Cantor also used generally the term “manifold” simply as synonym for “*Menge*” (quantity), or “*Inbegriff*” (collection), thereby laying the foundations for set-theory. Indeed, in the 1890s Cantor started calling his work *Mengenlehre* (“theory of quantities”), instead of *Mannigfaltigkeitslehre (“*theory of manifolds”).

Husserl was well aware of the differences between Cantor and Riemann, during the period of the *Philosophy of Arithmetic* and well awere of the different use of the concept in Cantor and Riemann. According to Husserl, by manifold, Cantor means a simple collection of elements

[357] Alain Badiou, Joël Bellassen and Louis Mossot , “The Rational Kernel of the Hegelian Dialectic”, See: (b) On the interior and the exterior. - Hegelian topology.

that are in some way united, and however, this conception does not coincide with that of Riemann and as used elsewhere in the theory of geometry, according to which a manifold is a collection not of merely united, but also ordered elements, and on the other hand not merely united, but continuously connected elements.

Husserl's *Mannigfaltigkeitslehre* is nothing to do with the Cantorian set-theory, but come rather closer to topology. In "*Prolegomena"*, Husserl introduces the idea of a pure *Mannigfaltigkeitslehre* as a meta-theoretical enterprise which studies the relations among theories, e.g. how to derive or found one upon another. When Husserl announces that in fact the best example of such a pure theory of manifolds is what we actually already have in mathematics, this sounds slightly odd and is a bit misleading. The pure theory of theories cannot simply be the mathematics underlying topology, but should rather be considered as a *mathesis universalis*.

Husserl is the founder of formal ontology (Husserl, E. 1929),[358], which is the natural environment of topology. Husserl's *"Logical Investigations" (1900/01)* contain a formal theory of part, whole and dependence that is used by Husserl to provide a framework for the analysis of mind and language of just the sort that is presupposed in the idea of a topological foundation for cognitive science. (Smith, B. 1994) [359] The roots of topology in Husserl can be found in the forerunners of the school of formal ontology - Bernard Bolzano and Franz Brentano (Brentano, F. 1988) [360], within the origins of the main traditions of contemporary ontology: the Phenomenological, the Analytical, and the Austro-Polish (with the main exponents as Adolf Reinach, Roman Ingarden and Nicolai Hartmann). After Kant's rejection

[358] E. Husserl, Formal and Transcendental Logic (1929), English translation: The Hague: Martinus Nijhoff, 1969, 27, p. 86.): "The idea of a formal ontology makes its first literary appearance in Volume I of Logische Untersuchungen (1900), [Chapter 11, The Idea of Pure Logic] in connection with the attempt to explicate systematically the idea of a pure logic -- but not yet does it appear there under the name of formal ontology, which was introduced by me only later. The Logische Untersuchungen as a whole and, above all, the investigations in Volume II ventured to take up in a new form the old idea of an apriori ontology -- so strongly interdicted by Kantianism and empiricism -- and attempted to establish it, in respect of concretely executed portions, as an idea necessary to philosophy."

[359] Barry Smith, Topological Foundations of Cognitive Science, a revised version of the introductory essay in C.Eschenbach, C. Habel and B. Smith (eds.), Topological Foundations of Cognitive Science, Hamburg: Graduiertenkolleg Kognitionswissenschaft, 1994, the text of a talk delivered at the First International Summer Institute in Cognitive Science in Buffalo in July 1994.: http://ontology.buffalo.edu/smith/articles/topo.html And Barry Smith, (ed.) 1982 Parts and Moments. Studies in Logic and Formal Ontology, Munich: Philosophia.)

[360] Brentano, Franz 1988 Philosophical Investigations on Space, Time and the Continuum, translated by Barry Smith, London/New York/Sydney: Croom Helm.

of the possibility of a general ontology [361], the first philosopher who contributed to the new ontological turn was Bernard Bolzano. In his 1837 *"Theory of Science" (Wissenschaftslehre)*, Bolzano attempted to provide logical foundations for all sciences, building on abstractions like part-relation, abstract objects, attributes, sentenceshapes, ideas and propositions in themselves, sums and sets…The logical theory of Bolzano developed in his work "Theory of Science" has come to be acknowledged as groundbreaking.

Bolzano was mainly concerned with three realms: (1) The realm of language, consisting in words and sentences; (2) The realm of thought, consisting in subjective ideas and judgements; (3) The realm of logic, consisting in objective ideas (or ideas in themselves) and propositions in themselves. The distinction between parts and wholes play a prominent role in Bolzano's system. Bolzano's work "The Paradoxes of the Infinite" was greatly admired by Charles Sanders Peirce, Georg Cantor and Richard Dedekind.

Bolzano's works was rediscovered by Edmund Husserl[362] and the Polish philosopher and logician Kazimierz Twardowski[363], both students of Franz Brentano. Through Husserl and Twardowski, Bolzano became a formative influence on both phenomenology and analytic philosophy. The influence of Bolzano on Heidegger is witnessed by Heidegger himself. (Heidegger, M. 1963) [364]

According to Husserl, the object of formal ontology is the study of the genera of being, the leading regional concepts, the categories; the true method - the eidetic reduction coupled with

[361] I. Kant, Critique of Pure Reason (A247/B304), Cambridge: Cambridge University Press, 1998, pp. 358-359 : "The Transcendental Analytic accordingly has this important result: That the understanding can never accomplish a priori anything more than to anticipate the form of a possible experience in general, and, since that which is not appearance cannot be an object of experience, it can never overstep the limits of sensibility, within which alone objects are given to us. Its principles are merely principles e of the exposition of appearances, and the proud name of an ontology, which presumes to offer synthetic a priori cognition of things in general in a systematic doctrine (e.g., the principle of causality), must give way to the modest one of a mere analytic of the pure understanding."

[362] Hermes Scholz, Concise History of Logic (1931), English translation: New York: Philosophical Library, 1961, p. 47. : "With such illogicality did things happen in the history of logic which we are pursuing here that this great, born logician fell prey to a fate which beats the fate of Joachim Jungius. For the latter at least was read, and read by a Leibniz; but that cannot even be said of Bolzano. Hence we cannot even maintain in his case that he was forgotten. All the greater is the merit of Edmund Husserl who discovered Bolzano."

[363] the founder of the Lvov-Warsaw School of logic, together with Alfred Tarski and Jan Lukasiewicz, Kazimierz Twardowski formed the troika which made the University of Warsaw, during that period, the most important research center in the world for formal logic.

[364] Martin Heidegger, Preface to: William Richardson, Heidegger. Through Phenomenology to Thought, The Hague: Martinus Nijhoff, 1963. p. X.: "The first philosophical text through which I worked my way, again and again from 1907 on, was Franz Brentano's dissertation: On the Manifold Sense of Being in Aristotle."

the method of categorial intuition. The phenomenological ontology is divided into two: (I) Formal, and (II) Regional, or Material, Ontologies. The former investigates the problem of truth on three basic levels: (a) Formal pophantics, or formal logic of judgments, where the a priori conditions for the possibility of the doxic certainty of reason are to be sought, along with (b) the synthetic forms for the possibility of the axiological, and (c) "practical" truths.

The genealogy of Derrida's philosophical algebra, especially his algebra of undecidables, could be traced trough Godel back in its roots in Leibniz. Derrida devoted to Leibniz an important section of *"Of Gramatology"* entitled *"Algebra: Arcanum and Transparence"*. This work of Derrida deals with the logical algebra …of writing. The strange term is typical for Derrida sense. Leibniz's ideas concerning the possibility of making topology into a rigorous mathematical discipline were among his great contribution to mathematics. In the 19 century Leibnitz's topology was developed into modern topology in the works of Karl Friedrich Gauss, Bernhard Riemann, Henri Poincare and others. In his "The Double Session", devoted to Deleuze's topology, Derrida offers us philosophically geometrical topological perspective on or approach to fold. This perspective is Derida's philosophical algebra, which entails, as Arkady Plotnitsky asserts a certain topology or spatiality.

### 1.8. Topological Fourfold of infinities in Hegel and The Fourfold of Hegelian Judgments

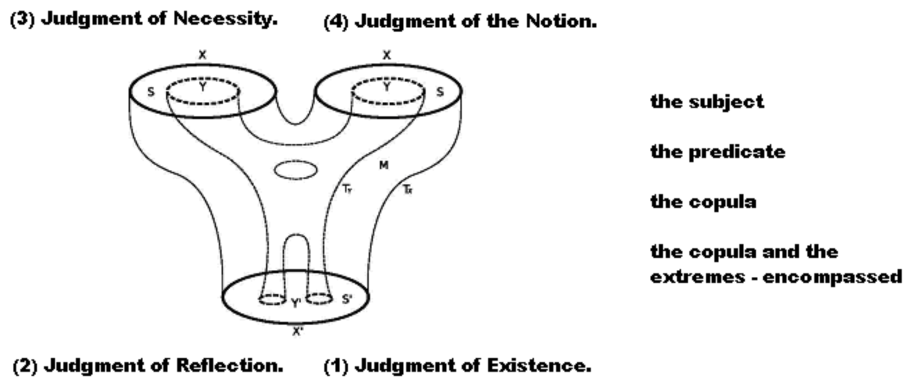
The true topological character of Hegel's logic, method and subject matter, the topological character of the double negation, where the Understanding and its negation, Dialectical Reason, and the Negation of the Negation – Speculative Reason, can be seen in Carlson's conclusion of Why Are There Four Hegelian Judgments?.

As Carlson asserts, "In Hegel's very last chapter, method and subject matter supposedly conjoin. Method is the one and only subject. We have the Understanding, its negation, Dialectical Reason, and the Negation of the Negation--Speculative Reason. But when all is said and done, there is a hole in the whole. Negation of the negation is not the restoration of the positive thing originally negated. It is less than that. This very absence is the silent fourth--the non-notional individual which guarantees that Logic never ends. It is only by virtue of the silent fourth that Logic is a circle."[365] (Carlson 2005). If the 'whole', developed from the starting theses of Being – Nothing – Becoming are constructed as torus (see . . .my diagram) there is 'a hole in the whole" from the very beginning of the Science of Logic (the part I, Chapter 1 – Being, Nothing, Becoming), and this 'hole' is 'developed' through the speculative reason/logic to the very end of the Science of Logic as double negation, the 'void', the 'very absence" of the 'silent fourth', that made 'the Logic is a circle'. Actually the 'logic is (not) a circle of circles' (Hegel), it is the torus and topology.

**Carlson concludes that "**Hegel's method is traditionally viewed as the passage from immediate Understanding to mediated Dialectical Reason to Speculative Reason, which holds the prior two positions in tension.Yet there is always a fourth. Method must work on *something*. This something is an irrational, non-methodical material without which the Heracleitan flux cannot flow. In the judgment chapter, this "silent fourth" finally speaks. In judgment, not only must the notion objectify itself in a notional way, it must judge its non-self--say what this *is*. The three notional moments, together with the non-notional self, comprise Hegel's four judgments." [366] (Carlson 2005).

[365] David Gray Carlson, 2005, Why Are There Four Hegelian Judgments?, p.114:125, in Hegel's theory of the subjects, David G. Carlson, ed. 2005, Palgrave Macmillan 2005
[366] David Gray Carlson, 2005, Why Are There Four Hegelian Judgments?, p.114:125, in Hegel's theory of the subjects, David G. Carlson, ed. 2005, Palgrave Macmillan 2005

Diagram based on Rene Thom's cobordism and
David Gray Carlson, 2005, Why Are There Four Hegelian Judgments?, p.114:125, in
Hegel's theory of the subjects, David G. Carlson, ed. 2005, Palgrave Macmillan 2005




**The four judgments in Hegel**

**the four judgments replicate the logic of quality, quantity, actuality and notionality.**

**"Across the four judgments, individuality travels around. It starts as the subject, it travels to the predicate, it travels to the copula, and it encompasses the copula and the extremes, while still preserving the necessity of non-notional contingency."** David Gray Carlson, 2005, Why Are There Four Hegelian Judgments?, p.114:125, in Hegel's theory of the subjects, David G. Carlson, ed. 2005, Palgrave Macmillan 2005

...............the circle of circles or the fourfold of infinities in Hegel's logic, presented through the model of cobordism, correspond strongly to the proposition made by David G. Carlson in his paper Why Are There Four Hegelian Judgments? ( Carlson 2005).[367]

As David Carlson asserts, "Hegel is the philosopher of threes." (Carlson 2005:114) Carlson remarks that "In the *Encyclopedia* system, there is logic-nature-spirit. Within logic, there is being-essence-notion. Within notion, there is subject-object-idea. Within subjectivity, there is notion-judgment-syllogism. Yet, as everyone notices, when it comes to *judgment*, the structure is tetrachotomous", confirms Carlson and adds that "here we find **existence-reflection-necessity-notion. Why should there by *four* judgments when there are only three of everything else?"** (Carlson 2005:114).

For Carlson, the four judgments replicate the logic of quality, quantity, actuality and notionality. (Carlson 2005:114). Across the four judgments, individuality travels around. "It starts as the subject, it travels to the predicate, it travels to the copula, and it encompasses the

367 David Gray Carlson, 2005, Why Are There Four Hegelian Judgments?, p.114:125, in Hegel's theory of the subjects, David G. Carlson, ed. 2005, Palgrave Macmillan 2005

copula *and* the extremes, while still preserving the necessity of non-notional contingency.” (Carlson 2005:114).

The four judgments in Hegel are:

(1) *Judgment of Existence*.
(2) *Judgment of Reflection*.
(3) *Judgment of Necessity*.
(4) *Judgment of the Notion*.

(1) *Judgment of Existence*. In the judgment of existence, some property of the subject is singled out arbitrarily: Hegel's example is "the rose is fragrant."[13] It has the form $A = \{A, B, C\}$, but this is misleading. The rose is still a rose even if not fragrant. In this first judgment, $A$ is abstract and self-sufficient. It has no need of the predicate. Speculatively, $A$ is the *lack of identity* between itself and the notion. Therefore, $A = \{A, B, C\}$, but also $A \dots \{A, B, C\}$.[14] At first, $A$ (the subject of notion's self judgment) is everything; the predicate $\{A, B, C\}$ is nothing. Individuality rests with $A$, the abstract universal.

(2) *Judgment of Reflection*. Whereas the judgment of existence plucked some inessential predicate of the subject (the rose's fragrance), the judgment of reflection makes the predicate universal; the subject is merely an instance of the grander predicate: this thing is useful, or this man is mortal.

In the judgment of existence, $A$ was what Hegel would call *diverse* from {A, B, C}. Diverse things are finite immediate beings. Finite Beings must, on their logic, pass away. If $A$ is diverse, $A$'s fate to become *nothing*.[15] Accordingly, in the judgment of reflection, the subject becomes nothing. The predicate $\{A, B, C\}$ becomes *everything*.[16] Since the predicate is fixed,[17] it now claims for itself the state of individuality, at the expense of the subject. One way of expressing this is that abstract $A$ (the subject) turns into notional $B$ (particularity), so that now $B = \{A,B, C\}$.

(3) *Judgment of Necessity*. In the judgment of necessity, the genetic requirements of the subject are emphasized. Instead of "this individual is mortal," we have "*all* individuals are mortal."

In the judgment of reflection, the subject (*B*) stated, "I do not exist. I am not the predicate {*A*, *B*, *C*}." Yet if {*A*, *B*, *C*} is diverse, it too must pass away as a finite being. But now *B* and {*A*, *B*, *C*} have a commonality. They both must pass away. This is their *necessary connection*. *Connection* (or *copula*) is the only thing that has staying power. Subject and predicate have no "being for self." Individuality now resides in the *unity* of subject and predicate. *B* morphs into *C*. Now *C* = *{=, …}*. *C* represents "the unity of self and other," in our colloquial formula. But shall the copula be {=} or {…}? This is a matter of blind accident. All we know is that subject and predicate are related *positively* or *negatively*.

(4) *Judgment of the Notion*. The judgment of the notion is normative: this individual is as she should be, or this house is good.

The copula was front and center in the judgment of reflection. But copulae cannot do without subject and predicate. These are the means by which *C*, the individual, expresses itself as copula. Individuality as copula now subsumes subject and the predicate. Notion now knows itself to be fully present in *all* its moments. Key here is the idea that there are notional moments and non-notional moments. But how can we tell which is which? Nothing in these moments betrays their true nature. About these moments there is nothing but doubt--*except* that *either* the moments are notional *or* they are not. Meanwhile, *C* = *{A, B, C}* and *C* … *{A, B, C}*.

As Carlson suggests, across the four judgments, then, individuality travels around. It starts as the subject, it travels to the predicate, it travels to the copula, and it encompasses the copula *and* the extremes, while still preserving the necessity of non-notional contingency. By this means, the four judgments replicate the logic of quality, quantity, actuality and notionality. (Carlson 2005:114).

Being – Essence – Notion (unity of Being and Essence) – Essence (is the twice-told tale) – (the realm of mediation) – Essence is doubled (must be – Hegel states), so judgment must be immediate, twice mediated, and notional (i.e., triune).

According to Hegel, the three kinds of judgement are parallel to the stages of Being, Essence, and Notion, yet the second of these kinds, as required by the character of Essence, which is the stage of differentiation, must be doubled . . . [W]hen the Notion, which is the unity of Being and Essence in a comprehensive thought, unfolds . . . it must reproduce these two stages in a transformation proper to the notion . . .

Carlson directs to Hegel own explanation in the *Science of Logic and* the *Encyclopedia Logic. In the Science of Logic* Hegel does not allude very directly to the change, but in the *Encyclopedia Logic*, Hegel explains:

the different species of judgement derive their features from the universal forms of the logical idea itself. If we follow this clue, it will supply us with three chief kinds of judgement parallel to the stages of Being, Essence, and Notion. The second of these kinds, as required by the character of Essence, which is the stage of differentiation, must be doubled . . . [W]hen the Notion, which is the unity of Being and Essence in a comprehensive thought, unfolds . . . it must reproduce these two stages in a transformation proper to the notion . . . (In Carlson 2005:114)

In this passage, Hegel suggests that it is the function of judgment to replay the objective logic, which had sublated itself at the end of essence. In the course of this dumb show for the sake of subjective notion, essence is the twice-told tale. Essence is the realm of mediation, so that judgment must be immediate, *twice* mediated, and notional (*i.e.*, triune).

Carlson asks – "Can someone explain to me the quadruplicity concept instead of the triplicity in the dialectics? Or point me in a direction that has a written answer." (Carlson 2005:114)

Carlson asserts that "Hegel does not limit the above remark to judgment. Perhaps he is saying that, throughout the subjective logic, where the notion reestablishes its own reality, there is *always* quadruplicity, since mediation (*i.e.*, negativity) is always both a mediation and an immediacy. If so, the question arises why *only* the judgment chapter and, we should add, the first third of syllogism, are overtly tetradic in form." (Carlson 2005)

In the introduction to the *Science of Logic*, Hegel suggests that the only valid exposition of philosophy is one that conforms to the "simple rhythm" of method, which is arguably triune. The divisions, headings, sections and chapters serve only "to facilitate a preliminary survey and strictly are only of *historic*value. They do not belong to the content and body of the science but are compilations of an external reflection which has already run through the whole of the exposition and consequently knows and indicates in advance the sequence of its moments before these are brought forward by the subject matter itself."

In other words, Hegel *himself*, having worked through the system, inserts the headings solely for expositional convenience. The headings have nothing to do with the logic itself. This leads one to believe that perhaps we should make *nothing at all* out of the quadripartite headings in Judgment.

Shall we say that tetrachotomy is simply an error by Hegel? There is some reason to think so. In Measure, Hegel denounces Kant's table of categories precisely *because* they are tetrachotomous.6 No triplicity inheres between Kant's quantity, quality, relation and modality, Hegel complains. For this very reason, Hegel writes, Kant "was unable to hit on the third to quality and quantity."[368] Hegel implies that "modality" was Kant's true third --a term Hegel equates with Measure.

All of Hegel's judgments can be found in Kant's table of categories and judgments. Of course, Hegel reverses Kant's priority and analyzes "qualitative judgments" first, consistent with the general priority of quality over quantity. He also renames the major headings. Instead of quantity-quality-relation-modality, Hegel gives us existence-reflection-necessity-notion.

In Kant we have Quantity-Quality-Relation-Modality. In Hegel there are Existence-Reflection-Necessity-Notion.

Carlson states that "It is certainly odd that Hegel should criticize the quadripartite Table of Categories while following the related Table of the Logical Functions of Judgment." (Carlson 2005:114). He recalls how Slavoj Žižek, as brilliant reader of the *Science of Logic*, defends Hegel: "Let us immediately show our cards: the three judgments actually acquire a fourth because 'Substance is Subject'; in other words, the 'lack of identity' between subject and predicate is posited as such in the fourth judgement (that of the Notion). (In Carlson 2005: supranote 12).

[368] SL, *supra* note 1, at 327; 1 WL, *supra* note 1, at 337.

Carlson shares Žižek's defence of Hegel's tetrachotomous judgment, but in different terms. Carlson argues that it is not the *last* but the *first* judgment, the judgment of existence or inherence—that stands for the diversity of subject and predicate. The last judgment in fact vindicates a *unity* between identity and difference. In Carlson's proposition we can find true topological notion both in mathematical topology manner and in rhetorical (Metalepsis). The "diverse" subjectivity on display in the first of the judgments (which reappears in the last of the judgments) is an acknowledgement of an external reflection that haunts all parts of the *Science of Logic,* asserts Carlson.

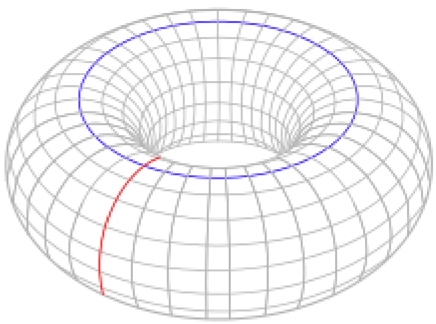
In ansering the question - *Why are there four judgments?-* Carlson emphasize on the "the silent fourth that energizes the logic", bringing to our attention what Hegel writes, that quantity depends upon an other for its determination. Who is this "other" but the silent fourth? Of quantity, Hegel writes that "determinateness in general is outside itself." Quantity is "posited as self-repelling, as in fact having the relation-to-self as a determinateness in another something (which is *for itself*)." This means that quantitative distinction is externally imposed. Quantity requires a silent fourth--an external mathematician who *counts*.

Measure likewise points to a *measurer* - a silent fourth external to the logic itself that raises the temperature of things in order to produce a qualitative change. Qualitative change is accomplished by quantitative change, which is defined as that which comes from the outside. "As a quantum [Measure] is an indifferent magnitude open to external determination and capable of increase and decrease. But as a measure it is also distinguished from itself as a quantum, as such an indifferent determination, and is a limitation of that indifferent fluctuation about a limit." Thus, measure stands for susceptibility to outside manipulation. A silent fourth is directly implied by measure.

Calson states that "the disjunctive syllogism is the point that the universal subject is all its predicates, but this subject still requires a non-notional object--a non-universal that constitutes a fourth to triune subjectivity. The subject's object must eventually be rendered notional. Through the dialectic of objectivity (mechanism-chemism-teleology), the silent fourth is further developed until, in Teleology, the silent fourth is revealed to be the subject's very own self. Two subjects face each other in Teleology. The silent fourth itself becomes three. … That is the very Idea of Hegel's *Science of Logic*." (Carlson, 2005:125).

Carlson's conclusion suggests topological notion and analogy with the torus - "**There is a hole in the Whole…"**(Carlson, 2005:125).

In Hegel's very last chapter, method and subject matter supposedly conjoin. Method is the one and only subject. We have the Understanding, its negation, Dialectical Reason, and the Negation of the Negation--Speculative Reason. But when all is said and done, there is a hole in the whole. Negation of the negation is not the restoration of the positive thing originally negated. It is less than that. This very absence is the silent fourth--the non-notional individual which guarantees that Logic never ends. It is only by virtue of the silent fourth that Logic is a circle. (Carlson, 2005:125).



The torus is defined as a Cartesian product of two circles $T = S^1 \times S^1$. The torus has a single path-connected component, two independent one-dimensional holes (indicated by circles in red and blue) and one two-dimensional hole as the interior of the torus. The corresponding homology groups are

$$H_k(T) = \begin{cases} \mathbb{Z} & k = 0, 2 \\ \mathbb{Z} \times \mathbb{Z} & k = 1 \\ \{0\} & \text{otherwise} \end{cases}$$

The two independent 1D holes form independent generators in a finitely-generated abelian group, expressed as the Cartesian product group $\mathbb{Z} \times \mathbb{Z}$.

Hegel's tetrachotomous judgment suggest topological logic and notion of **Homology**. Homological theory can be said to start with the Euler polyhedron formula and Euler characteristics, followed by Riemann's definition of genus and *n*-fold connectedness numerical invariants in 1857 and Betti's proof in 1871 of the independence of "homology numbers" from the choice of basis.

## 2. Topological Model of Cobordims and Categories of Hegel's multiplicity

The first language of infinity is the image. The image is the form of recollection. (Verene 2007:xiv-xv).[369]

Donald Phillip Verene emphasizes on the role of the image in philosophical text, stating that "Any philosophical text depends upon images; they are always present. The reader can look first not for arguments in the work but for these root images. Once found, the reader can look for the questions that can be drawn forth from the images. The reader will then see how the image is directing and providing support for the question, which carries the reasoning process of the text forward. What are the images? What are the questions embedded in them?" (Verene 2007:xiv-xv).[370]

The visual side of topological metaphorical speculative thinking and reasoning in Hegel is presented in the term Vorstellung (picture-thinking, figurative thinking)[371] Verene points out that "This is Hegel's contrasting term to thought done in the true philosophical form of the concept (*Begriff*)."[372] Hegel also characterizes the stage of consciousness that he calls "Religion," the  stage immediately before that of "Absolute Knowing," as a stage in which thought has the shape of *Vorstellungen* (picturethoughts). Verene asserts that "*Vorstellung* is not easily translated by any single English term that will suit all or most contexts. It is a flexible and easily used term in German. Verene emphasized on the use of Vorstellung

[369] Donald Phillip Verene, 2007, Hegel's Absolure: An Introduction to Reading the Phenomenology of Spirit, State University of New York Press, Appendix, p. 117

[370] Donald Phillip Verene, 2007, Hegel's Absolure: An Introduction to Reading the Phenomenology of Spirit, State University of New York Press, Appendix, p. 117

[371] Donald Phillip Verene, 2007, Hegel's Absolure: An Introduction to Reading the Phenomenology of Spirit, State University of New York Press, Appendix, p. 117

[372] p. 118

(Picture-thinking, Figurative thinking) also in his *Hegel's Recollection: A Study of Images in the 'Phenomenology of Spirit.*

Following the above assertion, it seems natural for the contemporary commentators of Hegel to elaborate such a picture thoughts within images and diagrams.

In the series of papers and *A Commentary on Hegel's Science of Logic* considered not only as the first English language full commentary on the monumental The Science of Logic*,* but as a major advancement in the study of Hegelian philosophy, David Gray Carlson has devised a system for diagramming every single logical transition that Hegel makes, many of which have never before been explored in English. The topological approach in Hegel's Science of Logic is evident in Carlson's diagrams for clarifying the argumentative structure of each move of the text in the fashion of complicated Venn diagram called a Borromean Knot (Lacan's famous Borromean knots). Carlson presents Hegel's logic in the form of pictorial triads of overlapping concepts, in the quite topological mode of Lacanian knots (Carlson 2003a).

The "inapparent" topological (in) Hegel is revealed only through the pentimento offered by Carlson, as we read in the publisher's review of the book: "The author has devised a system for diagramming every single logical transition that Hegel makes, many of which have never before been explored in English. This reveals a startling organizational subtlety in Hegel's work which heretofore has gone unnoticed. In the course of charting Hegel's logical progress, the author provides a vigorous defence and thorough explication of unparalleled scale and scope." Carlson's diagrams are really evoking the topological qualitative quantity of the Lacan's famous Borromean knot, where no one of the rings is directly tied to the other, but if you cut *one* of the rings the other two slip away." (quotation…)

Carlson's diagrams are constructed with three rings representing triads of circles, where the movement to the new element in development of Hegel's logic as inclusion and extraction is illustrated with segments and vectors. Such representation of development of Hegel's though remains captured to Lacanian topological presentation of Borromean knots and lack topological dimensionality since the segments from the overlapping circles. Such visual representation is accompanied with the text where Carlson addresses the segments only with numbers, for example 7 is the in-self of 3 which is mediation of 4, 5, 6. The lack of naming these segments is quite distracting. There is a lack of true topological sense with the above mentioned extraction of segments, for example the qualitative quantity is the third ring linked with quality and quantity in the diagram of measure and if one cut off the qualitative quantity, the notion of the two – quality and quantity will remain incomplete and non-topological.

As it is mentioned in Christopher Yeomans's review of *A Commentary on Hegel's Science of Logic (2007)*: "Carlson's primary device for clarifying the argumentative structure of each move of the text is complicated Venn diagram called a Borromean Knot. This is a figure that Jacques Lacan has found significant and made prominent, but though there is also some discussion of Lacan in the commentary (to my mind the most interesting discussions in the text are those in which Hegel's concepts are compared with Lacan's), there is no general discussion on the comparative significance of the Knot for both thinkers . . "[373]

The picture thoughts within images and diagrams is exhibited by Julie E. Maybee in her *Picturing Hegel: An Illustrated Guide to Hegel's Encyclopaedia Logic (2009).*[374] If Carlson exhibits Hegel's logic in the form of pictorial triads of overlapping concepts (2003a: 93-101), Maybee in her *Picturing Hegel: An Illustrated Guide to Hegel's Encyclopaedia Logic* offers her own diagrams.

My proposition and interpretation of Hegel's Logic implements two topological constructs – cobordism and simplicial complex.

As William Lawvere observes and discusses in his paper *Unity and identity of opposites in calculus and physics* (Lawvere, 1996) [375] - A significant fraction of dialectical philosophy can be modeled mathematically through the use of "cylinders" (diagrams of shape Δ) in a category, wherein the two identical subobjects (united by the third map in the diagram) are "opposite". In a bicategory, oppositeness can be very effectively characterized in terms of

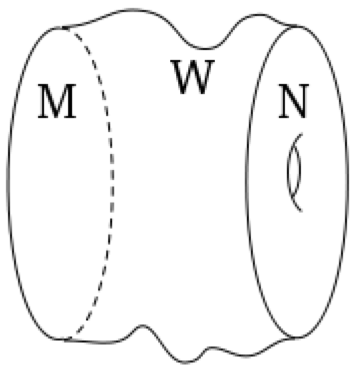
[373] C. Yeomans, Mind, Volume 119, Issue 475, pp. 783-786, Oxford Journals, Art and Humanities and Social Sciences http://mind.oxfordjournals.org/content/119/475/783.extract#

[374] Julie E. Maybee. "Picturing Hegel: An Illustrated Guide to Hegel's Encyclopaedia Logic", Lexington Books, 2009, 34

[375] F. William Lawvere, 1996, Unity and identity of opposites in calculus and physics, Applied Categorical Structures, June 1996, Volume 4, Issue 2, pp 167-174

adjointness, but even in an ordinary category it may sometimes be given a useful definition. . . . The description in engineering mechanics of continuous bodies that can undergo cracking is clarified by an example involving lattices, raising a new questions about the foundations of topology.” (Lawvere, 1996).

Following Lawvere’s suggestion about the use of “cylinders” (diagrams of shape Δ) I will direct attention to two topological constructs. The first based on cylinders is the cobordism and the second based on the shape Δ addresses the simplicial complex.




A cobordism (*W*; *M*, *N*).
"A sample cobordism, between a sphere and a torus."

From the two circles (or annuli [376]) of the *Pure Being* and *Pure Nothing*, Hegel constructs the triad of Becoming just to deconstruct it, deriving the Determinate Being (Quality), demonstrating the move from Becoming (the Determinate Being) to the Determinate Being (Quality).

In Hegel’s Logic the manifolds of *Pure Being* (M) and *Pure Nothing* (N) are *cobordant* (aborder) if their disjoint union is the *boundary* of a manifold one dimension higher. The notion of cobordism is simple - two manifolds M and N are said to be cobordant if their disjoint union is the boundary of some other manifold (W) or the “Third”. The third is *Becoming.*

[376] In mathematics, an annulus (the Latin word for "little ring", with plural *annuli*) is a ringshaped geometric figure, or more generally, a term used to name a ring-shaped object. Or, it is the area between two concentric circles.

Being and Noting, Quality and Quantity are in fundamental equivalence relation in Hegel's Logic, forming classes of compact manifolds of the same dimension, which is set up using the concept of the boundary if a manifold. Any of the two manifolds from each of Hegel's triads – Being and Noting and Quality and Quantity (for example), are cobordant since their disjoint union is the boundary of a manifold and forms the boundary of the third – Becoming and Qualitative quantity. The boundary – the third manifold is one dimension higher.

The boundary of an $(n + 1)$-dimensional manifold $W$ (Becoming) is an $n$-dimensional manifold $\partial W$ that is closed, i.e., with empty boundary. In general, a closed manifold need not be a boundary: cobordism theory is the study of the difference between all closed manifolds and those that are boundaries. The theory was originally developed for smooth manifolds (i.e., differentiable), but there are now also versions for piecewise-linear and topological manifolds.

A *cobordism* between manifolds $M$ and $N$ is a compact manifold $W$ whose boundary is the disjoint union of $M$ and $N$, $\partial W = M \sqcup N$.

The mathematical concept of mani*fold* brings together geometry and topology and is critical for philosophers in the tradition of Continental philosophy, as well as the roots of this tradition in Ancient Greek philosophy and geometry.

Hegel's The Phenomenology of the Spirit [Phänomenologie des Geistes] first appeared in 1807 under the title System of Science, and in the text Hegel called Knowledge [Wissen/Savoir] or Science [Wissenshaft] - a System or Manifold (Mannigfaltigkeit), which has both a topological and philosophical sense. The proposition of Topological Hegel is developed by Robert Groome, from P.L.A.C.E., Santa Monica[377]. Groome is the author of the original study "Formalization of Hegel's Phenomenology of the Spirit". [378]

In his study Groome proposed and developed the thesis of topological Hegel. According to Groome the Phenomenology of the Spirit and Hegel's conception of phantasy are constructed in a topological structure, while showing the correspondence with Lacan's topological project.

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377 http://lacanlosangelespsychoanalysis.com/

378 Robert Groome, "Formalization of Hegel's Phenomenology of the Spirit" http://www.lacanlosangelespsychoanalysis.com/classes/course/info.php?id=21

The mathematical concept of manifold could be traced to Leibniz, but it was Reimann who elaborated the concept. As Arkady Plotnitsky recalls Deleuze in his influential work "Algebras, Geometries and Topologies of the fold: Deleuze, Derrida and Quasi-Mathematical thinking with Leibniz and Mallarme" – "**the noun "manifold" marked the end of dialectics and the beginning of topology".**

The mathematical concept of maniforld is cruicial to the philosophy of Deleuze, who took his topology from Leibniz (Deleuze, The Fold: Leibniz and the Baroque", 1993)

The manifold is topological space, a kind of patchwork of (local) spaces, each of which can be mapped by a (flat) Euclidean, or Cartesian coordinate map, without allowing for a global Euclidean structure or a single coordinate system for the whole except in the limited case of a Euclidean homogenous space itself. Every point has a small neighbourhood that can be treated as Euclidean, while the manifold as a whole cannot.

The concept of manifold is present in Husserl as *Mannigfaltigkeit* (variously translated as "manifold" or "multiplicity"). Husserl's inspiration for elaborating the concept of *Mannigfaltigkeit* came from the father of the term maniforld - Bernhard Riemann. Cantor also used generally the term "manifold" simply as synonym for "*Menge*" (quantity), or "*Inbegriff*" (collection), thereby laying the foundations for set-theory. Indeed, in the 1890s Cantor started calling his work *Mengenlehre* ("theory of quantities"), instead of *Mannigfaltigkeitslehre* (*"*theory of manifolds").

Husserl is well aware of the differences between Cantor and Riemann, during the period of the *Philosophy of Arithmetic* and well awere of the different use of the concept in Cantor and Riemann. According to Husserl, by manifold, Cantor means a simple collection of elements that are in some way united, and however, this conception does not coincide with that of Riemann and as used elsewhere in the theory of geometry, according to which a manifold is a collection not of merely united, but also ordered elements, and on the other hand not merely united, but continuously connected elements.

Husserl's *Mannigfaltigkeitslehre* is nothing to do with the Cantorian set-theory, but come rather closer to topology. In "*Prolegomena"*, Husserl introduces the idea of a pure

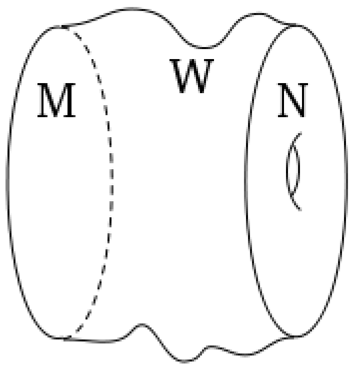
*Mannigfaltigkeitslehre* as a meta-theoretical enterprise which studies the relations among theories, e.g. how to derive or found one upon another. When Husserl announces that in fact the best example of such a pure theory of manifolds is what we actually already have in mathematics. , this sounds slightly odd and is a bit misleading. The pure theory of theories cannot simply be the mathematics underlying topology, but should rather be considered as a *mathesis universalis*.

Husserl will later explicitly tie together the notions of pure theory of manifolds and *mathesis universalis*. The *mathesis universalis* in this sense is formal, a priori and analytic, as theory of theory in general. It is an analysis of the highest categories of meaning and their correlative categories of objects.

Lets move back to Hegel's development of the Logic to see how a series of sample cobordisms, between a sphere and a torus is constructed.

The torus presents the dialectics between *Pure Being* and *Pure Nothing*, where the *vectors* illustrate the relations (magnitudes) *between* the two forming a mobius strip. This mobius strip is *Becoming.* The two manifolds of *Pure Being* (M) and *Pure Nothing* (N) are *cobordant* (aborder) since their disjoint union is the *boundary* of a manifold one dimension higher – the third manifold is *Becoming.*

From the two circles (manifolds) of the *Pure Being* and *Pure Nothing*, Hegel constructs the manifold is *Becoming* - diagrams of shape Δ – the triad of Becoming just to deconstruct it, deriving cobordantly the Determinate Being (Quality), demonstrating the move from Becoming (the Determinate Being) to the Determinate Being (Quality).



Next step on the stage is the two fold interplay between the manifolds of *Quality* (M) and *Negation (N)*, where from their middle Something (W) appears and we have the new triad – new manifold constructed between Quality, Negation and Something, just to dissolve it, moving from W to Something/Other. Here again, the interplay is twofold entity, two manyfolds between *Something/Other* (M) and *Being for Other* (N), where Being for Other is also new posited as Being-for-other/Being-for-itself.

In order to move (one dimension higher) to the new manifold, new triad, here Hegel includes the Determination (W) between Something/Other (M) and Being-for-itself (N). The cobordism (and the triad - diagrams of shape Δ) of the Determination of the In-itself is done, just to be deconstructed again – excluding the middle intersection (W) from the triad and moving to Constitution.

Then *Constitution* (M) is opposed to *Determination* (N) yet these two are in relation of the two fold again. From the relation between the two manifolds – Constitution (M) and Determination (N), the third is born (W) – the Limit (Determinateness as Such).

The new reborn manifold of Determination as Determinateness as Such is the crown of the new triad, new manifold posited as Limit (W). The crown of the Limit is down in the emerged triad of Finitude.

Finitude (W) is born from the toposes, the middle intersection, the cobordism between Constitution (M), Determination (N) and Limit (W). Limit is in play with Limitation. The two spheres have something in common, their middle (W) is the seed of Ought, and the new triad is here, new cobordism constructed between the Finitude (M), Limitation (N) and the Ought (W). If Finitude before was the manifold W, now is the manifold (M).

Again deconstruction is active and Hegel chaining one cobordism with the next, derives from this deconstructed triad the Enriched Finitude, born from the topos of the tree components. The new dual structure between *Enriched Finite* (M) and *Another Finite* (N) is established, just to lead us to Infinity (W).

Infinity - both (W) becoming now (M) - is the crown of the interplay between Enriched Finite (M) and Another Finite (N). The middle three intersections from the cobordant (W) of this triad move to construct the manifold of Spurious Infinity (W) that will become (M).

Again the Spurious Infinity (M) is involved in the twofold game with Another Finite (N). The posited two are enriched with the True Infinity (W) and the triad/manifold of True Infinity (W) is constructed between the Spurious Infinity (M), Another Infinity (N) and True Infinity (W). Here the journey of Being-for-self begin from the transformed Spurious Infinity as part of the triad.

Again the twofold interplay is witnessed between the *Being-for-self* (M) and *Being-for-other* (N). This double enclosure is named Being-for-one (W). From here, Hegel builds the new manifold, new triad, the triad of One (W) from Being-for-self (M), Being-for-Other (N), The One(W). From this triad the shadow of new entity emerge – the One in its own self.

The dual structure of interplay between the *One* (M) and the *Void* (N) emerge from the cobordism and Repulsion (W) is included. The very new triad – the manifold of the One/Many Ones (M), the Void/Many Ones (N) and the Repulsion (W) is done.

The pathway of Attraction is derived from the middle intersection and relation between the One/Many, the Void/Many and Repulsion, and from the Repulsion itself. *Attraction* (M) and *Repulsion* (N) are here in the double action. From their interrelation the new triad is born, the triad of Quantity (W), the Quantity (W) will become the Quantity (M). The triad of Quantity is constructed by Quantity (M) – Attraction/One (N) – Repulsion/Many (W). This triad is the nest of Continuity.

Continuity (W) is in move, implementing in its journey the middle again. The middle between the three circles from the triad/manifold of Quantity needs new face, new role and get this through the interplay between Continuity (M) and Dicreteness (N), their middle. From the topos between the Continuity and Discreteness springs the Enriched Quantity (W), the head of the new manifold/triad embedding the Continuity and Discreteness.

The very new face and role of Quantity here is the Magnitude. Magnitude (W) is constructed from the toposes of Continuity (M) and Discrteness (N) and the Enriched Quantity (W). The name of this newborn Quantity is the Continuous Magnitude.

The Continuous Magnitude (M) is free from the old triad of Enriched Quantity and now face his counterpart – Discrete Magnitude (N). The cobordant topos of the two Magnitudes is the seed of new triad, the triad of the Quantum, from which the Amount is derived, constructed from the toposes of the three. Amount is not left untouched and the twofold interplay between the Unit (M) and Amount (N) is here. Their topos introduces the new triad of Number (W) – The Number (M) – Amount (M) – Unit (W). From the common intersection of these three circles, the manifold of Extensive Magnitude (Extensive Quantum) emerges.

The Extensive Magnitude (M) meets its counterpart - the Intensive Magnitude (N), and Degree (W) is here as their topos. The topos between the Extensive and Intensive Magnitude establishes the new triad, the triad of Quality of Quantum (W) that will become Quality of Quantum (M).

Intensive Magnitude (Degree) – (W) is born again from the next triad, from the toposes of the previous three. In contrast with the Intensive Magnitude (as twofold between Extensive and Intensive Magnitude), here is the new twofold of Extensive Magnitude (W) which combine the Degree (M) and Extensive Magnitude (N). The topos of the last two leads to the Qualitative Something (W).

The threefold of Qualitative Something (W) give birth to the Quantitative Infinity. Quantitative Infinity (M) will play with the Quantitative Infinity Progress (N) and produce from their common new triad, where the head will be Infinitely Great/Small. With this new triad, Hegel presents the Direct Ratio (W).

And again, the Direct Ratio (M) will be involved in twofold action with Inverse Ratio (N). The product from their between will be Ratio of Powers (W). These three ratios will constitute the new triad of Ratio of Powers (W). This triad will lead to the Immediate Measure.

Immediate Measure (M) and Mediated Measure (N) will form new relation, the triad of the Mediated Immediate Measure. Immediate Measure and Mediated Measure impose a topos, the new triad of Specifying Measure (W). From Specifying Measure the move of the Rule is established.

The Rule as Limit (M) and Specifying Measure as Amount (N) construct the Rule Measuring its Other. From this double the new topos is born – The Ratio of Measures (Realized or Specifyed Measure) – (W).

The topos of the new triad will construct the Combination of Measures (W). Hegel sees here the new topological twofold of relations between the Externality of Measure (M) and Measure as Series (N). From the intersection of these two the Series of Measure Relations (W) appears. These two involve as third the Elective affinity (W) and construct new triad from which Continuity of Affinity appears.

Again, the two - Continuity (M) and Indifference Subtrate (N) - presents the new double of Indifference of Affinity (Substrate) – (W). The topos of these two are the seed of the Nodal Line triad.

Hegel will deconstruct the triad of the Nodal Line, deriving from it the three circles of the Abstract Measureless (W) moving from the dual interplay between Quality (M) and Abstract Measureless (N). The topos of these two will introduce new triad, the triad of Infinity for Itself (W), from where Hegel will derive Absolute Indifference.

With the new born Hegel attracts again the ratio, this time as Inverse Ratio and will establish the relation between two – Inverse Ratio of Factors.

Finaly we arrive at the manifold of Essence constructed as triad containing the Absolute Indifference (M) and Inverse Ratio (N).

The above presentation shows how Hegel construct true topological manifold from which the notion of multiplicity emerges.

In his speculative logic and semantic Hegel uses the pair of tropes, cobords the pair of terms and categories not only in opposition but in manifold within their twofold relation, where the emphasis is on the middle, the topos of the logical and semantological space between these tropes, terms, and categories. Merging metaphorical power of rethoric with the charge of speculative reason, it is striking how Hegel creates the blueprints of something, the mode and construct that centuries later will be defined as a *cobordism* . . . between the pair of manifolds *M* and *N,* where the third will appear as a compact manifold *W* whose boundary is the disjoint union of *M* and *N*, $\partial W = M \sqcup N$. From this topos and betweenness something new arises as meaning and fruit of speculative thinking, as the core of Hegel's dialectical method, where mathematical (topological) and philosophical (categorical) emerges. This mode of developing both text and though can be recognized as a double-entendre – the cobordant (W) between (M) and (N). The **double-entendre** in Hegel's logic is recognized by Andrew Hass in his 'Hegel and the Problem of Multiplicity' (Haas, 2000).[379]

As Haas suggests: "The particular quality of quantification, the reproduction and restoration is taken care of in the dialectic of qualitative-quantitative concept – and in Hegel, *multiplicity means the quantification of quality and the qualification of quantity, a multiplicity of the double-entendre, of the inevitable double-meaning.* (Haas, A. 2000: 113). [380]

In the Science of Logic, Hegel develop his triads as "diagrams of shape Δ" (Lawvere, 1996) [381], deriving the new element for the each triad from the twofold pairs of tropes posited in opposition and distilating the new trope, term from the common place, the intersection between these two, from their topos. This rethorical mode used by Hegel represents not only the metonymy but double metonymy – two metonymies, one contained in the other, but only one expressed. For example from the twofold interplay between ***Quality*** (M) and ***Negation*** **(N)**, from the middle of their relationship, from their topos, the semantological place between these, **Something (W)** as the third trope and category emerge. When **Something** appears, we

[379] Haas, A. 2000. Hegel and the Problem of Multiplicity. SPEP Studies in Historical Philosophy. Evanston: Northwestern University Press

[380] Haas, A. 2000. Hegel and the Problem of Multiplicity. SPEP Studies in Historical Philosophy. Evanston: Northwestern University Press

[381] F. William Lawvere, 1996, Unity and identity of opposites in calculus and physics, Applied Categorical Structures, June 1996, Volume 4, Issue 2, pp 167-174

have the new triad constructed between **Quality (M)**, **Negation (N)** and **Something (W).** The third element emerges from the double meaning and the triads are constructed from the twofold relation between the two tropes, from their common space, from the intersection between the two circles.

The cobordism - between manifolds M and N is a compact manifold W whose boundary is the disjoint union of M and N, $\partial W = M \sqcup N$ - provides conceptual model and explanation of the concept of Aufhebung, or sublation, as it arises in G. W. F. Hegel's dialectical logic. As it is stated by Ryan Krahn – "the concept shall not be understood to function simply as a mere negation of negation, where that would mean an assimilatory determinate negation of a prior moment of abstract negation." (Ryan Krahn, 2014) Instead, Ryan Krahn argues that "both abstract and determinate negation function at the level of sublation as such and that the concept should thereby be understood not only as a synthesis that combines a term with its antithesis (invokes the popular caricature of a simple thesis-antithesis-synthesis ) , i.e., a unifying third term, but also as a fourth that treats these terms in their difference, holding them apart as oppositional (Ryan Krahn, 2014)

Ryan Krahn assertion of quadruplicity of sublation (Ryan Krahn, 2014) is in compliance with David Gray Carlson's thesis of four Hegelian judgments. (Carlson 2005:114). Carlson's asks – "Can someone explain to me the quadruplicity concept instead of the triplicity in the dialectics? Or point me in a direction that has a written answer." (Carlson 2005:114)

In ansering the question - *Why are there four judgments?*- Carlson emphasize on the "the silent fourth that energizes the logic", bringing to our attention what Hegel writes, that quantity depends upon an other for its determination. Who is this "other" but the silent fourth? Of quantity, Hegel writes that "determinateness in general is outside itself." Quantity is "posited as self-repelling, as in fact having the relation-to-self as a determinateness in another something (which is *for itself*)." This means that quantitative distinction is externally imposed. Quantity requires a silent fourth--an external mathematician who *counts*.

Measure likewise points to a *measurer* - a silent fourth external to the logic itself that raises the temperature of things in order to produce a qualitative change. Qualitative change is accomplished by quantitative change, which is defined as that which comes from the outside. "As a quantum [Measure] is an indifferent magnitude open to external determination and capable of increase and decrease. But as a measure it is also distinguished from itself as a quantum, as such an indifferent determination, and is a limitation of that indifferent fluctuation about a limit." Thus, measure stands for susceptibility to outside manipulation. A silent fourth is directly implied by measure.

Calson states that “the disjunctive syllogism is the point that the universal subject is all its predicates, but this subject still requires a non-notional object--a non-universal that constitutes a fourth to triune subjectivity. The subject's object must eventually be rendered notional. Through the dialectic of objectivity (mechanism-chemism-teleology), the silent fourth is further developed until, in Teleology, the silent fourth is revealed to be the subject's very own self. Two subjects face each other in Teleology. The silent fourth itself becomes three. … That is the very Idea of Hegel's *Science of Logic*.” (Carlson, 2005:125).

Carlson’s conclusion suggests topological notion and analogy with the torus - “**There is a hole in the Whole…”**(Carlson, 2005:125). In Hegel's very last chapter, method and subject matter supposedly conjoin. Method is the one and only subject. We have the Understanding, its negation, Dialectical Reason, and the Negation of the Negation--Speculative Reason. But when all is said and done, there is a hole in the whole. Negation of the negation is not the restoration of the positive thing originally negated. It is less than that. This very absence is the silent fourth--the non-notional individual which guarantees that Logic never ends. It is only by virtue of the silent fourth that Logic is a circle. (Carlson, 2005:125).

The quadruplicity of sublation - the thesis supported by Ryan Krahn. (Ryan Krahn, 2014) is in compliance with our thesis of Topological Notions of Multiplicity in Hegel’s Fourfold of Infinities. Our thesis is strongly supported by the research of Andrew Haas, in particular his book “Hegel and the Problem of Multiplicity”. (Haas, A. 2000)

This fourfold of infinities (in Hegel) could be expressed in by the term or category of Cobordism - A cobordism (pair of pants) between a single circle (at the top) and a pair of disjoint circles (at the bottom). In topological terms, the notion of Hegel’s Qualitative quantity could be represented with the single circle (at the top) and ‘quality’, and ‘quantity’ a pair of disjoint circles (at the bottom). Cobordisms form a category whose objects are closed manifolds and whose morphisms are cobordisms.

Cobordism had its roots in the attempt by Henri Poincare in 1895 to define homology purely in terms of manifolds. Poincaré simultaneously defined both homology and cobordism, which are not the same, in general. However, Rene Thom, in his remarkable, if unreadable, 1954 paper “Quelques proprietes globales des varietes differentiables”, gave the full solution to this problem for unoriented manifolds, as well as many powerful insights into the methods for solving it in the cases of manifolds with additional structure. It was largely for this work that Thom was awarded the Fields medal in 1958.

David Aubin, provides an intriguing discussion on ‘topology and meaning’ in Rene Thom, stating that “Having pointed out the relevance of topological concepts and practices for the modeling of biological phenomena, Thom saw no reason to stop there. Since the early 1970s, his main field of research, besides philosophy, have been linguistics and semiotics. With his incursion into the human science, Thom was bound to confront structuralism. Never himself a structuralist per se, but trained in the mathematical structuralism of Bourbaki, he was attracted by this movement. With some adjustments, his theories could be made to fit into structuralist model of thought. But because he began to work on linguistics so late, catastrophe theory was only mildly affected by structuralism in practice. Increasingly faced with strong opposition to his ideas about modeling, Thom pondered the epistemological foundations of catastrophe theory. In attempting to articulate the kind of knowledge that the theory produced, he used structuralist resources most obviously.”[382] (Aubin, 2004)

Thom’s work on cobordism clearly illustrates his intuitive approach as allied with the profound knowledge of Bourbakist methods that guided most of his mathematical work. David Aubin, remind us that Hopf testified, how cobordims was important because of the way it mixed topological and algebraic approaches in the classification of manifolds. (Aubin, 2004:101)

[382] David Aubin, (2004), Forms of explanation in the catastrophe of Rene Thom: topology, morphogenesis, structuralism, in *Growing Explanations: Historical Perspective on the Sciences of Complexity*, ed. M. N. Wise, Durham: Duke University Press, 2004, 95-130.

The manifold composed of two circles is cobording with the manifold consisting of a single circle because there is a “pant-shaped” smooth surface joining them. Since this is true for manifolds combining any number of circles, the group ‘delta’ is the one-element trivial group. (Aubin, 2004:102)

For Thom, “The problem of cobordism . . . is of knowing when two manifolds can be deformed one into other without encountering a singularity in the resulting space, at the moment in this deformation.” The example of a circle becoming two circles can, very crudely of course, model cell division. (Aubin, 2004:101)

The link or topos between Rene Thom’s catastrophe theory (and topological syntax) and Hegel’s logic and mathematics can be found within the concept of singularities, in particular in Hegel’s discussion on Leibniz and Hegel’s disagreement with Leibniz’ concept of singularity.[383] Leibniz was the first to introduce the proto topological ideas in his Analysis Situs, and the concept of singularities was critical in his attempt. Rene Thom calls catastrophe theory the *application* of specific mathematical results in the field of differential topology and the theory of singularities.[384] For Hegel, concept of singularities is central concept in his formulation of infinities, both ‘bad infinity’ and ‘good infinity.’ Hegel believes that in Leibniz’s philosophy the categories of ‘singularity,’ ‘totality,’ ‘infinite,’ ‘being/reality,’ and ‘possibility’ do collapse on one another. Hegel’s criticized Leibniz’s concept of ‘singularity’ as leading to what for Hegel is ‘bad infinity’.[385] Antonio M. Nunziante suggests some remarks on the concepts of ‘singularity’ and ‘infinite’ in Leibniz and Hegel, and on how it is possible to find a world of semantic connections that bind Hegel and Leibniz together. Choosing to put ‘singularity’ in relation to the concept of ‘infinite’ was drawn by Hegel’s notation that in Leibniz’s ontology singularity “*geht in seiner eigenen Totalität zu Grunde*.”[386] (Nunziante, 2015)

[383] See Hegel’s Jena writings from 1804, *Logik, Metaphisik, Naturphilosophie* (Jena 1804/05), in HEGEL, G.W.F. Jenaer Systementwürf II. In Hegel, G.W.F. Gesammelte Werke. Bd. 7. hrsg. von R.-P. Horstmann und J.H. Trede, 1971, p. 144. From now on JS II.

[384] Thom, R., 1973: Langage et catastrophes: elements pour une semantique topologique, in: Peixoto, M. M. (ed.) *Dynamical Systems.* Proceedings of the Symposium at Salvador, Brazil, 619—654.

[385] Antonio M. Nunziante, (2015), Infinite vs. Singularity. Between Leibniz and Hegel, University of Padova http://revista.hegelbrasil.org/iii-nunziante-singularity-vs-infinite-print/

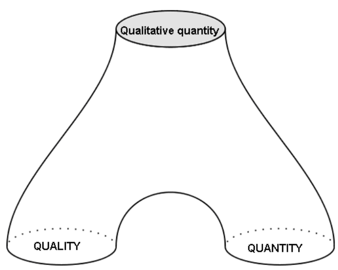
[386] See HEGEL, G.W.F. Jenaer Systementwürf II. In Hegel, G.W.F. Gesammelte Werke. Bd. 7. hrsg. von R.-P. Horstmann und J.H. Trede, 1971, p. 144. From now on JS II.

Topology is a generalization of geometry that studies spaces with the degree of generality appropriate to a specific problem. One central concern of topology is to study the properties of spaces that do not change under a continuous transformation, that is, translation, rotation, and stretching without tearing.  One such concept is expressed by the concept of dimension: a curve is one dimensional; a surface has two dimensions; ordinary space, three; and the space-time of general relativity, four. Mathematicians faced with the problem of characterizing a space locally to a Euclidean space use the notion of manifold. An n-dimensional manifold is a space M, such that a neighborhood V exists around each point p of M in one-to-one correspondence with a subset W of the n-dimensional Euclidean space R. The study of manifolds is called differential geometry, and the classification of all manifolds of a given dimension is an important problem of topology. . (Aubin, 2004:101)

In mathematics, cobordism is a fundamental equivalence relation on the class of compact manifolds of the same dimension, set up using the concept of the boundary of a manifold. Two manifolds are *cobordant* if their disjoint union is the *boundary* of a manifold one dimension higher. The name comes from the French word bord for boundary.

In mathematics, a ‘pair of pants’ is a simple two-dimensional surface resembling a pair of pants: topologically, it is a sphere with three holes in it. ‘Pairs of pants’ admit hyperbolic metrics, and their isometry class is determined by the lengths of the boundary curves (the cuff lengths), or dually the distances between the boundaries (the seam lengths). In hyperbolic geometry all three holes are considered equivalent – no distinction is made between "legs" and "waist". *In cobordism theory the holes are not equivalent* – a pair of pants is a cobordism between one circle (the "waist") and two circles (the "legs").

TOPOLOGICAL MODEL OF QUALITATIVE QUANTITY




In topological terms, the category of Qualitative quantity
is a cobordism between a single circle (at the top)
and a pair of disjoint circles (at the bottom).

In Topology the “pair of pants” is important because it shows how one can "morph" a single circle (the waist) into two circles (the legs); this is technically known as a *cobordism*. Putting together all these horizontal cross-sections gives the final surface.[387]

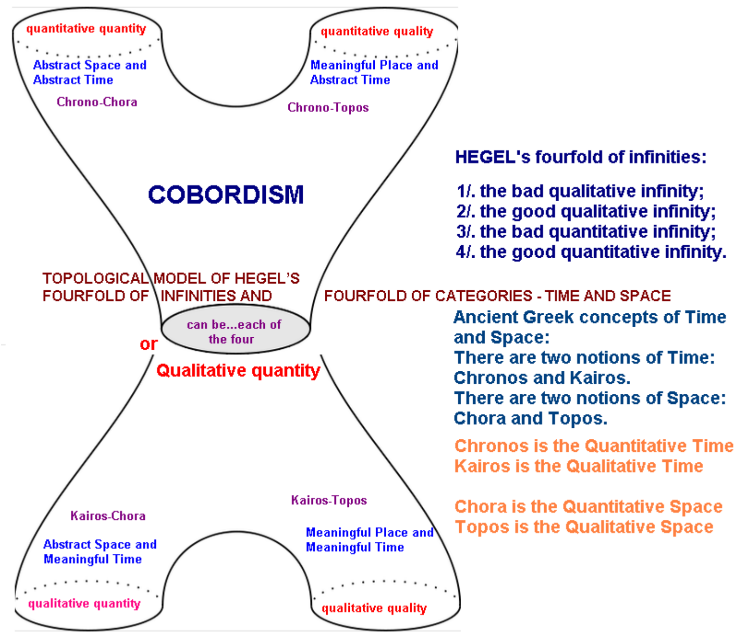
In Philosophical or Dialectical Hermeneutics the “pair of pants” is important because it shows how one can “morph” not only the categories of the “quality” and “quantity” into the fourfold of categories in Hegel, or how one can “morph” Hegel’s fourfold of infinities, but also how one can “morph” the Understanding of human finitude in infinitude and reverse the human infinitude into the finitude, how Hermeneutical Circle can be morph into the Topological Cobordism Ring – how the true Hermeneutical “fusion of horizon” could found in Topology and Topological Hermeneutics.

The Cobordism of Hermeneutical Circle shows how Dialectical Hermeneutics could retrieve itself in Topology. Each of the categories, presenting Hegel’s fourfold of infinities:

1. Quantitative quantity;
2. Quantitative quality;
3. Qualitative quantity;
4. Qualitative quantity,

could be thought as circle or four circles placed in a 3-manifold.

[387] http://www.quora.com/What-is-the-mathematical-expression-which-when-plotted-lookslike-a-pair-of-pants




Hegel's four categories derived from the relation of the "quality" and "quantity" constructing his fourfold of infinities could be seen and represented as cobordism illustrated with the 3-manifolds with corners in the Figure of next page. The cobordism from the so called in topology "pair of pants", illustrates the pair of categories as circles or annuli [388] at the top to the two-punctured disc at the bottom.

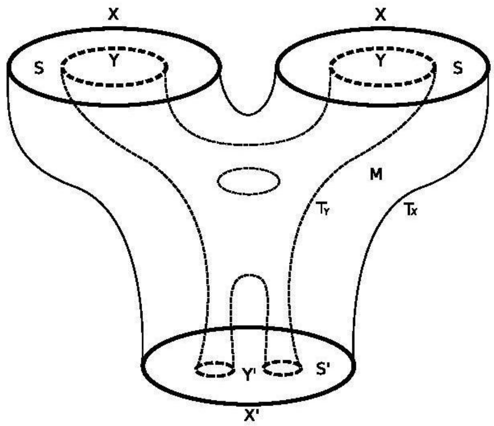
One of the most important question in my reinterpretation of Hegel's Logic is How we are moving from Hegel's triplicity or triadic structure - diagrams of shape Δ – to the quadruplicity of sublation (Ryan Krahn, 2014) and quadruplicity of the four Hegelian judgments (Carlson 2005:114), from the unifying third term to the 'fourth' that treats these terms in their difference, holding them apart as oppositional (Ryan Krahn, 2014). How we are moving to Hegel's fourfold of infinities implemented in 1. Quantitative quantity; 2. Quantitative quality; 3. Qualitative quantity; 4. Qualitative quantity.

Hegel's concept of sublation can be thought of, respectively, as cobordism from one pair of circles to another, and from one circle to two circles. The large cobordism has other boundary components: the outside boundary is itself a cobordism from two circles to one circle; the inside boundary (in dotted lines) is a cobordism from one pair of circles to another pair. We

[388] In mathematics, an annulus (the Latin word for "little ring", with plural *annuli*) is a ringshaped geometric figure, or more generally, a term used to name a ring-shaped object. Or, it is the area between two concentric circles.

could “compose” this with another such cobordism with corners by gluing along any of the four boundary components: top or bottom, inside or autside.

This involves attaching another such cobordism along corresponding boundary components by a diffeomorphism.




As we can see here the concept of sublation is not be understood to function simply as a mere negation of negation, as assimilatory determinate negation of a prior moment of abstract negation. Instead, both abstract and determinate negation function at the level of sublation as such and that the concept should thereby be understood not only as a synthesis that combines a term with its antithesis, i.e., a unifying third term, but also as a fourth that treats these terms in their difference, holding them apart as oppositional. The quadruplicity of sublation can be thought and interpreted as 3-manifold, where to the presented cobordism of M- N – W there is attached another such cobordism along corresponding boundary components by a diffeomorphism. Cobordism and Diffeomorphism provides the ground for reinterpretation of Hegel’s theory of negation and Hegel’s the four Hegelian judgments.

Topological notions in Hegel’s concept of becoming are expressed in the logic *betweenness*, the structure of an *either/and*. As Krahn asserts – “From the concept of becoming we are able to discern some general characteristics of sublation (such as *betweenness*, the structure of an *either/and*, and a fourth element that frustrates the oversimplified, traditional picture of synthetic unity in a third term), each of which will contribute to a preliminary definition of the concept. In so doing, this section will give a preview of the form sublation will take as we later encounter it in the *Phenomenology's* master-slave dialectic and the *Logic's* conclusion with the Absolute." (Ryan Krahn, 2014)

If we trace again the development of Hegel's Logic as presented above interpreted as cobordism, we can see how Hegel applies diffeomorphism, attaching to his triads or three dimensional manifolds another such cobordism along with corresponding boundary components by diffeomorphism. We can see how the middle element (concept) marked as manifold of "W" from the between "M" and "N" becomes the new "M" in the new manifold introduced. The manifold of W contains always the silent fourth its (M). Here is the entrance and introduction (transmission) of new cobordism between manifolds M and N is a compact manifold W.  whose boundary is the disjoint union of M and N. The (W) that will become (M) is the attachment of another such cobordism along corresponding boundary components by a diffeomorphism.There is an internal 'pair of pants' inside the initial one. This internal pair of pants is two headed, having two annuli, two circles on (W). This is the **double-entendre** in Hegel's logic as recognized by Andrew Hass in his 'Hegel and the Problem of Multiplicity' (Haas, 2000).[389]

The multiplicity of the double-entendre, of the inevitable double-meaning. (Haas, A. 2000: 113) [390] in rhetorical mode is the mechanism of Hegel's metalepsis – the metonymy of metonymy.

In the Science of Logic, Hegel develop his triads as "diagrams of shape Δ" (Lawvere, 1996) [391], deriving the new element for the each triad from the twofold pairs of tropes posited in opposition and distilating the new trope, term from the common place, the intersection between these two, from their topos. This rethorical mode used by Hegel represents not only

[389] Haas, A. 2000. Hegel and the Problem of Multiplicity. SPEP Studies in Historical Philosophy. Evanston: Northwestern University Press

[390] Haas, A. 2000. Hegel and the Problem of Multiplicity. SPEP Studies in Historical Philosophy. Evanston: Northwestern University Press

[391] F. William Lawvere, 1996, Unity and identity of opposites in calculus and physics, Applied Categorical Structures, June 1996, Volume 4, Issue 2, pp 167-174

the metonymy but double metonymy – two metonymies, one (M) contained in the other (W), but only one expressed.

For example, from the two circles (manifolds) of the manifolds of *Quality* (M) and *Negation (N)*, where from their middle Something (W) appears, we have the new triad – new manifold constructed between Quality, Negation and Something. Something which is (W) appears in the new manifold as Something/Other (M). Here again, the interplay is twofold entity, two manifolds between *Something/Other* (M) and *Being for Other* (N)*,* where Being for Other is also new posited as Being-for-other/Being-for-itself (W). The diffeomorphism or the two head manifold is presented within the two cobordism where Something is both – first (W) and second (M). Here is the metalepsis and this metalepsis is the chain or attachment of the new cobordism. In order to move (one dimension higher) to the new manifold, new triad, here Hegel includes the Determination (W) between Something/Other (M) and Being-for-itself (N). The diffeomorphism of the new triad of the Determination of the In-itself is done, just to be deconstructed again not only excluding the middle intersection (W) from the triad and moving to Constitution, but renaming and positing this concept. Here the Determination wich before was (W) is now (N). Then *Constitution* (M) is opposed to *Determination* (N) yet these two are in relation of the two fold again. From the relation between the two manifolds – Constitution (M) and Determination (N), the third is born (W) – the Limit (Determinateness as Such). In the new manifold the Determination (which was W and then become N, now is Determinateness as Such (the new, enhanced W), the crown of the new triad, new manifold posited as Limit (W). The crown of the Limit is down in the emerged triad of Finitude.

**The figure of Metalepsis,** the trope Met-a-lep´sis is constructed from μετά (mĕta), behind, and λείπω (leipô), to leave, a leaving behind. The Figure is so called, because something more is deficient than in Metonymy, which has to be supplied entirely by the thought, rather than by the association or relation of ideas, as is the case in Metonymy. This something more that is deficient consists of another Metonymy, which the mind has to supply. Hence Metalepsis is a double or compound Metonymy, or a Metonymy in two stages, only one of which is expressed.[392]

[392] Bullinger, E. W., D.D. Entry for 'Metalepsis; or double metonymy'. Bullinger's Figures of Speech Used in the Bible. http://www.studylight.org/lexicons/fos/view.cgi?n=132.

Hegel’s language, syntax, concepts and notions have topological nature. The topological is presented in Hegel both as mathema (in his philosophy of mathematics) and as rhetoric. If topological nature of Hegel’s mathesis is presented within his fourfold of infinities (multiplicities) and Hegel’s four type of judgments, Hegel’s rhetoric is bearing the four basic tropes of rhetoric: metaphor, metonymy, synecdoche, ironi as equally presented in his manifold (Mannigfaltigkeit) of infinities, quality and quantity, time and space.

David G. Carlson, in his Hegel’s Theory of Measure, in particular in his discussion of (b) Measure as a Series of Measure Relations), states that “**Metonymy** is the theme of this new section's tongue. Metonymy is the inability to name the thing directly, but only the context of the thing. In metonymy, if the entire context is described, the unnameable thing becomes a ghostly space the existence of which is simply inferred from context.” (David G. Carlson, 2003, Hegel’s Theory of Measure:47)

Both Carlson and Maybee represent in their drawing the diagram of Hegel’s main notions as circles, perhaps following Hegel’s statement that the logic is circle – circle of circles .Yet from the circles we can derive the three partite structure of Hegel’s logic – the triangle and move from the triangle to the simplex – or simplexes, just to end up with the proposition that the structure of Hegel’s Logic can be presented and seen as simplicial complex.

The topological notion of Hegel’s logic contains the seed of topological hermeneutics with “genuine infinite, a circle closed on itself” …The infinite that wants to be unlimited, because as Hegel points out – “**there are two worlds, one infinite and one finite, and in their relationship the infinite is only the limit of the finite and is thus only a determinate infinite, an infinite which is itself finite."** What the understanding should not forget is that these alternating determinations of the finite and the infinite are ultimately determinations of one unified something.

Topological cobordis is just another dialectical and hermeneutical view on the betweennes of these two somethings – the finite and the infinite.

## 3. Topological Language of Being: Linguistics and Catastrophe retrieved in Dialectics

**The following three sections (3, 4, 5) deals with the topological language of Being, where linguistic and Catastrophe theory are seen as 'retrieved' in Hegel's dialectics, Rene Thom's topological theory of language and topological syntax, and the thema of Dialectics and Catastrophe, where cobordism of Hegel's fourfold of multiplicities (infinities) are discussed, seen and modeled through bifurcation diagrams.**

**The following discussion emphasize on philosophical program of Rene Thom** (Rene Thom topological theory of language and topological syntax) with his aim of geometrization of thought and linguistic activity in topological, qualitative way, where Gadamer's claim, established with the last words of his essay "The Idea of Hegel's Logic" [393], that "Dialectic must retrieve itself in hermeneutics" (Gadamer 1976) shall be recalled, again. Also on the following works (Thom, R. 1975), (Peter Tsatsanis, P. 2012: 223-224) (Bruce, B. and D.N. Mond .1999) (Zwick, M. 1978)

In this sense, the following discussion **builds on and contributes to works of** (Gadamer 1971/1976), Lucien Tesnière's *Elements of Structural Syntax*[394] (Wildgen and Brandt 2010:57), Rene Thom's "Structural stability and Morphogenesis (1975), Thom's concept of versal unfolding, [395] Rene Thom's work "From Catastrophes to Archetypes: Thought and Language", and Thom's catastrophe theory model of language, his visual representation of the verbs associated with spatio-temporal activity, his classified syntactical structures into 16 categories, where he claimed that "the topological type of the interaction determines the syntactical structure of the sentence wich describes it."

In addition, the present study builds on the concept of cobordism deeply explored by René Thom, after Henry Poincaré's Analysis Situs, Thom's idea of homology and the homology classes, first defined rigorously by Henry Poincaré in his seminal paper "Analysis situs", *J.*

[393] "Die Idee der Hegelschen Logik (1971) in Hans-George Gadamer, "Hegel's Dialectic: Five Hermeneutical Studies", translated into English by P. Christopher Smith and collected in (New Haven: Yale University Press, 1976). These five essays are "Hegel and Heidegger,"; "Hegel's Dialectic of Self-consciousness"; "Hegel and the Dialectic of the Ancient Philosophers,"; "Hegel's 'Inverted World,'; and "The Idea of Hegel's Logic," 75-99

[394] Wildgen, W., & Brandt, P. A. (2010). *Semiosis and catastrophes: René Thom's semiotic heritage*. Bern: Peter Lang., p.57

[395] Bruce, B. and D.N. Mond, eds. Singularity Theory, Cambridge, England: Cambridge U.Press,1999, page xi.

*Ecole polytech.* (2) 1. 1–121 (1895), D'Arch Thompson's concept of homology presented in his "On Growth and Form" (1917).

The topological notion of Hegel's logic contains the seed of topological hermeneutics with "genuine infinite, a circle closed on itself" …The infinite that wants to be unlimited, because as Hegel points out – "there are two worlds, one infinite and one finite, and in their relationship the infinite is only the limit of the finite and is thus only a determinate infinite, an infinite which is itself finite." (Miller,A.V. trans.,1990. Hegel's Science oif Logic)

Once visualized in topological space through the notion and model of Rene Thom's 'cobordism', Hegel's fourfold model of multiplicities (infinities) build on the logical interrelations and interdependence between the categories such as quality and quantity, the model that unfolds the 'qualitative quantity' notion of place within the time, could be exhibited also by the bifurcation diagrams.

The result of the interpretation offered here, demonstrates the striking resemblance between the philosophical or dialectical hermeneutics of the categories of quality and quantity, seen through the model of 'cobordism" - the "pair of pants" – or - the cobordism of hermeneutical circle, showing how dialectical hermeneutics (must) retrieve itself in topology, namely the Hegel's fourfold of infinities:1. Quantitative quantity; 2. Quantitative quality; 3. Qualitative quantity; 4. Qualitative quantity, could be thought as **circle or four circles placed in a 3-manifold, and the pitchfork bifurcation diagrams**, where the components of **supercritical** p**itchfork bifurcation diagrams (b < 0)** and **subcritical pitchfork bifurcation diagrams (b > 0)** draw the **two half planner cilinder** from our example of **cobordism**.

In addition, the presented result of discussion demonstrates how the model of cobordism of topological notion of the categories of quality and quantity – Hegel's Qualitative quantity dialectic and logic, could be expressed (having an exhibit form) as the surface that consists of infinitely many pairs of pants (cobordisms) sewn together in the way that the waist line to leg opening, a pair of pants (Hegel's fourfold of infinities) is homeomorphic to the Cantor set.

Thinking of philosophical program of Rene Thom with his aim of geometrization of thought and linguistic activity in topological, qualitative way, one shall recall Gadamer's claim, established with the last words of his essay “The Idea of Hegel’s Logic” [396], that “Dialectic must retrieve itself in hermeneutics” (Gadamer 1976).

Gadamer’s “The Idea of Hegel’s Logic” appeared in 1971. Since the early 1970s, Rene Thom’s main field of research, beside philosophy, have been lingustics and semiotics.

With his essay Gadamer answers the question – how is possible For Dialectic to retrieve itself in hermeneutics. Gadamer found this answer in the language, by establishing the relevance of Hegel’s logic to the problem of the language, language which Heidegger called “the house of Being”, language through which is possible the understanding of the Being, according to Gadamer’s “Being that can be understood is language”.

After the introductional part of his essay, Gadamer sets off his plan to complete the task for the revivel of Hegel’s Logic, with the following steps: 1. to treat the idea of Hegel’s logic generally; 2. to explore the method of Hegel’s logic; 3. to examine the starting point of Hegel’s logic; 4. In conclusion to establish the relevance of Hegel’s logic to the problem of the language.

In these three steps Gadamer establishes his hermeneutics as dialectical and phenomenological form or dialectical and phenomenological hermeneutics.

In his examination of Hegel’s starting point of Hegel’s logic, Gadamer focuses on the language and logical instinct of language. According to Gadamer, when Hegel speaks of the logical instinct of language he is thus pointing out the direction and object of thought — its tendency towards “the logical.” In the grammar there is a reflection of these logical structures. But Hegel’s talk of the “logical instinct” of language obviously implies more than that. It means that language leads us to logic because in the logic the categories naturally at work in language are focused on as such.

According to Gadamer, for Hegel, language reaches its perfection in the idea of logic since in the latter thinking goes through all of the determinations of thought occurring within itself and

[396] "Die Idee der Hegelschen Logik (1971) in Hans-George Gadamer, “Hegel's Dialectic: Five Hermeneutical Studies”, translated into English by P. Christopher Smith and collected in (New Haven: Yale University Press, 1976). These five essays are "Hegel and Heidegger,"; "Hegel's Dialectic of Self-consciousness"; "Hegel and the Dialectic of the Ancient Philosophers,"; "Hegel's 'Inverted World,'; and "The Idea of Hegel's Logic," 75-99

operating in the natural logic of language, and relates these to each other in thinking the Concept as such. But the question arises whether language is in fact only an instinctive logic waiting to be penetrated by thought and conceptualized.

Hegel notes the correspondence between logic and grammar and compares — without heed to the differences between languages and their grammatical bases — the life which a "dead" grammar assumes in the actual use of a language to the life which logic assumes when one gives content to its dead form through use of it in positive sciences. But as much as logic and grammar might correspond to each other in that both are what they are in concrete use, the natural logic lying in the grammar of every language is by no means exhausted in the function of being a prefiguration of philosophic logic.

Of course, logic in its traditional form is a purely formal science, and thus in any specific use made of it in the sciences or elsewhere, it is one and the same; the life which it assumes for the knower in such use is its proper life.

On the other hand, the idea of logic which Hegel develops within the tradition of Kant's transcendental analytics, is not formal in this sense.

Kantian transcendental analytics or the formal logic, however have a consequence which Hegel would not desire. Specifically, their use in the sciences is by no means the only concretion of this logic. (Indeed the one-sidedness of neo-Kantianisrn lay in the fact that it turned the given fact of science into a monopoly.) On the contrary, in the "variety of human language structures" lies a range of very different anticipations of what is logical, which are articulated in the most diverse schemata of linguistic access to the world. And the "logical instinct," which most assuredly does lie in language as such, can for that reason never be comprehensive enough to include all of what is prefigured in this vast number of languages. Thus it could never really be elevated to its "concept" by being transformed into logic.

Our human nature is so much determined by finitude that the phenomenon of language and the thinking wherein we seek to get hold of it must always be viewed as governed by the law of human finitude.

Hegel’s notion of “speculative logic” and “speculative statement” goes beyond Kantian transcendental analytics and the formal logic. According to Gadamer, Hegel’s grant relevance for today is embedded indeed in what Hegel calls “speculative logic” and “speculative statement”.

For Gadamer, the speculative statement is not so much a statement as it is language. The speculative statement maintains the mean between the extremes of tautology on the one hand and self-cancellation in the infinite determination of its meaning on the other.

Here lies Hegel’s great relevance for today: the speculative statement is not so much a statement as it is language. It calls for more than objectification in dialectical explication. The speculative statement brings dialectical movement to a standstill. Through speculative statement thought is made to see itself in relationship to itself.

In the language form (not of a judgment as a statement, but in the judgment as it *is spoken* in a verdict, for example, or in the curse) the event of its being said is felt, and not merely what is said.

What Gadamer calls *Mutatis mutandis* is the event of thinking presented in the speculative statement. Gadamer’s point is that the speculative statement is not a judgment restricted in the content of what it asserts any more than a single word without a context or a communicative utterance torn from its context is a self-contained unit of meaning.

The words which someone utters are tied to the continuum in which people come to understand each other, the continuum which determines the word to such an extent that it can even be “taken back.”

Similarly, the speculative statement points to an entirety of truth, without being this entirety or stating it. Hegel conceives of this entirety which is not in actual existence as the reflection in itself through which the entirety proves to be the truth of the concept.

Having been compelled by the speculative statement to follow the path of conceptual comprehension, thought unfolds “the logical” as the immanent movement of its content.

Recalling beyond Heidegger's use of Aletheia, Gadamer establishes that language is an "element" within which we live in a very different sense than reflection. Language completely surrounds us like the voice of home which prior to our every thought of it breathes a familiarity from time out of mind.

Heidegger refers to language as the "house of being". Gadamer concludes at the end of his essay, that Hegel brought the development of traditional logic into a transcendental "logic of objectivity" — a development which began with Fichte's "Doctrine of Science, but "the language-ness of all thought (that) continues to demand" (Hans-George Gadamer HG., 1976), and "Dialectic must retrieve itself in hermeneutics".

The relationship between Logic and Hermeneutic are evident on the ground of social science, as Gadamer himself discusses the link between hermeneutics and social science in his 1975th paper "Hermeneutics and Social Science".

Logical arguments are usually classified as either 'deductive' or 'inductive'. The current logic of social science is the deductive logic. The mode of deductive logic is to start from general statement to the detail. The inductive logic's mode is going on from detail to the general.

It seems that both types of logics are playing in the hermeneutic circle, which describes the process of understanding and reffers to the idea that one's understanding of the text as a whole is established by reference to the individual parts and one's understanding of each individual part by reference to the whole. Neither the whole text nor any individual part can be understood without reference to one another, and hence, it is a circle. Deduction and induction by themselves are inadequate for a scientific approach. The development of the scientific method involved a gradual synthesis of these two logical approaches.

One of the possible synthesis of these two mode of logical reasoning is the third main logic - the hermeneutic logic.

When hermeneutical approaches are applied in social sciences, they normally use qualitative and interpretative methods.

These constellations between Logic, Hermeneutics and Topology lead us to Rene Thom.

## 4. Rene Thom topological theory of language and topological syntax

Wildgen and Brandt assert that in the developing topological theory of language, Thom is standing on the shoulders of the founding father of modern syntax, Lucien Tesnière (1893-1954), who in 1934 published the article ”Comment construire une syntaxe” which preceded his monumental work on structural syntax, posthumously published in 1959 with the title *Eléments d’une syntaxe structurale (*Elements of Structural Syntax).[397] (Wildgen and Brandt 2010:57). Tesnière’s *Elements of Structural Syntax* proposes a sophisticated formalization of syntactic structures.He developed the concept of valency in detail, and the primary distinction between arguments (actants) and adjuncts (*circumstants*, French *circonstants*), which most if not all theories of syntax now acknowledge and build on. Tesnière argued that syntax is autonomous from morphology and semantics. Some of the central concepts in Tesnière's approach to syntax are 1) connections, 2) autonomous syntax, 3) verb centrality, 4) stemmas, 5) centripetal (head-initial) and centrifugal (head-final) languages, 6) valency, 7) actants and circonstants, and 8) transfer.

Lucien Tesnière begins the presentation of his theory of syntax with the ‘**connection’**, the central concept for him. Connections are present between words of sentences. They group the words together, creating units that can be assigned meaning. Tesnière writes:

"Every word in a sentence is not isolated as it is in the dictionary. The mind perceives connections between a word and its neighbors. The totality of these connections forms the scaffold of the sentence. These connections are not indicated by anything, but it is absolutely crucial that they be perceived by the mind; without them the sentence would not be intelligible. ..., a sentence of the type *Alfred spoke* is not composed of just the two elements *Alfred* and *spoke*, but rather of three elements, the first being *Alfred*, the second *spoke*, and the third the connection that unites them – without which there would be no sentence. To say that a sentence of the type *Alfred spoke* consists of only two elements is to analyze it in a

[397] Wildgen, W., & Brandt, P. A. (2010). *Semiosis and catastrophes: René Thom's semiotic heritage*. Bern: Peter Lang., p.57

superficial manner, purely morphologically, while neglecting the essential aspect that is the syntactic link."[398]

The connections that Tesnière describes in this passage are now more widely called *dependencies*, hence the term *dependency grammar.* Two words that are connected by a dependency do not have equal status, but rather the one word is the superior, and the other its subordinate. Tesnière called the superior word the *governor*, and the inferior word the *subordinate*. By acknowledging the totality of connections between the words of a sentence, Tesnière was in a position to assign the sentence a concrete syntactic structure, which he did in terms of the stemma.

Tesnière defines syntax as a system of dependence relations with superordinate and subordinate lexical entities. Syntactical analysis, thus, consists in laying down the hierarchy of connections between lexical entities in a sentence. In ’my old friend’, ’friend’ is the superordinate word, whereas ’my old’ are the subordinate terms (’old’ being the superordinate of ’my’). In ’my old friend Alfred reads a book’, the highest superordinate term is ’reads’, whereas ’Alfred’ and ’book’ are the immediate subordinate words with each their subordinate expressions. Now, key to Tesnière’s theory of syntax is the notion of *valency*.

Tesnière defines syntax as a system of dependence relations with superordinate and subordinate lexical entities. Syntactical analysis of Hegel’s language demonstrate strong dependence relations between superordinate and subordinate lexical entities. In Hegel, there is a hierarchy of connections not only between lexical entities in a sentence, but also between different terms, categories, concepts and notions. Something more, the structure and relations between the core terms in Hegel demonstrate not only hierarchy but heteronomy – horizontal relation between lexical and logical entities. This structure of syntax in Hegel’s text revieals both vertical and horizontal direction organized in topological mode. There is a strong emphasis on relations between lexiacal entities in

[398] The passage cited here is taken from the first page of the Éléments (1959[1969]) in Wildgen, W., & Brandt, P. A. (2010). *Semiosis and catastrophes: René Thom's semiotic heritage*. Bern: Peter Lang., p.57

Hegel, where dependence and subordination is mutually oriented in simultaneous way, both in vertical (hierarchical) and horizontal (heterarchical) order.  In Hegel, language is structured in terms of dependency relations between superordinate and subordinate parts (with the verb as the kernel and other word classes. The structural configurational order of Hegel's language, the topological space of narrativ, prevails over the linear, combinatorial order. The schematization of sentence structure in Hegel does not simply mirror or reproduce its linear order.

For example, Hegel's category of 'qualitative quantity' reviels exactly the superordination between two entities – qualitative' and 'quantity'. The lexical entity 'quantity' is the superordinate word, whereas 'qualitative' is the subordinate term. In Hegel's syntax we can find the key notion of Tesnière's theory of syntax, the notion of *valency*. In Hegel's text we can find the topological notion of 'verbal knot', central concept in Tesnière's theory. Tesnière places the verb or the verbal ”knot” on top of the hierarchy of connections.  It is the core element of linguistic expressions and Tesnière defines it in terms that are readily transposable to recent grammars' use of argument-role structure and the like. ”The verbal knot,” Tesnière says, ”expresses a real small drama. As a drama it indeed comprises a process, and most often actors and circumstances.”[399] [400]

Thom's linguistic theory could be characterized as a topological schematization of the concept of valency. Thom construes Tesnière's connexion between actants in terms of positional connexions between places occupied by actants in an abstract space.[401]

Rene Thom called in his "Structural stability and Morphogenesis - An Outline of a general theory of models" (1975) -  '*the malignity of the human attractor'*.

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399 Wildgen, W., & Brandt, P. A. (2010). Semiosis and catastrophes: René Thom's semiotic heritage. Bern: Peter Lang., p.57

400 Wildgen, W., & Brandt, P. A. (2010). Semiosis and catastrophes: René Thom's semiotic heritage. Bern: Peter Lang., p.57

401 (to this, cf. Petitot Thom and modern evolutionary linguistics 18 1985, 1992, 1995). in Wildgen, W., & Brandt, P. A. (2010). Semiosis and catastrophes: René Thom's semiotic heritage. Bern: Peter Lang., p.57

The exact words of Thom are the solution that both person and society needs to resolve the challenges in the period of *transition*, the advise contained within the suggestion “We need to slow down: as the possible meaningful way, the malignity of human attractor.” [402] Rene Thom’s second principle of morphogenesis states that “what is interesting about morphogenesis locally, is the *transition*, as the parameters varies from a single state of the vector field to an unstable state by means of process which we use to model a system’s local morphogenesis.[403]

It was Thom who contributed to the idea of versal unfolding.[404] The term ‘versal’ is the intersection of ‘universal’ and ‘transversal’, and one of the Thom’s insights was that the singularities of members of families of functions of mappings are versally unfolded if the corresponding family of jet extension maps is transverse to their orbit (equivalence classes) in jet space.”[405]

Angel Lopez Garcia, who introduced the “topological linguistics”, gives a rather critical reading of Thom`s proposals related to verbal semantics and the structure of basic propositions. Garcia compares Thom’s specific analysis of verbs with the tradition of structuralism (Hjelmslev, Jakobson, Halliday, Chomsky) and the models: “Liminar Grammar” and “Topological Linguistics” proposed by himself. [406] (Garcia, 1990)

**5. Dialectics and Catastrophe: Cobordism of Hegel’s fourfold model of multiplicities (infinities) exhibited in bifurcation diagrams Pitchfork Bifurcation Diagrams - Supercritical Pitchfork Bifurcation Diagrams (b < 0) and Subcritical Pitchfork Bifurcation Diagrams (b > 0)**

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[402] Peter Tsatsanis, On Rene Thom Sinificance for Mathematics and Philosophy, Scripta Philosophicae Naturalis 2:213-229 (2012), p.223-224.

[403] Peter Tsatsanis, On Rene Thom Sinificance for Mathematics and Philosophy, Scripta Philosophicae Naturalis 2:213-229 (2012), p.223-224.

[404] Bruce, B. and D.N. Mond, eds. Singularity Theory, Cambridge, England: Cambridge U.Press,1999, page xi.

[405] Peter Tsatsanis, On Rene Thom Sinificance for Mathematics and Philosophy, Scripta Philosophicae Naturalis 2:213-229 (2012), p.223-224.

[406] Garcia, Angel Lopez, (1990), Introduction to Topological Linguistics - Annexa-LynX. Valencia-Minnesota, 1990.

Back in 1989, as a Research Associate at Institute for Philosophical Research at Bulgarian Academy of Science, I have established Hegel's category and notion of 'qualitative quantity' (Dimitrov, 1989a) as vocamen of my research interests in philosophy, in particular within the area that can be defined as philosophical topology. With my first publications (Dimitrov, 1989, Dimitrov, 1990), applicable to 'dialectics and the problem of novelty', I introduced the claim about the topological notion of Hegel's category of 'qualitative quantity', where 'topological' implies gradual transformation and continuous change without leap or abrupt changes, thus exhibit form of this category is topological homeomorphism.

I have supported my argument about topological homeomorphism as exhibit form of Hegel's Qualitative quantity with exploration of D'Arcy W. Thompson's "Growth and Form" (1917), and Hermann Haken (Haken, 1983) discussion on D'Arcy W. Thompson's transformation, concluding that Haken's finding of structural stability and homology exhibited by such transformation of the forms, explicitly state the notion of qualitative quantity.

In his book "Synergetics: Introduction and Advanced Topics", 41 in the Chapter 1.13. "Qualitative Changes: General approach", Hermann Haken explores and illustrate the structural stability with an example /figure 1.13, p.434 in Haken/ given by the Scottish biologist, mathematician and classics scholar D'Arcy W. Thompson, the author of the book, On Growth and Form, /1917/. My assertion is that Hegel's category of qualitative quantity is illustrated with Herman Haken's citation of D'Arcy W. Thompson.

Homology is related with the works of D'Arch Thompson, especially his "On Growth and Form" (1917). In the last chapter of "On Growth and Form", D'Arcy Thompson's illustrates his "cartesian transformations" of animal forms. Thompson's mappings are referred to as "rubber sheet" mappings. D'Arch Thompson suggested that one should study the change from one biological form to another by examining the unique mathematical object that maps between them in accord with biological homologies.

Exploring the invariance in deformation and transformation of the forms against spatial or temporal deformation, Haken wrote:

“Figure 1.13, p.434 /“Synergetics: Introduction and Advanced Topics”/ shows two different kind of fish, namely, porcupine fish and sun fish. According to the studies by D'Arcy W. Thompson of the beginning of the twentieth century, the two kinds of fish can be transformed into each other by a simple grid transformation. While from the biological point of view such a grid transformation is a highly interesting phenomenon, from the mathematical point of view, we are dealing here with an example of structural stability. In a mathematician’s interpretation the two kinds of fish are the same. They are just deformed copies of each other. A fin is transformed into a fin, an eye into an eye and etc. In other words, no new qualitative features such as a new fin, occur. In the following we shall have structural changes /in the widest sense of word/ in the mind.” (Haken, H. 1983)

Under the illustration set in Figure 1.13, p.434 /“Synergetics: Introduction and Advanced Topics”/, Haken wrote – “the porcupine fish and the sun fish can be transformed into each other by a simple grid transformation. After D'Arcy W. Thompson: On Growth and the Form, ed. By J.T. Bonner, University Press, Cambridge, 1981/.” (Haken, H. 1983)

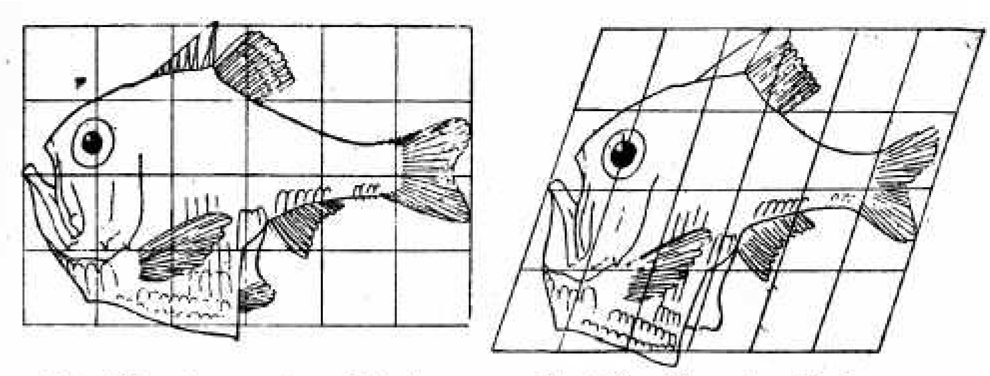

**Fig. 517.** *Argyropelecus Olfersi.* **Fig. 518.** *Sternoptyx diaphana.*

The original Thompson’s illustration of the transformation of the fish Argyropelecus olfersi into the fish Sternoptyx diaphana by applying a 70° shear mapping.

Hermann Haken’s example we are illustrating here with the original Thompson’s illustration of the transformation of the fish Argyropelecus olfersi into the fish Sternoptyx diaphana by applying a 70° shear mapping. The reverse transformation is possible simply with manipulating the grid and shear mapping.

The example illustrated this transformation actually is a good example of homeomorphism. Two objects are homeomorphic if they can be transformed /or deformed/ into each other by a continuous inverible mapping, continuous one-to-one and having continuous inverse. The two fish are two objects with the same topological properties. They are said to be homeomorphic. There are properties that are not destroyed by stretching and distorting an object.

The claim I made with my first two publications (Dimitrov, 1989, Dimitrov, 1990), contradicted the well established, until the time of 'persetroika', paradigm of dialectical materialism.[407]

From his reading of Hegel's The Science of Logic,[408] Engels identifies the "law" of dialectics as being reducible to three basic concept: 1) the transformation of quantity into quality and vice versa, 2) the interpenetration of opposites, and 3) the negation of the negation.[409] Engels's "law" for the transformation of quantity into quality and vice versa, states that continuous quantitative development results in qualitative "leaps" in nature whereby a completely new form or entity is produced. This is how "quantitative development becomes qualitative change". The new quality develops quantitatively through a step-by-step process of quantitative changes, qualitative changes begin with the quantitative introduction of the new quality into the quantitative development of the old measure, thus qualitative changes occur as leaps. In Anti-Dühring, Engels[410] (Engels, 1954:67) identifies this with Hegel's example of the boiling or freezing of water at specific temperatures, qualitative (discontinuous) leaps arising from quantitative (continuous) changes. [411] (Hegel, G.W.F. 1842: 217) Engel's "dialectics" of quality and quantity presented in his Anti-Duhring, became the founding text of dialectical materialism and orthodox for the Marxism. Until the present time there are political consequences related to the problem of Engels's appropriation of Hegel's Science of Logic.

[407] The term dialectical materialism usually associated with Engels's concept of 'modern materialism' from Anti-Duhring, was actually coined by Plekhanov, completely taken over by him. See: Z.A.Jordan, The Evolution of Dialectical Materialism: A Philosophical and Sociological Analysis (New York: St. Martins Press, 1967), 66-67.), for an account of the high esteem held for Engels's Anti-Duhring by key practioners of Second International Marxism like Plekhanov, Kautsky and Lenin.
[408] G.W.F. Hegel, The Science of Logic, trans. A. V. Miller (NJ: Humanities Press International, 1969), Vol. 1, Book 1, §3, 335.
[409] Engels, F. 1940. The Dialectics of Nature, New York, International Publishers, 1940, p. 26

[410] Engels, F. 1954. Anti-Dühring: Herr Eugen Dühring's Revolution in Science, Moscow, Foreign Languages Publishing House
[411] Hegel, G.W.F. 1842. Enzyclopadie der Philosophischen Wissenschaften im Grundrisse, Part 1, Logik, Vol. VI, Berlin, Duncken und Humblot - 1842, p. 217

For both Marx and Engels (1848), the "law" of transformation of quantity into quality was the central key to the change from one mode of production to another. Dialectical materialism approach sees history as unfolding in qualitatively distinct stages such as ancient slavery, feudalism, and capitalism. (Barkley Rosser, J.,Jr., 1998/2000:5) Engels (1940, pp. 18-19)[412] confronted the contradiction between the apparently simultaneous acceptance of discontinuity arising from the idea of qualitative leaps and of continuity arising from the 'fuzziness' implied by the interpenetration of opposites in the dialectical approach. He dealt with this by following Darwin (1859) in accepting a gradualistic view of organic evolution in which species continuously change from one into another, while arguing that in human history, the role of human consciousness and choice allow for the discontinuous transformation of quantity into quality as modes of production discontinuously evolve. (Barkley Rosser, J.,Jr., 1998/2000:5)

Whereas Marx largely used these concepts to analyze historical change, Engels drew on Kant and Hegel to extend such approach to science. Although Engels's discussion in *The Dialectics of Nature* was reasonably current with regard to science for the time of its writing (the 1870s and early 1880s), much of its content is seen to be scientifically inaccurate by today's standards, and many of its examples thus hopelessly muddled and wrongheaded. (Barkley Rosser, J.,Jr., 1998/2000:5) In contrast, Hegel's though and writing does not suffer the distance of time and appear scientifically accurate to the subject of contemporary topology as specific discipline of modern mathematics, although topology was just emerging in her protophenomenal form from Euler and Leibniz during Hegel's time.

Engels's law of transformation and the passage of quantitative changes into qualitative changes, as all of his three laws of dialectics become cliché in the mode of thinking of quality and quantity. These three principles established by Engels are not only oversimplified, but also misleading at best, establishing something quite self-evident, trivial and common. The notion of gradualness and gradual changes were criticized from the standing point of dialectical materialism as not leading to turning point and new quality, thus the new quality and qualitative change may appear in the new measure only through abrupt changes and qualitative leap. Due to this assumption, the notion of Qualitative quantity in Hegel's dialectics, remained inapparent.

[412] Engels, F. 1940. The Dialectics of Nature, New York, International Publishers, 1940, p.18-19)

Approaching the domain of topology from the standpoint of the dialectics of qualitative quantity, we should conclude that the interplay of quality and quality is associated with the development and growth. Both the classical and non-classical approaches to the dialectics of quality and quantity are addressing the dialectical nature of change.

The known quality, defined by Hegel as determined quality, implies discontinuous transformation and change through a leap. The exhibit form of such determined quality is abrupt displacements in the equilibrium – in social term this change is known as **revolution**.

In contrast, Hegel's qualitative quantity implies gradual and continuous transformation and change is topological. The exhibit form of qualitative quantity is topological continuous transformation without leap or abrupt displacements in the equilibrium – in biological and social term this change is known as **evolution**.

The aim of Dialectical materialism's orthodox idea of qualitative/quantitative transformation was to reconcile Marxism's notion of the historical emergence of one social form out of another with the emphasis on the leaps, abrupt changes in political situation, namely revolutions. Such aim imposes epistemic limits upon knowledge itself.

Engels's focus on quantity/quality transformation supplements Marxist historical materialism with a Hegelian-inspired concept of rupture, linking the schemas of gradual change between modes of production with a notion of the radical changes taking place in a revolution. The attempt of dialectical materialism to separate the concept of quantity-quality transformations from the historicist system of Hegel, falls apart. (Coombs, 2013:29).

Once dialectical materialism imports the idea of quantity-quality transformation from physical mechanics to the social phenomenon, it signals need for a singular nomination of what the quantity is from which a change in quantity emerges. The implication is that change must take place at a single point of transformation at which the linear development of quantity (for example, productive forces) gives way to a respective qualitative transformation (for example, relations of production). (Coombs, 2013:29). Instead of seeing the event as a unity of overlapping processes (as Althusser will see it) or as a multiple (as Badiou will conceive it), the reduction of revolutionary change to a single point of transformation, (Coombs argue) reduces the potential to think through the processes which create genuine novelty/discontinuity. This leaves the idea of quantity-quality transformations ensnared in a notion of change being effectuated by a transformation taking place through a dialectical switch – an idea intimately connected with a linear concept of history unable to properly think novelty. (Coombs, 2013:30).

The problem of quantitative-qualitative transformation of economic systems from one mode to another is among the deepest problems in political economy. Something more, dialectic of quantitative transformation leading to qualitative change or the issue of dialectic and novelty, is a key to the analysis of systemic social, political, and economic transformation. (Barkley Rosser, J.,Jr., 1998/2000) The attempts in classical Marxism to conceptualize a novelty-bearing event out of Hegelian dialectics necessarily reach an impasse. [413] (Coombs, 2013:29).

Similar conclusion provides Barkley Rosser, stating that "A long tradition, based on Marx, argues that this can be explained by a materialist interpretation of the dialectical method of analysis as developed by Hegel. Although Marx can be argued to have been the first clear and rigorous mathematical economist (Mirowski, 1986), this aspect of his analysis generally eschewed mathematics." (Barkley Rosser, J.,Jr., 1998/2000).

Hegel's dialectics of qualitative and quantitative demonstrates two notions. The first is related with Hegel's determined quality, where discontinuous transformation implies and change

[413] Coombs, Nathan. (2013), Politics of the event after Hegel, PhD dissertation, University of London, Department of Politics and International Relations.: "The basic idea behind this orthodox concept is that at a certain quantitative tipping point the linear accumulations taking place within any social phenomena radically transform into a new qualitative state. Hegel's example is the transformation of water into ice. Running down a linear (quantitative) temperature scale it is only when the liquid water hits precisely 0 degrees centigrade that it suddenly (qualitatively) transforms into a solid state. Even though for Hegel this idea was solely meant to exhibit the limits of quantitative determination within physical mechanics, dialectical materialism converted it into a concept that could be used as a model for the discontinuities taking place within social transformation. Unlike the gradualist take on history proposed by historical materialism, where, in Marx's analysis, capitalism emerges over centuries as a gradual process of consolidation between the value form and the separation of workers from the means of production, the idea behind quantity-quality transformations supplied by dialectical materialism is that they are meant to demonstrate how events like revolutions radically interrupt evolutionary processes. Furthermore, the transformation of those attributes of the social world that can be analyzed quantitatively into a new qualitative state signals an epistemic break in knowledge. At the very least, the criteria used to evaluate phenomenon ex-ante have to be significantly readjusted ex-post." (Coombs, 2013:29).

occurs through a leap. The second notion is the topological notion of qualitative quantity, where gradual and continuous transformation implies and change is topological. The exhibit form of the qualitative quantity is topological continuous transformation without leap or abrupt displacements in the equilibrium.

These two notions of Hegel's qualitative and quantitative dialectics can be revealed through the catastrophe theory and chaos theory.

The word catastrophe comes from Greek tragic drama and refers to the sudden twist of development in the plots. The catastrophe is related with the rhetorical figure of 'metalepsis'. Catastrophe theory is a method for describing the evolution of forms in nature and it is particularly applicable where gradually changing forces produce sudden effects. Catastrophe theory is interdisciplinary in character linking mathematics, biology, social sciences and philosophy. Catastrophe theory is represented by using topology since one of the central concerns of topology is to study the properties of spaces that do not change under a continuous transformation, that is, translation, rotation and stretching without tearing.

The catastrophe theory will be applied as moments of catastrophe by overlapping two opposing situations as a twist in the narrative, and the physical fold on the surface.

The different moments will be unrolled and unified on a singular surface to create multiple moments of catastrophes. Both the stable and unstable conditions of the continuous surface will be addressed by negotiating between shifts in unstable narratives on stable physical folds and shifts in the stable narrative on the unstable physical folds.

There is only one kind of singularity that could occur in the catastrophe theory and that is called a fold. The fold made possible the definition of flux equilibrium by combining a series of different surfaces to define multiple points of equilibrium.

Catastrophe models come in both dynamic and static forms, the static forms being simply the equilibrium (stable and unstable) of the dynamic forms. Multiple stable equilibriums are inherent in catastrophe models.

Linking mathematical definition of catastrophe models, such as the dynamic and static forms with the two notions of Hegel's dialectic of qualitative and quantitative, we could associate Hegel's dialectic of quanity and qualitative quantity with the structural stability, where a model is structurally stable if its qualitative behaviour is unchanged by small perturbations of the parameters, and with structural stability, where a model is structurally stable if its qualitative behaviour is unchanged by small perturbations of the parameters. Hegel's dialectic of quantity and determined quality can be associated with the catastrophe, where a sudden change in state is presented.

The issue of Catastrophe Theory and Dialectics is discussed by Barkley Rosser (Barkley Rosser, J.,Jr., 1998/2000; 2004) and Martin Zwick, in his paper "Dialectics and Catastrophe". (Zwick, M. 1978).

The key idea for analyzing discontinuities in nonlinear dynamical systems is *bifurcation*, and was discovered by Poincaré who developed the qualitative theory of differential equations to explain more-than-two-body celestial mechanics. (Barkley Rosser, J.,Jr., 1998/2000:10) The name "bifurcation" was first introduced by Henri Poincaré in 1885 in the first paper in mathematics showing such a behavior. Henri Poincaré also later named various types of stationary points and classified them.

Bifurcation theory is the mathematical study of changes in the qualitative or topological structure of a given family, such as the integral curves of a family of vector fields, and the solutions of a family of differential equations. Most commonly applied to the mathematical study of dynamical systems, a bifurcation occurs when a small smooth change made to the parameter values (the bifurcation parameters) of a system causes a sudden 'qualitative' or topological change in its behaviour.

Bifurcations occur in both continuous systems, and discrete systems (described by maps).

Catastrophe theory, which originated with the work of the French mathematician René Thom in the 1960s, and became very popular due to the efforts of Christopher Zeeman in the 1970s, considers the special case where the long-run stable equilibrium can be identified with the minimum of a smooth, well-defined potential function (Lyapunov function). Small changes in certain parameters of a nonlinear system can cause equilibria to appear or disappear, or to change from attracting to repelling and vice versa, leading to large and sudden changes of the behaviour of the system. However, examined in a larger parameter space, catastrophe theory reveals that such bifurcation points tend to occur as part of well-defined qualitative geometrical structures.

For René Thom this becomes the mathematical model of morphogenesis, of qualitative transformation from one thing into something else, following the analysis of D'Arcy Thompson (1917) of the emergence of organs and structures in the development of an organism. Furthermore, Thom explicitly links this to dialectics, albeit of an idealist sort:

> "Catastrophe theory...favors a dialectical, Heraclitean view of the universe, of a world which is the continual theatre of the battle of between 'logoi,' between archetypes." [414] [415] (Thom, 1975A, 1975B:382). (Barkley Rosser, J.,Jr., 1998/2000:12)

More generally, Thom argues that catastrophe theory showed how qualitative changes could arise from quantitative changes as in Hegel's dialectical formulation.[416] (Barkley Rosser, J.,Jr., 1998/2000:12)

Rene Thom's work "From Catastrophes to Archetypes: Thought and Language" aimed to extend the techniques and assumption of catastrophe models of morphogenesis to human processes and societies. Thom used mathematical notations and language only to express vague correspondences among neurobiologycal states, thought and language. R. Thom established that "the sequence of our thought and our acts is a sequence of attractors, which succeed each other in catastrophes". According to Thom language translates the mental attractors of our brain. When one wishes to formulate a sentence expressing idea, it was

[414] Thom, R. 1975A. Structural Stability and Morphogenesis: A General Outline of a Theory of Models, Reading, W.A. Benjamin; and Thom, R. 1975B. Catastrophe theory: its present state and future perspectives, in Dynamical Systems-Warwick 1974, Lecture Notes in Mathematics No. 468, Berlin, Springer-Verlag

[415] Thom, René, with response by E. Christopher Zeeman. 1975. "Catastrophe Theory: Its Present State and Future Perspectives," in *Dynamical systems-Warwick 1974*. Lecture Notes in Mathematics No. 468. Anthony Manning, ed. Berlin: Springer-Verlag, pp. 366-389.

[416] See Rosser (2000b) for further discussion.

mathematically projected onto a space of admissible sentences, where several attractors prompeted. One was eventually chosen and the sentence was uttered.

In 1970, Thom presented sophisticated catastrophe theory model of language. He developed a visual representation of the verbs associated with spatio-temporal activity. This was, Thom would say 20 yars later, a “geometrization of thought and linguistic activity”.

Thom classified syntactical structures into 16 categories. He claimed that “the topological type of the interaction determines the syntactical structure of the sentence wich describes it.”

According to Thom, meaning and structure were no more independent. Rene Thom constructed a modeling practice which, roughly speaking, used topologically informed means of transformation, biologically inspired raw materials that he adapted to mathematical practice.

The idea of cobordism was deeply explored by René Thom, but the roots of cobordism is due to Henry Poincaré in his Analysis Situs and in its complementary papers. His idea of homology is very close to the modern framework elaborated by Thom. Homology classes were first defined rigorously by Henry Poincaré in his seminal paper "Analysis situs", *J. Ecole polytech.* (2) 1. 1–121 (1895).

Recalling beyond Heidegger’s use of Aletheia, Gadamer establishes (Gadamer HG., 1976), [417] that language is an “element” within which we live in a very different sense than reflection. Language completely surrounds us like the voice of home which prior to our every thought of it breathes a familiarity from time out of mind. Heidegger refers to language as the “house of being”. Gadamer concludes at the end of his essay, that ..But the language-ness of all thought continues to demand … Dialectic must retrieve itself in hermeneutics (Gadamer). (Gadamer HG., 1976)

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[417] "Die Idee der Hegelschen Logik (1971) in Hans-George Gadamer, “Hegel's Dialectic: Five Hermeneutical Studies”, translated into English by P. Christopher Smith and collected in (New Haven: Yale University Press, 1976). These five essays are "Hegel and Heidegger,"; "Hegel's Dialectic of Self-consciousness"; "Hegel and the Dialectic of the Ancient Philosophers,"; "Hegel's 'Inverted World,'; and "The Idea of Hegel's Logic," 75-99, “The Idea of Hegel’s Logic” is one of the five essays by Hans-Georg Gadamer known in his “Hegel’s dialectics: Five Hermeneutical studies” (1971).

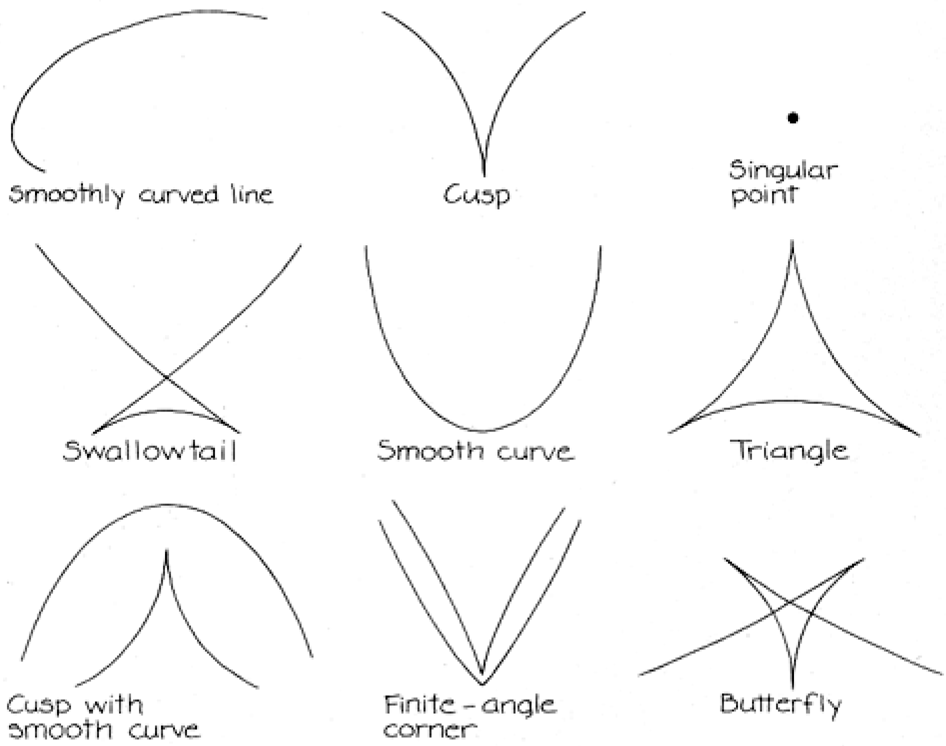
Today we are experiencing something that Rene Thom called in his "Structural stability and Morphogenesis - An Outline of a general theory of models" (1975) - '*the malignity of the human attractor'*. (Thom, R. 1975) (Peter Tsatsanis, P. 2012: 223-224)

The exact words of Thom are the solution that both person and society needs to resolve the challenges in the period of *transition*, the advice contained within the suggestion "We need to slow down: as the possible meaningful way, the malignity of human attractor." (Thom, R. 1975) (Peter Tsatsanis, P. 2012: 223-224) Rene Thom's second principle of morphogenesis states that "what is interesting about morphogenesis locally, is the *transition*, as the parameters varies from a single state of the vector field to an unstable state by means of process which we use to model a system's local morphogenesis. (Thom, R. 1975) (Peter Tsatsanis, P. 2012: 223-224)

It was Thom who contributed to the idea of versal unfolding. (Bruce, B. and D.N. Mond . 1999) The term 'versal' is the intersection of 'universal' and 'transversal', and one of the Thom's insights was that the singularities of members of families of functions of mappings are versally unfolded if the corresponding family of jet extension maps is transverse to their orbit (equivalence classes) in jet space." (Peter Tsatsanis, P. 2012: 223-224)

This insight of Thom led him to the Catastrophe theory with identified by him seven orbits of function singularities which can be met transversally in families of fewer parameters – the seven elementary catastrophes, which meant to underlie all abrupt changes (bifurcation) in generic four parametrers families of given dynamical systems.

These seven are: fold, cusp, swallowtail, butterfly, hyperbolic umbilic, elliptic umbilic, and parabolic umbelic. Rene Thom used transversality as the main tool to prove the existence of universal unfolding. Thom created a mathematically rigorous theory that showed "the true complementary nature of the seemingly irreconcilable notions of versality and stability, that is, preserving identity in spite of development.




Hegel's dialectic of quantity and determined quality can be associated with a catastrophe form that shows most of the phenomena occurring in catastrophe models is that of the three dimensional cusp catastrophe, Behaviour observable in such a dynamical system can include bimodality, inaccessibility, sudden jumps, hysteresis, and divergence. If what one wishes to do is to examine the structural stability of a particular pattern of bifurcation, or perhaps more specifically to compare the topological characteristics of two distinct patterns of discontinuities in economics, then proper catastrophe theory is clearly the most appropriate method to use for sufficiently low dimensional systems with gradient dynamics derived from a potential function. (Barkley Rosser, J.,Jr., 2004) The idea that huge, sudden, and revolutionary changes might happen had considerable widespread appeal, especially among more dissident intellectuals. But widespread applications of the theory that were inappropriate either theoretically or methodologically undermined its credibility. A counterattack came in the late 1970s, and as the 1980s wore on, fewer and fewer applications of catastrophe theory were seen, especially in economics, although catastrophe theory always retained more respectability among mathematicians as a special case of bifurcation theory.

Nevertheless, there were many applications of catastrophe theory in economics that were properly done before the counterattack's influence was fully felt. (Barkley Rosser, J.,Jr., 2004)

Barkley Rosser, provides a comprehensive review about the wide application of catastrophe theory not only in economics but in various areas and fields.[418] (Barkley Rosser,J.,Jr., 2004) These application highlights the application of Hegel's dialectical logic of qualitative quantity as well. Barkley Rosser provides also an important assertion about the current importance of catastrophe theory, stating that "Although there are serious limits to its proper application in economics, there remain many potential such proper applications. Economists should no longer shy away from its use and should include it with the family of other methods for studying dynamic discontinuity.... A reasonable middle ground can and should be found. (Barkley Rosser, J.,Jr., 2004)

I believe that such reasonable middle ground for evaluation and overcoming the limits to the proper application of catastrophe theory in economics and even political science can be found in Hegel's topological notion of qualitative quantity.

The applicability of Hegel's topological notion of qualitative quantity to the catastrophe theory or in reverse, of catastrophe theory to Hegel's logic as topology as logic of toposes of change, a logic which with it's categories and notions can provide both understanding and models for explanation of how small changes in certain parameters of a nonlinear system can cause equilibria to appear or disappear, or to change from attracting to repelling and vice versa, leading to large and sudden changes of the behaviour of the system or gradual and continuous change of system's behaviour.

The two notions of Hegel's dialectic of quanity with the determined quality and the qualitative quantity, can be associated with description of two types of dialectics as proposed by Martin Zwick (Zwick, M. 1978).

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418 Barkley Rosser, J.,Jr., (2004), The rise and fall of Catastrophe theory applications in economics: was the baby trown out with the bathwater?

Martin Zwick, in his paper "Dialectics and Catastrophe" (Zwick, M. 1978), assert, that there are two distinguishable types of dialectics, one which results in victory of one of the opposing forces, and second which gives rise to a compromise or synthesis. Some dialectical phenomena are best modeled with the cusp, but others are more complex and more appropriately grasped with the butterfly, the butterfly of reconcillation, where the struggle of opposites within the cusp bifurcation set is itself negated.

The first notions of Hegel's dialectic of quanity with the determined quality can be associated as logic of the cusp catastrophe, and the second notion of Hegel's qualitative quantity can be associated as logic of the butterfly, the butterfly of reconcillation.

Perhaps, dialectic would retrieve itself through the language and hermeneitics in catastrophe theory as well. If the 'cusp' catastrophes could be well illustrated and explained through the well known "law" of dialectics of transformation of quantity into quality, thus through Hegel's notion of quantity and determined quality, the 'butterfly' catastrophe not only illustrates but embeds the logic and topology of Hegel's notion of qualitative quantity.

There is striking analogy between Hegel's fourfold of infinities
[1/. the bad qualitative infinity;
2/. the good qualitative infinity;
3/. the bad quantitative infinity;
4/. the good quantitative infinity]

or the fourfold cobordism of the categories of:

- quantitative quantity (in the domain of Chronochora – Abstract Space and Abstract Time);
- quantitative quality (in the domain of Chronotopos –Meaningful Place and Abstract Time);
- qualitative quantity (in the domain of Kairochora – Abstract Space and Meaningful Time);
- qualitative quality (in the domain of Kairotopos – Meaningful Place),

and **the four parameters of the butterfly catastrophe**, where the volatile dyad is changed into a precarious triad and then into a stable tetrad.

The topological notion of Hegel's logic contains the seed of topological hermeneutics with "genuine infinite, a circle closed on itself" …The infinite that wants to be unlimited, because as Hegel points out – "there are two worlds, one infinite and one finite, and in their relationship the infinite is only the limit of the finite and is thus only a determinate infinite, an infinite which is itself finite." (Miller,A.V. trans.,1990. Hegel's Science oif Logic)

Once visualized in topological space through the notion and model of Rene Thom's 'cobordism', Hegel's fourfold model of multiplicities (infinities) build on the logical interrelations and interdependence between the categories such as quality and quantity, the model that unfolds the 'qualitative quantity' notion of place within the time, could be exhibited also by the bifurcation diagrams.

The striking resemblance between the philosophical or dialectical hermeneutics of the categories of quality and quantity, seen through the model of 'cobordism" - the "pair of pants" – or - the cobordism of hermeneutical circle showing how dialectical hermeneutics (must) retrieve itself in topology, namely the Hegel's fourfold of infinities:

1. Quantitative quantity;
2. Quantitative quality;
3. Qualitative quantity;
4. Qualitative quantity,

could be thought as **circle or four circles placed in a 3-manifold, and the pitchfork bifurcation**, presented in the figure bellow, where the **supercritical** and **subcritical** components in the diagram, draw the **two half planner cilinder** from our example of **cobordism**.

**The dialectics of whole and parts implies topological notions.** [419] **(see Smith, B. 1994;Smith, B., (ed.) 1994). Barkley Rosser, emphasizes on the dialectics of whole and parts (Barkley Rosser, J.,Jr., 1998/2000),** stating that "Finally there is the idea of wholes

[419] Barry Smith, Topological Foundations of Cognitive Science, a revised version of the introductory essay in C. Eschenbach, C. Habel and B. Smith (eds.), *Topological Foundations of Cognitive Science,* Hamburg: Graduiertenkolleg Kognitionswissenschaft, 1994, the text of a talk delivered at the First International Summer Institute in Cognitive Science in Buffalo in July 1994.: http://ontology.buffalo.edu/smith/articles/topo.html
And Barry Smith, (ed.) 1982 Parts and Moments. Studies in Logic and Formal Ontology, Munich: Philosophia.)

consisting of related parts – implied by this formulation. For Levins and Lewontin (1985)[420] this is *the most important aspect of dialectics* (emphasis added by me, BD) is and they use it to argue against the mindless reductionism they see in much of ecological and evolutionary theory, Levins (1968)[421] in particular identifying holistic dialectics with his 'community matrix' idea. This can be *seen as working down from a whole to its interrelated parts, but also working up from the parts to a higher order whole.* (emphasis added by me, BD) This latter concept can be identified with more recent complex emergent dynamics ideas of self-organization (Turing, 1952[422]; Wiener, 1961)[423], autopoesis (Maturana and Varela, 1975)[424], emergent order (Nicolis and Prigogine, 1977,[425] Kauffman, 1993)[426], . . . It is also consistent with the general social systems approach of the dialectically oriented post-Frankfurt School (Luhmann, 1982, 1996;[427] Habermas, 1979, 1987;[428]. . .). (Barkley Rosser, J.,Jr., 1998/2000:8)

**Pitchfork Bifurcation Diagrams**

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420 Levins, R. and R. Lewontin. 1985. *The Dialectical Biologist*, Cambridge, Harvard University Press

421 Levins, R. 1968. *Evolution in Changing Environments*, Princeton, Princeton University Press

422 Turing, A.M. 1952. The chemical basis of morphogenesis, *Philosophical Transactions of the Royal Society B,* vol. 237, no. 1

423 Wiener, N. 1961. Cybernetics: or Control and Communication in the Animal and the Machine, 2nd edition, Cambridge, MIT Press

424 Maturana, H.R. and F. Varela. 1975. *Autopoietic Systems*, Report BCL 9.4, Urbana, Biological Computer Laboratory, University of Illinois

425 Nicolis, G. and Prigogine, I. 1977. Self-organization in Nonequilibrium Systems: From Dissipative Structures to Order Through Fluctuations, New York, Wiley-Interscience

426 Kauffman, S.A. 1993. The Origins of Order, Self-Organization and Selection in Evolution, Oxford, Oxford University Press

427 Luhmann, N. 1982. The world society as social system, *International Journal of General Systems*, vol. 8, 131-8; and Luhmann, N. 1996. Membership and motives in social systems, *Systems Research*, vol. 13, 341-8

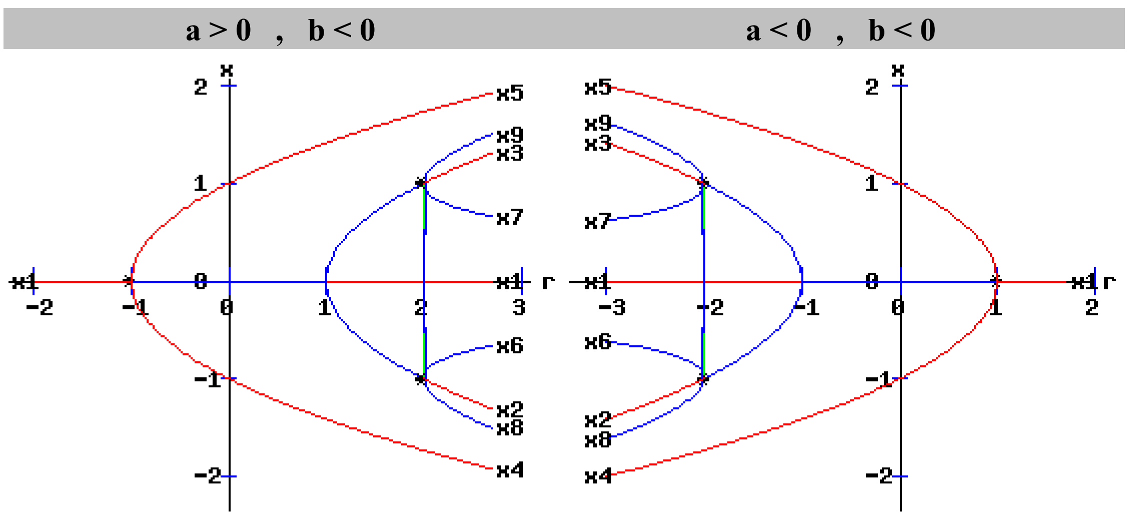
428 Habermas, J. 1987. *The Philosophical Discourse of Modernity: Twelve Lectures*, translated by F.G. Lawrence, Cambridge, MIT Press

The bifurcation diagrams for the two types of supercritical and two types of subcritical pitchfork bifurcations are displayed below. The fixed points of **f** are **$x_1$, $x_2$, and $x_3$**, whereas the fixed points of **$f^2$** are **$x_1$, $x_2$, $x_3$, $x_4$, $x_5$, $x_6$, $x_7$, $x_8$, and $x_9$**. These fixed points are graphed as functions of the parameter, **r**.

**Supercritical Pitchfork Bifurcation Diagrams (b < 0)**

The discrete time, dynamic process undergoes a pitchfork bifurcation at r = 1 / a where $x_1$ bifurcates into $x_1$, $x_2$, and $x_3$. A **subcritical flip bifurcation** occurs at r = -1 / a where the stability of the fixed point $x_1$ flips, and the unstable fixed points $x_4$ and $x_5$ of **$f^2$** emerge bracketing the stable fixed point $x_1$ of **f**. Supercritical flip bifurcations occur at r = 2 /a where the stabilities of $x_2$ and $x_3$ flip from stable to unstable. Concurrently, the stable fixed points $x_6$ and $x_8$ of **$f^2$** emerge bracketing $x_2$, and the stable fixed points $x_7$ and $x_9$ of **$f^2$** emerge bracketing $x_3$.

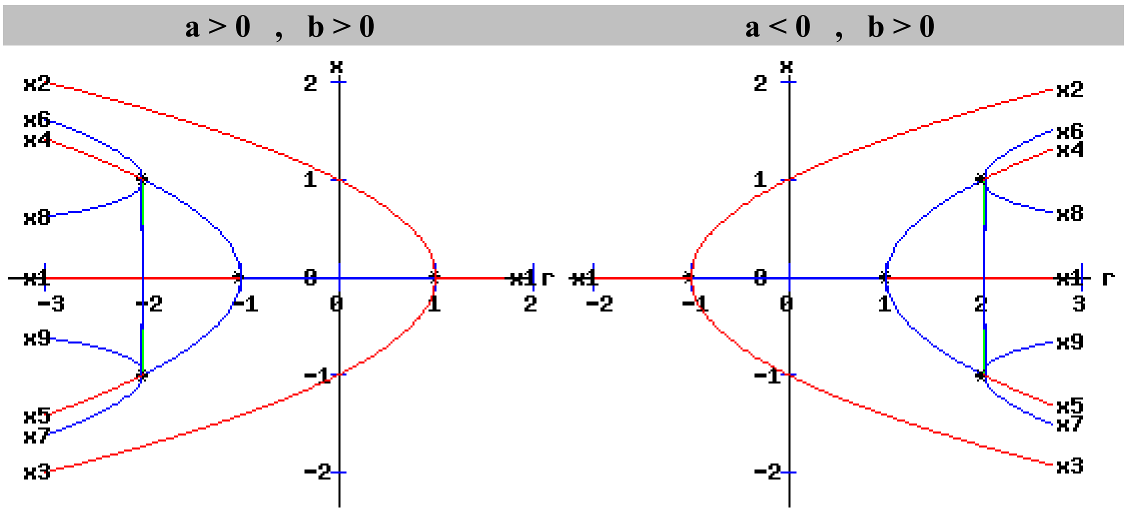
The blue curves represent the stable fixed points, whereas the red curves represent the unstable fixed points. A bifurcation point is marked with an asterisk, *.



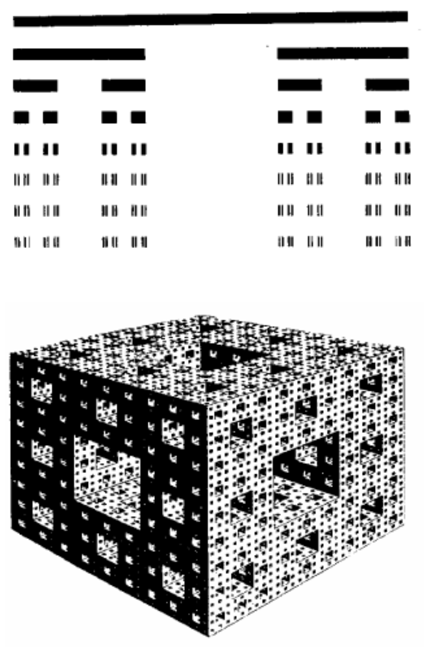

**Subcritical Pitchfork Bifurcation Diagrams (b > 0)**

The discrete time, dynamic process undergoes a pitchfork bifurcation at r = 1 / a where $x_1$ bifurcates into $x_1$, $x_2$, and $x_3$. A supercritical flip bifurcation occurs at the critical value of r = -1 / a where the stability of the fixed point $x_1$ flips. Concurrently, fixed points of $\mathbf{f}^2$ emerge bracketing $x_1$ with $x_4$ and $x_5$. At r = -2 / a, the fixed point $x_4$ of $\mathbf{f}^2$ bifurcates into $x_4$, $x_6$ and $x_8$; meanwhile the fixed point $x_5$ of $f_2$ bifurcates into $x_5$, $x_7$ and $x_9$.

The blue curves represent the stable fixed points, whereas the red curves represent the unstable fixed points. A bifurcation point is marked with an asterisk, *.

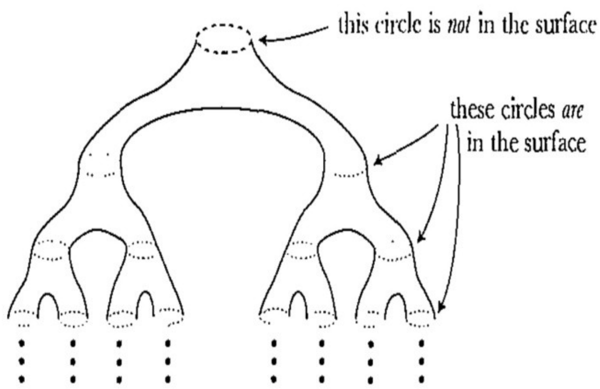



Hegel's fourfold of infinities (1. Quantitative quantity; 2. Quantitative quality; 3. Qualitative quantity; 4. Qualitative quantity), could be thought as circle or four circles placed in a 3-manifold, and the Cantor dust or Cantor set:



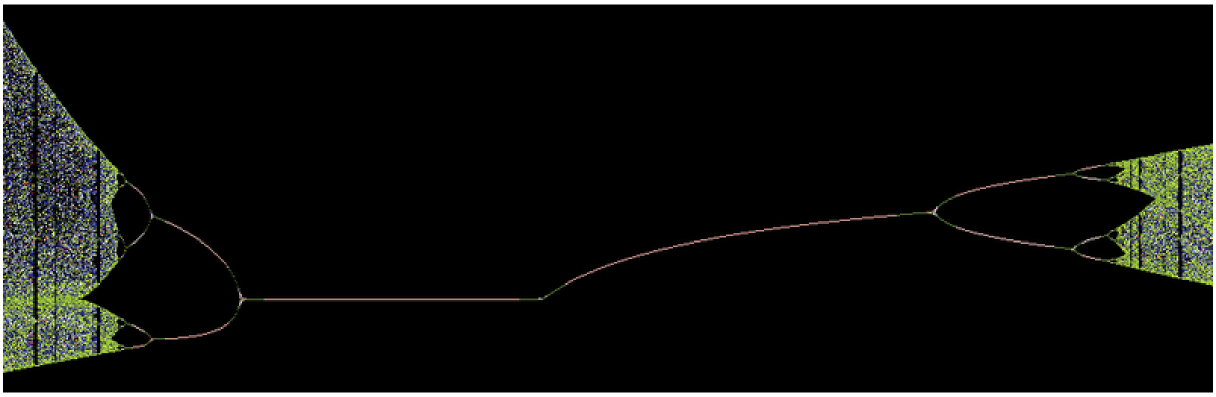
- **Figure 7. Cantor set (Modified from Gleick, 1987).**

The Cobordism models of Topological notion of the categories of quality and quantity – Qualitative quantity dialectics and logic, could be expressed (having an exhibit form) as the surface that consists of infinitely many pairs of pants sewn together in the way shown above: waist line to leg opening. A pair of pants (Hegel's fourfold of infinities) is homeomorphic to the Cantor set.




Hegel's fourfold of infinities as fractal and

The Cobordism of Hegel's fourfold of infinities in the Feigenbaum Diagrams



In particular, in the Period-halving bifurcations leading to order, followed by period doubling bifurcations leading to chaos.

The novel ‘topological’ reading of Hegel’s notions of Qualitative quantity is fundamental in re-thinking of the evolution of hierarchical systems. (Dimitrov, B. 2014)

**6. Dialectics and Chaos: Rethinking of the evolution of hierarchical systems through Hegel’s fourfold model of multiplicities exhibited in Feigenbaum Diagram - The relations between the algebraic topology and evolution - The Role of Heterarchy and Heteronomy in Evolution**

The present discussion on the subject of Chaos theory and Hegel’s dialectics of qualitative and quantitative builds on the following works (Rosser, J.B.,1998/2000:15), (Rosser, J.B., 1991), (Rosser, J.B., 1995), (Rosser, J.B., 1997), (Rosser, J.B., 1991)[429], (Feigenbaum, M.J. 1978)[430], (Aaron Klebanoff, A., Rickert, J. 1998), (Chan JM, Carlsson G, Rabadan R, 2013), Topological Data Analysis (TDA). (Carlson, G. 2009), and contributes, in relation with Hegel’s statements exposed in “The “nodal Line of Measure Relations” (Hegel, G.W.F. 1812, 1813, 1816. tr. /1969. Miller, A. V., foreword by Findlay J. N.: § 777), with novel

[429] Rosser, J.B., Jr. 1991. From Catastrophe to Chaos: A General Theory of Economic Discontinuities, Boston, Kluwer Academic Publishers; Rosser, J.B., Jr. 1995. Systemic crises in hierarchical ecological economies, Land Economics, vol. 71, 163-72; Rosser, J.B., Jr. 1997. Complex dynamics in post Keynesian and new Keynesian models, pp. 288-302, in Rotheim, R. (ed.), New Keynesian Economics/Post Keynesian Alternatives, London, Routledge; Rosser, J.B., Jr., C. Folke, F. Günther, H. Isomäki, C. Perrings, and T. Puu. 1994. Discontinuous change in multilevel hierarchical systems, Systems Research, vol. 11, 77-94; Rosser, J.B., Jr. and M.V. Rosser. 1994. Long wave chaos and systemic economic transformation, World Futures, vol. 39, 197-207; Rosser, J.B., Jr. and Rosser, M.V. 1996. Endogenous chaotic dynamics in transitional economies, Chaos, Solitons & Fractals, vol. 7, 2189-97; Rosser, J.B., Jr. and Rosser, M.V. 1997. Complex dynamics and systemic change: how things can go very wrong, Journal of Post Keynesian Economics, vol. 20, 103-22

[430] Feigenbaum, M.J. 1978. Quantitative universality for a nonlinear class of transformations, Journal of Statistical Physics, vol. 19, 25-52

interpretation of Hegel's emphasis on the leap and quality (that breaks in) *per* saltum, namely with the thesis that Hegel's notion of discontinuous transformation of quantity-determined quality can be located on the Feigenbaum Diagram within the intervals, where the parameter 'a' increase over the value from 2.4 to 3.0. Here we can apply measure and measurement in the meaning of Engels's "law' of transformation and the passage of quantitative changes into qualitative changes, elucidated from his reading of Hegel. Quantitative development becomes qualitative change only prior the so called Cantor's dust (Aaron Klebanoff, A., Rickert, J. 1998), the dust of dialectical logic, where the notion of Hegel's qualitative quantity remained inapparent. Hegel's claim about "the attempt to explain coming-to-be or ceasing-to-be on the basis of gradualness of the alteration" as "tedious like any tautology" (Hegel, G.W.F. 1812, 1813, 1816. tr. /1969. Miller, A. V., foreword by Findlay J. N.) is correct only for the the value from 2.4 to 3.0 of the Feigenbaum Diagram.

The study of chaotic dynamics also originated with Poincaré's qualitative celestial mechanics. As argued in Rosser (1991, Chaps. 1 and 2)[431] catastrophe theory and chaos theory represent two distinct faces of discontinuity, and hence arguably of dialectical 'quantity leading to quality.' The common theme is bifurcation of equilibria of nonlinear dynamical systems at critical values. (Barkley Rosser, J.,Jr., 1998/2000:15)

Although there remain controversies regarding the definition of chaotic dynamics (Rosser, ibid), the most widely accepted sine qua non is that of sensitive dependence on initial conditions (SDIC), the idea that a small change in an initial value of a variable or of a parameter will lead to very large changes in the dynamical path of the system. This is also known as the 'butterfly effect,' from the idea that a butterfly flapping its wings could cause hurricanes in another part of the world (Lorenz, 1963). (Barkley Rosser, J.,Jr., 1998/2000:15)

Although there are systems that are everywhere chaotic, many are chaotic for certain parameter values and are not for others. In such cases there may be a 'transition to chaos' as a parameter value is varied and a system experiences bifurcations of its equilibria. A pattern exhibited by many well known systems is for there to be a zone of a unique and stable

[431] Rosser, J.B., Jr. 1991. From Catastrophe to Chaos: A General Theory of Economic Discontinuities, Boston,

equilibrium, then beyond a critical parameter value there emerges a two-period oscillation, then beyond another point emerges a four-period oscillation, an eight-period oscillation, and so forth, a sequence known as a period-doubling cascade of bifurcations (Feigenbaum, 1978). (Barkley Rosser, J.,Jr., 1998/2000:16-17)

The present section is based on my publication Philosophical topology and Topological philosophy as the mode of thinking of Evolution of Hierarchical Systems: The Role of Heterarchy and Heteronomy in Evolution. [432]

Here, the evolution of hierarchical systems is approached topologically as specific 'problem situation' for mathematical and philosophical mind, in the term of Philosophical Analisis Situs, where philosophical aspect is present through the categories of Hegel's multiplicity – the dialectics and logic of 'qualitative' and 'quantitative', spatial and temporal, and mathematical aspect is presented through the models, such as Cantor Set, logistic map, bifurcation diagrams and topological notion of Cobordism (from French word 'bord' for 'boundary') after Henri Poincare and Rene Thom. Hegel's fourfold of infinities - the bad qualitative infinity; the good qualitative infinity; the bad quantitative infinity; the good quantitative infinity, implementing the four groups of categories such as quantitative quantity - quantitative quality - qualitative quantity - qualitative quality, is proposed trough the topological model and notion of 'cobordism'.

Novel 'topological' reading of Hegel's notions of spatial and temporal, qualitative and quantitative, is proposed as fundamental in re-thinking of the evolution of hierarchical systems.

In addition to the Hierarchical (vertical evolution) relations, an emphasis on the topological notion of qualitative quantity (quality of quantity) in Hegel, reveals the role of Heterarchy and heteronomy in Evolution (horizontal evolution), since the exhibit form of Qualitative quantity is related with continuous changes and smooth, gradual, topological transformations. The

[432] Borislav Dimitrov, 2014, Philosophical topology and Topological philosophy as the mode of thinking of Evolution of Hierarchical Systems: The Role of Heterarchy and Heteronomy in Evolution, Conference Edition: Evolution of Hierarchical Systems, Sofia, Faber Publishing House, September 2014, p.285-318

gradualness of such transformations demonstrates topological homeomorphism as exhibit form of the category of Qualitative quantity, which could be successfully implemented in mathematical, indeed topological models and methods.

Hegel's notions of manifold presented in the model of Rene Thom's cobordism is discussed and implemented through the logistic map of Feigenbaum bifurcation Diagram accepted as universal scenario of development, change and evolution.

The proposed outcome demonstrate that within the interval of chaos, marked in the Feigenbaum Diagram, where the parameter 'a' increase over the value of 3.0 in the higher octaves of 4.33 , the 'voices' of Hegel and Cantor are present within the region of chaos known as Cantor dust. In the zone of chaos and Cantor dust, Hegel's multiplicity and four measures works in progress and logic breaths the thin air of being retrieving itself in metaphysic. Beyond the octaves of the values 4.33 in Feigenbaum bifurcation Diagram, within the chaos, there are the heads of Canto comets or the divergence diagrams, where can be found the seeds of the new orders and multiplicity (manifolds) of new bifurcations and Feigenbaum diagrams. The visible 'white' corridors of homeostasis are windows open for new hierarchies of order and possibilities of development. Based on the said proposition, the focus of the present paper is on the evolutionary scenario, where in addition to the currently accepted paradigm of hierarchy of evolutionary systems where the core of representation is through the phylogenetic tree structure (vertical evolution), the thesis of reticulate (horizontal evolution) is asserted and discussed as exhibiting the heterarchy and heteronomy of evolutionary systems.

The standard evolutionary representation, the phylogenetic tree, and the notion of hierarchy of evolutionary systems, faithfully represents the vertical evolution, but cannot capture horizontal, or reticulate, events, which occur when distinct clades merge together to form a new hybrid lineage. Both hierarchy and heterarchy of the tree structure and the structure of reticulate events could be mathematically investigated, modeled and represented through the field of algebraic topology known as Topological Data Analysis, where the primary mathematical tool considered is a homology theory for point-cloud data sets—persistent homology—and a novel representation of this algebraic characterization— simplicial complex and barcodes.

The persistent homology in evolution, which characterizes global properties of a geometric object that are invariant to continuous deformation, such as stretching or bending without tearing or gluing any single part of it, and the properties that includes such notions as connectedness, is the current method of implementation of Hegel' topological logic of multiplicity and Qualitative quantity.

The novel 'topological' reading of Hegel's notions of Qualitative quantity is fundamental in re-thinking of the evolution of hierarchical systems. [433](Dimitrov, B. 2014)

The evolution of hierarchical systems could be approached topologically as specific 'problem situation' for mathematical and philosophical mind, in the term of Philosophical Analisis Situs, where philosophical aspect is present through the categories of Hegel's multiplicity – the dialectics and logic of 'qualitative' and 'quantitative', spatial and temporal, and mathematical aspect is presented through the models, such as Cantor Set, logistic map, bifurcation diagrams and topological notion of Cobordism (from French word 'bord' for 'boundary') after Henri Poincare and Rene Thom. Hegel's fourfold of infinities - the bad qualitative infinity; the good qualitative infinity; the bad quantitative infinity; the good quantitative infinity, implementing the four groups of categories such as quantitative quantity - quantitative quality - qualitative quantity - qualitative quality, is proposed trough the topological model and notion of 'cobordism'.

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The gradualness of such transformations demonstrates topological homeomorphism as exhibit form of the category of Qualitative quantity, which could be successfully implemented in mathematical, indeed topological models and methods. Hegel's notions of manifold presented

[433] Dimitrov, B. (2014), Philosophical topology and Topological philosophy as the mode of thinking of Evolution of Hierarchical Systems: The Role of Heterarchy and Heteronomy in Evolution, Conference Edition: Evolution of Hierarchical Systems, Sofia, Faber Publishing House, September 2014, p.285-318

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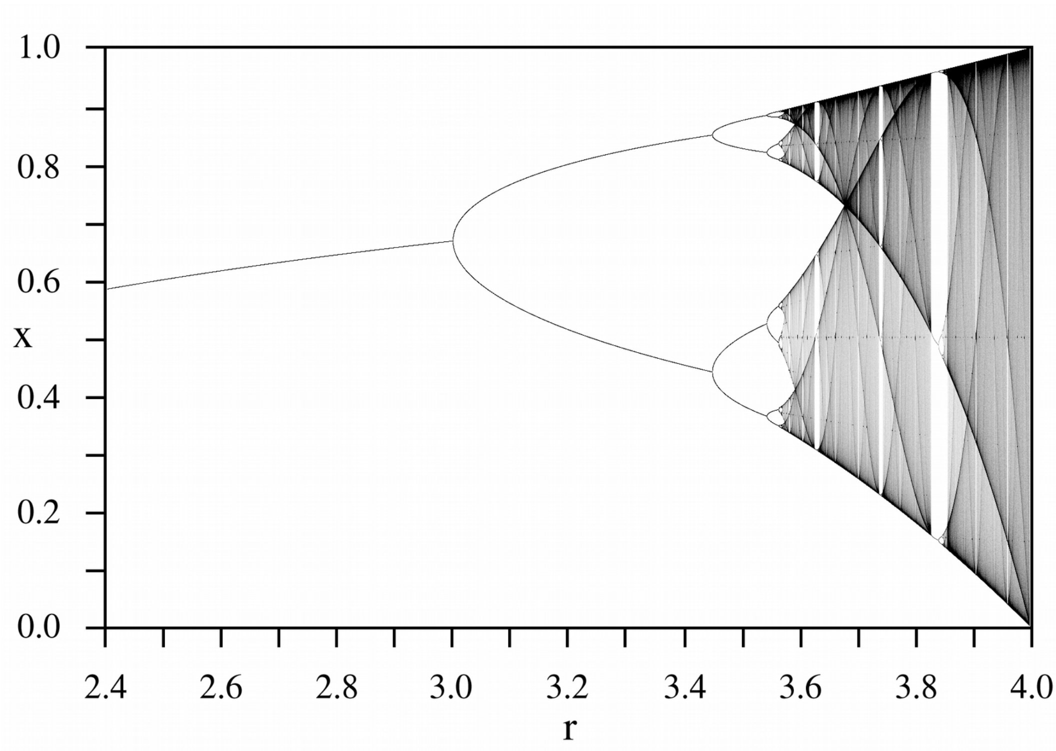


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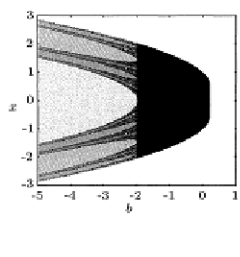
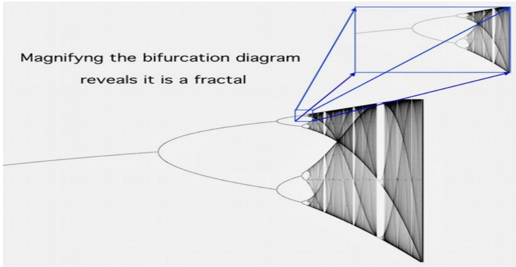


Cantor comet or the divergence diagram, the seed of the new order and multiplicity (manifolds) of new bifurcations within the Feigenbaum diagram. (Aaron Klebanoff, A., Rickert, J. 1998)

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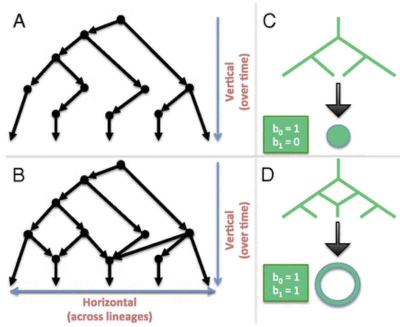
The “nodal Line of Measure Relations” (Hegel, G.W.F. 1812, 1813, 1816. tr. /1969. Miller*,* A. V., foreword by Findlay J. N.: § 777), and Hegel’s emphasis on the leap and quality (that breaks in) *per* saltum can be located on the Feigenbaum Diagram within the intervals, where the parameter ‘a’ increase over the value from 2.4 to 3.0. Here we can apply measure and measurement and the second law of dialectics, elucidated by Engels from his reading of Hegel, the law of transformation and the passage of quantitative changes into qualitative changes. Quantitative development becomes qualitative change only before the so called Cantor’s dust, the dust of dialectical logic, where the notion of Hegel’s Qualitative quantity remained inapparent. Hegel’s claim about “the attempt to explain coming-to-be or ceasing-to-be on the basis of gradualness of the alteration” as “tedious like any tautology” (Hegel, G.W.F. 1812, 1813, 1816. tr. /1969. Miller*,* A. V., foreword by Findlay J. N.) is correct only for the the value from 2.4 to 3.0 of the Feigenbaum Diagram.

The discussion offered by Hegel about 'gradualness' and the "the attempt to explain coming-to-be or ceasing-to-be on the basis of gradualness of the alteration" is valid and correct only before the entrance of the system into the chaos, where the value of 3.0 increase to the higher octaves of 4.33. The 'smallness' that is *not yet perceptible only because of its smallness* is the seed of the Cantor's comet, or the multiplicity of the comets within the Cantor's dust. Beyond the octaves of the values 4.33 in Feigenbaum bifurcation Diagram, within the chaos, there are the heads of Cantor comets or the divergence diagrams, where can be found the seeds of the new orders and multiplicity (manifolds) of new bifurcations and Feigenbaum diagrams. (Aaron Klebanoff, A., Rickert, J. 1998)




The relations between the algebraic topology and evolution
(Chan JM, Carlsson G, Rabadan R, 2013)

A. the phylogenetic tree illustration of the vertical evolution.

B. reticulate structure of the horizontal evolution.

C. A tree that can be compressed to a point. In the vertical evolution this is possible.

D. A tree that can not be compressed to a point – postulate of the horizontal evolution – without destroying the hole in the center.

The standard evolutionary representation, the phylogenetic tree, and the notion of hierarchy of evolutionary systems, faithfully represents the vertical evolution (where the well known dialectics of quality and quantity occurs), but cannot capture horizontal, or reticulate, events, which occur when distinct clades merge together to form a new hybrid lineage. Both hierarchy and heterarchy of the tree structure and the structure of reticulate events could be mathematically investigated, modeled and represented through the field of algebraic topology known as Topological Data Analysis (TDA). (Carlson, G. 2009). In TDA where the primary mathematical tool considered is a homology theory for point-cloud data sets —persistent homology—and a novel representation of this algebraic characterization — simplicial complex and barcodes.

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## Chapter 5 Topological (in) Hegel's language, syntax and semantics, pictorial thoughts, pictorial thinking and topological metalepsis

**The present chapter build on the following works:** (Yovel 1981); (Berto 2007); (Lamb, 1979); (Pippin, 1988); (Pinkard, 1994); (Redding, 1996); (Flach, 1964); (Damsma, 2010); (Verene, 2007); (Coltman, 1998); (Maybee, 2009); (Haas, 2000); (Haas 2008), (Carlson, 2003); (Chen, 1982): (Chen, 2000); (Murphy, 2014): (Willats, 1997); (Voegelin, 1990/2000); (Hughes 1993); (Webb 1981); (Curtius, 2013); (Ostheeren, 1998); (Gelley, 1974); (Malina, 2002); (Lorenz, 2007); (Verene, 2009); (Kunze, 2013); (Žižek, 2012); (O'Regan, C, 1994).

### 1.1. Hegel's Dialectics as a Semantic Theory

According to Y. Yovel, Hegel's enterprise aims at 'creating a new philosophical glossary by exploiting existing ambiguities and connotations of ordinary language. In declaring this program [. . .], Hegel specifically offers to make systematic distinctions between terms that are usually considered to be synonyms, especially the set: Existenz, Dasein, Wirklichkeit, etc.' (Yovel 1981: 117).[434] [435]

**What is Hegel's dialectics?** With this question, Francesco Berto resumes his investigation of Hegel's Dialectics as a semantic theory, proposing an analytic reading. (Berto 2007:20) Berto points out that in In 1964, Werner Flach claimed that the research on Hegel had not provided

[434] Yovel, Y. (1981), 'Hegel's Dictum that the Rational is Actual and the Actual is Rational. Its Ontological Content and Its Function in Discourse', in W. Becker and W. K. Essler (eds.)
[435] Francesco Berto, 2007, Hegel's Dialectics as a Semantic Theory: An Analytic Reading, Blackwell Publishing Ltd. 2007:24

an adequate reply to this question (Flach 1964:55–64)[436] Few years later, adds Berto, Hans-Friedrich Fulda admitted that, "despite the considerable efforts of scholars, our Auseinandersetzung with the famous Hegelian method 'has not led, so far, to any satisfactory result'[437] In his paper Berto expand some aspects of the current Anglo-American revitalization of Hegelian philosophy, as rooted in Wilfrid Sellars' Empiricism and the Philosophy of Mind, unfolded in John McDowell's justly celebrated Mind and World, and Robert Brandom's works on inferential semantics [438], Lamb [439], Pippin 1988[440], Pinkard 1994[441], Redding 1996[442]. Berto's proposition is on "the inferential intuition that an essential part of what it is to grasp a conceptual content, and to be able to apply it correctly to an object, consists in mastering its connections with the concepts it entails, and with the concepts that entail it. These connections can be expressed by meaning postulates. The suggestion that Hegel's dialectics operates on meaning postulates, as far as I know, has been largely ignored by traditional minded scholars."[443] Berto's work "provides a promising path towards a new Hegel—towards a better understanding of his philosophy, and in particular of its core, the dialectical method."[444]

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436 Flach, W. (1964), 'Hegels dialektische Methode', Hegel-Studien, 1: 55–64.

437 Fulda, H. F. (1973), 'Unzula¨ngliche Bemerkungen zur Dialektik', in R. Heede and J. Ritter (eds.) Hegel-Bilanz. Frankfurt a.M.: Klostermann, repr. in R.-P- Horstmann (ed.) Seminar: Dialektik in der Philosophie Hegels. Frankfurt a.M.: Suhrkamp.

438 Brandom, R. B. (1994), Making It Explicit. Cambridge, MA: Harvard University Press.
—— (1999), 'Some Pragmatist Themes in Hegel's Idealism: Negotiation and Administration in Hegel's Account of the Structure and Content of Conceptual Norms', European Journal of Philosophy, 7: 164–89.
—— (2000), Articulating Reasons. Cambridge, MA: Harvard University Press.
—— (2001), 'Holism and Idealism in Hegel's Phenomenology', Hegel-Studien, 36: 57–92.
—— (2002), Tales of the Mighty Dead: Historical Essays on the Metaphysics of Intentionality. Cambridge, MA: Harvard University Press.

439 Lamb, D. (1979), Language and Perception in Hegel and Wittgenstein. Avebury.

440 Pippin, R. B. (1988), Hegel's Idealism. The Satisfactions of Self-Consciousness. Cambridge: Cambridge University Press

441 Pinkard, T. (1994), Hegel's Phenomenology. The Sociality of Reason. Cambridge: Cambridge University Press.

442 Redding, P. (1996), Hegel's Hermeneutics. Ithaca & London: Cornell University Press.

443 Francesco Berto, 2007, Hegel's Dialectics as a Semantic Theory: An Analytic Reading, Blackwell Publishing Ltd. 2007:19

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It has been widely recognized, asserts Berto, that dialectics assumes as its starting point ordinary language (in the broad sense in which it includes scientific and somewhat technical philosophical terminology). In particular, it investigates the meanings of conceptual terms, shared by competent speakers and constituting their lexical competence.[445] In this regard, Berto quotes what Hegel says in the Preface to the second edition of the Science of Logic, remarking that what Hegel says "is the terminus a quo of dialectics":

The forms of thought are, in the first instance, displayed and stored in human language. Nowadays we cannot be too often reminded that it is thinking which distinguishes man from the beasts. Into all that becomes something inward for men, an image or conception as such, into all that he makes his own, language has penetrated, and everything that he has transformed into language and expresses in it contains a category— concealed, mixed with other forms or clearly determined as such, so much is logic his natural element, indeed his own peculiar nature. (WL: 31)[446]

Ordinary language is theory-laden, and dialectics explicates (as we shall see, in a form very similar to the one of meaning postulates) the presuppositions and theoretical correlations that underlie the semantic settlement of conceptual terms and govern their actual use. Such an idea of a 'logical explication of the implicit' comes from Robert Brandom's works." (See Brandom 1994:2000). In "Making It Explicit", Brandom argues that logic should play the expressive role of the organ of semantic self-consciousness (see Brandom 1994: 384).[447] Logical vocabulary provides the resources to express the inferential commitments articulating descriptive conceptual contents. As Brandom himself observes, the resulting structure of explanation is distinctively Hegelian (see Brandom 2000: 22).[448] Berto asserts, that "a few lines after the passage quoted above, Hegel says that the categories of the Logic are 'thoroughly familiar to educated people', 'determinations of thought which we employ on every occasion, which pass our lips in every sentence we speak'. Nevertheless, 'it does not follow [. . .] that they are intelligently apprehended'. Philosophical, or speculative, logic is the

[445] Francesco Berto, 2007, Hegel's Dialectics as a Semantic Theory: An Analytic Reading, Blackwell Publishing Ltd. 2007:20
[446] In Francesco Berto, 2007, Hegel's Dialectics as a Semantic Theory: An Analytic Reading, Blackwell Publishing Ltd. 2007:20
[447] Brandom, R. B. (1994), Making It Explicit. Cambridge, MA: Harvard University Press.
[448] Brandom, R. B. (2000), Articulating Reasons. Cambridge, MA: Harvard University Press.

intelligent apprehension of this familiarity, the explication of this implicit or, in the Hegelian jargon, the development of the 'ansich' in the 'fursich'. ([449] [450] [451]) A couple of pages later, we read that 'natural logic' is 'unconsciously busy' with the categories of language and thought. Speculative logic makes theoretical commitments, which are implicit in our ordinary language, explicit: 'as impulses the categories are only instinctively active', and 'the loftier business of logic therefore is to clarify these categories and in them to raise mind to freedom and truth' (WL: 36–7)."[452]

Philosophy, thus, has to begin with natural language. Hegel thoroughly opposes those authors, such as Spinoza, who start with a regimentation of it, or with stipulative definitions. But Hegel is obviously not a descriptive philosopher of ordinary language. For ordinary expressions can be vague, their meanings can be only partially determined and, most interesting for the dialectical procedure, the class of their synonyms can be incoherent, giving rise to inconsistencies. Therefore, philosophy also has to reshape meanings and intensional contents: it can criticize, control, and improve our linguistic business in order to introduce distinctions and rectifications where there was only confused, unconscious practice. But let Hegel say so himself—in a way one could hardly improve: Philosophy has the right to select

[449]Here is my reference to Donald Phillip Verene, Hegel's Recollection: a Study of Images in the Phenomenology of Spirit (Albany: SUNY Press, 1985). In the epilogue of "Hegel's recollection: A study of images in the Phenomenology of Spirit", Verene discusses "the great problem in the interconnection of Hegel's works" in Hegel's system of philosophy. Hegel's system of philosophy is represented first by "Phenomenology of Spirit", second by the "Science of Logic" and the last two – the philosophy of nature and the philosophy of spirit. The great problem on which Verene focuses his attention is the transition from the absolute idea to the nature. The movement from the idea to natur, asserts Verene after Hegel, is not a process of becoming, nor is transition. This passage should be understood through an absolute liberation and freedom. According to Verene, the key to the passage from Idea to nature lies in the passage from the Phenomenology to the Logic. This passage Verene calls "a movement that is usually taken for granted". The root of the absolute liberation of consciousness and the ground of its freedom is in the absolute "space" between the two moments – from what is "ansich" to what is "ansich for us", or the parallel passage from the standpoint of being from what is "*ansich*" to what is "*fursich*". The "and" that exists between them represents not the merger in a unity between these two moments, but only that being exists in a state of mutual attachment to itself and, ..the mind as subject exists in mutual attachment to its object. Once the illusion of unifying the two moments is overcome in absolute knowing in the "Phenomenology of Spirit", way for the Logic is made.

[450] For Hegel's method "*An sich* (in itself) and *fur sich* (for-itself)", See Donald Phillip Verene, Hegel's Recollection: a Study of Images in the Phenomenology of Spirit (Albany: SUNY Press, 1985), Second Chapter – The Method of In-itself.

[451] Donald Phillip Verene, 2007, Hegel's Absolure: An Introduction to Reading the Phenomenology of Spirit, State University of New York Press,p.13 (Hegel's "Introduction":The Double *Ansich)*

[452] Francesco Berto, 2007, Hegel's Dialectics as a Semantic Theory: An Analytic Reading, Blackwell Publishing Ltd. 2007:21

from the language of common life which is made for the world of pictorial thinking, such expressions as seem to approximate to the determinations of the Notion. There cannot be any question of demonstrating for a word selected from the language of common life that in common life, too, one associates with it the same Notion for which philosophy employs it; for common life has no Notions, but only pictorial thoughts and general ideas, and to recognize the Notion in what is else a mere general idea is philosophy itself. It must suffice therefore if pictorial thinking[453], in the use of its expressions that are employed for philosophical determinations, has before some vague idea of their distinctive meaning; just as it may be the case that in these expressions one recognizes nuances of pictorial thought that are more closely related to the corresponding Notions.[454]

The transition from Typological Mode to the Topological Mode proposed by Jay L. Lemke. [455]
Jay L. Lemke introduced the mixed-mode Semiosis - Typological vs. Topological Semiosis. [456]

According to Lemke there are two fundamentally different kinds of meaning-making: "typological" meaning-making - meaning-by-kind /natural language/ and “topological” meaning-making - meaning-by-degree /visual language/, “which is more easily presented by means of motor gestures or visual figures -- the meaning of continuous variation or "topological" meaning.” Meaning-by-kind is qualitative and meaning-by-degree is quantitative.

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453 For **Vorstellung (Picture-thinking, Figurative thinking),** See Donald Phillip Verene, 2007, Hegel’s Absolure: An Introduction to Reading the Phenomenology of Spirit, State University of New York Press, Appendix, p. 117 and Donald Phillip Verene, Hegel’s Recollection: a Study of Images in the Phenomenology of Spirit (Albany: SUNY Press, 1985.

454 Francesco Berto, 2007, Hegel’s Dialectics as a Semantic Theory: An Analytic Reading, Blackwell Publishing Ltd. 2007:21

455 Jay L. Lemke, “Topological Semiosis and the Evolution of Meaning” –
http://www-personal.umich.edu/~jaylemke/webs/wess/index.htm
456 Jay L. Lemke, Typological vs. Topological Semiosis –
.http://www-personal.umich.edu/~jaylemke/webs/wess/tsld002.htmSee: Jay L. Lemke, “Mathematics in the middle: measure, picture, gesture, sign, and word, and Opening Up Closure: Semiotics Across Scales”.

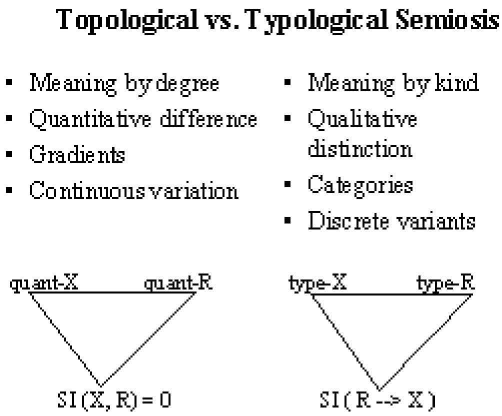


Figure by Jay L. Lemke, Typological vs. Topological Semiosis – .http://www-personal.umich.edu/~jaylemke/webs/wess/tsld002.htm

For Lemke Typological semiosis is qualitative semiosis and Topological semiosis is quantitative semiosis. Qualitative character of typological semiosis determines the discrete variant and Quantitative character of topological semiosis and the continuous variation. I support the idea that qualitative quantity is possible to appear as categorical gradiant /mixture from categories in typological semiosis and gradients in topological semiosis/. The topological and continuous notion of qualitative quantity as gradualness and continuous variation or "topological" meaning" can be seen in Figure above given in Lemke's work " Mathematics in the middle: measure, picture, gesture, sign, and word". In "Opening Up Closure: Semiotics Across Scales", Lemke proposes the "Mixed-mode Semiosis". In this mixed-mode of typological and topological semiosis, the domain of qualitative quantity could be seen, as "categorical gradiant" /Lemke/ of the "unified system for meaning-making" /Lemke/. The Qualitative quantity Method Research due to the gradual and nondiscursive notion of the category /qualitative quantity/ and the ability to exhibit topological homeomorphism is the method /within the system of meaning-making/ with the ability to transform the form or the structure from typology to topology. The aim of the The Qualitative quantity Method Research is to create "Topology of meaning". [457] [458]

[457] The term "topology of meaning" emerged at the proceedings of the "Einstein meets Magritte" Conference, Brussels, Belgium /1995/. The "topology of meaning" was introduced by R. Ian Flett and Donald H. McNeil in their paper "What's Wrong with this Picture? Towards a Systemological Philosophy of Science with Practice." McNeil and Ian Flett utilized systemology to topologically visualize the dynamic intra-relationship of art and sciences, offering an unconventional systemological illustrations as developments beyond the conventional scientific and artistic imagery

[458] Donald H. McNeil acknowledges that the allusions to the "topology of meaning" relate to the unpublished work by R. Ian Flett – See: Donald H. McNeil, "What's going on with the topology of recursion?" -

Typology of research topics in a specific area of research in research culture are necessary very useful to understand the relationships between the research topics. Since Aristotle, the organization of knowledge is based on the conceptualization, classification, typology or taxonomy. Typology is instrument of hierarchical system.

What is Hegel's dialectics?
Dialectic bears topological notions since it is concerned with the 'forms' and 'relations'.Hegel's dialectic refers to the *forms*, to the forms of consciousness, to the actual and necessary form of consciousness as it develops in *relation* to its object and to the *form* that speculative knowing takes when it makes this development explicit.

### 1.2. Hegel and Language

Hegel held that all that can be known about the world is known in the language and things that cannot be expressed in a form of language cannot actually be known at all, For the world to be represented in thought, it must be represented in language, the structure of language must be isomorphic with the structure of the world's knowability. Language has been developed as a tool to understanding the world and the categories on which set theory is based have a qualitative basis in language (see Damsma, 2010).

Hegel's logic relates to the most fundamental structural relationships between categories in language. Logic itself consists of categories, and without the categories the world would be unintelligible, distinctionless white noise. If 'Mathematical and formal thinking has a place in this structure of language' (Damsma, 2015:3) so Topology (as mathematical and formal thinking) has a place in the structure of language and the structure of categories need topological rethinking and considerations especially where structure of the world reveal topological. Same as mathematical and formal thinking, where quantitative is presented, topology (the qualitative geometry), cannot be directly applied at more concrete levels (e.g. the level of society) without elaboration of qualitative empirical considerations about these

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http://www.library.utoronto.ca/see/SEED/Vol4-1/McNeil.htm/ See: Science and Art: The Red Book of `Einstein Meets Magritte': The Red Book Vol 2 (Einstein Meets Magritte: An Interdisciplinary Reflection on Science, Nature, Art, Human Action and Society), Kluwer, 1999 - Flett, R. Ian, and Donald H. McNeil. 1995. "What's Wrong with this Picture? Towards a Systemological Philosophy of Science with Practice." Proceedings of the "Einstein meets Magritte" Conference, Brussels, Belgium.

fields. Here I disagree with Damsma who concludes that 'mathematical models may play a role in the empirical sciences, but not in dialectics' (Damsma, 2015:3).[459] The ground for my disagreement is that topology is qualitative, and in my arguments it is based on qualitative quantity. Topological modeling, models and tools such as Topological Data Analysis are applicable to empirical science and dialectics.

Mathematical Topology (as mathematical and formal thinking) has a place in the structure of language and the structure of language can inform us about the structure of the world we think about.

Logic ('the science of the Idea in and for itself' – §18), relates to the most fundamental (structural relationships between) categories in language, i.e. it consists of categories without which the world would certainly be unintelligible, distinctionless white noise (such as Being, Becoming, the One and its Other) without however considering the application of these to the world itself.

Hegel's main tenet is that all that can be known about the world is known in language. Things that cannot be expressed in a form of language cannot actually be known at all. The upshot of this is that the basic structures of language are the basic structures of intelligibility of the world. In other words: for the world to be represented in thought, it must be representable in language. If so, the structure of language must be isomorphic with the structure of the world's knowability (a thought also expressed by Hofstädter, 1979) and by mapping the basic systematic relationships between categories in language, the systematic of the world's intelligibility and the fundamental interrelations between everything we can claim about it, can be discovered.2 Hegel provides an overview of this project in his *Encyclopädie* (1830, 1817). (Damsma, 2010).[460]

[459] D. Damsma (2010). *Qualitative and quantitative analysis in systematic dialectics: Marx vs. Hegel and Arthur vs. Smith.* (Preprints). Amsterdam: University of Amsterdam, School of Economics.[go to publisher's site]

[460] D. Damsma (2010). *Qualitative and quantitative analysis in systematic dialectics: Marx vs. Hegel and Arthur vs. Smith.* (Preprints). Amsterdam: University of Amsterdam, School of Economics.[go to publisher's site]

### 1.3. The presence of 'topological' notions in Hegel's language and syntax:

The language of Hegel's Logic implies topological relationship between the main categories in language and categories in logic. Topological syntax can be traced in the semantic of such construction as Dialektik (Dialectic) - Vernunft (Reason) and Der Speculative Satz (speculative sentence or proposition) - Gesetzt (Posited) - System – Science – Circle – Circle of circles - An Sich (In Itself), Fur Sich (For Itself), Das Anundfursichsein (Being-In-And-For-Itself) - Aufhebung, Aufheben, Aufgehoben - Bildung (Formation, Education, Culture) - Entsprechen, Sich Entsprechen (Correspond to, Meet) - Gleich (like) - Die Bestimmte Negation (Determinate Negation) - Unendlichkeit (Infinity) - Schlecht-Unendliche (bad infinite) and Wahrhaft Unendliche (true infinity) - Verdopplung (Doubling) - Vermittlung (Mediation) and Die Mitte (the Middle) - Vorstellung (Picture-thinking, Figurative thinking) - Erinnerung (Recollection).

As Donald Phillip Verene assrets, ***Dialektik (Dialectic)*** is much discussed in relation to Hegel's philosophy but not much discussed as a term by Hegel.[461] Hegel uses the term, for example, *die dialektische Bewegung* (dialectical movement), referring to the manner in which consciousness develops in relation to its object. Hegel speaks more of the "self-movement of the concept [*Begriff*]," which is dialectical in form. Hegel associates this dialectical movement with the form of speculative thought, especially the "speculative sentence or proposition" (see Preface, par. 61ff). Hegel's dialectic is unlike dialectic as it is commonly understood as a simple posing of opposites, or the generation of probabilities. It is not connected to argumentation. Hegel's dialectic is also not a method in the ordinary sense of an instrument that can be applied externally to a subject matter.

Dialectic bears topological notions since it is concerned with the 'forms' and 'relations'.Hegel's dialectic refers to the *forms*, to the forms of consciousness, to the actual and necessary form of consciousness as it develops in *relation* to its object and to the *form* that speculative knowing takes when it makes this development explicit.

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[461] Donald Phillip Verene, 2007, Hegel's Absolure: An Introduction to Reading the Phenomenology of Spirit, State University of New York Press, Appendix, p. 104

Topological characteristics can be discovered in what Hegel regards as relationships between ***Vernunft (Reason)*** and ***Verstand (understanding).*** Hegel endorses the idea of *Vernunft* against *Verstand* ("the Understanding"). The understanding allows us to classify and order the world, to create typology and structure but it can give no knowledge of the whole. Hegel calls *Verstand (*the understanding) a "table of contents" mentality. The thema of relationship between parts and whole is topological. Transformation of Understanding to Reason is the transition from typology to topology. Hegel emphasizes on the form of this transformation in true topological mode of reasoning – the speculative reasoning. For Hegel thought has the shape and only reason can be dialectical in form. ***Der Speculative Satz (the speculative sentence or proposition)*** is the thought-form of reason. Giving the form of reason trough dialectics is the core of Vernunft. The understanding cannot get beyond the propositional form of ordinary logic that keeps the subject and predicate separate. (Appendix 117) Hegel's sense of the *Begriff* is closed to the formalism of the understanding but is open to and in fact is the life of reason. Following what is the 'speculative sentence or proposition (Der Speculative Satz) in/for Hegel, we can see that distinction between subject and predicate is destroyed in the speculative sentence. For Hegel, the speculative sentence is based on an inner dialectical movement that moves from subject to predicate and back in the direction of subject, but now in a new sense. The transformation of proposition, translated by Miller as Satz, yet accepted by Verene as 'sentence', [462] where the boundary between 'subject' and 'predicate' is gradually disappearing first and then appearing again in a new sense, bears topological nature. The speculative sentence defines what it means to think speculatively and provides Hegel's model of what philosophical language is. Hegel's *Phenomenology* is a construction of such philosophical sentences.[463]

The Understanding begins by making a proposition about the universe yet this proposition is one-sided. The Understanding refuses to see that everything is mediated. The Understanding is necessary to the logical system. It is the first step in logical reasoning. There must be some proposition before the next steps. The next step, the second step is that of Dialectical Reason. Dialectical Reason knows that the Understanding's proposition is insufficient and one-sided. It brings into the light what the Understanding has suppressed. Dialectical Reason

[462] Donald Phillip Verene, 2007, Hegel's Absolure: An Introduction to Reading the Phenomenology of Spirit, State University of New York Press, Appendix, p. 117

[463] Donald Phillip Verene, 2007, Hegel's Absolure: An Introduction to Reading the Phenomenology of Spirit, State University of New York Press, Appendix, p. 117

is the voice of *experience*. It retrieves what has been learned before and sets this suppressed material in /opposition to the proposition of the Understanding. As David Carlson asserts "Dialectical Reason brings forth what the Understanding has left out. But in positivizing this negated material, Dialectical Reason suffers from the same one-sidedness as the Understanding. Dialectical Reason is the pot calling the kettle black. It is as one-sided to emphasize what is *not* as it is to emphasize what *is*." (David G. Carlson, 2002, Hegel's Theory of Quantity:2) The third step is Speculative Reason. The Speculative Reason maintains that the positive (what is)and negative (what is not) must be thought together. As Carlson adds "Once Speculative Reason reconciles the positions of the Understanding and Dialectical Reason, the Understanding again takes the stage to formulate a proposition about what it has learned. It reduces the lesson to a one-sided proposition. Dialectical Reason once again critiques the effort, and Speculative Reason mediates the dispute. These three steps replicate themselves again and again and again--until mediation exhausts itself and confesses (at the very end of the *Science of Logic*) that mediation is an immediacy after all." (David G. Carlson, 2002, Hegel's Theory of Quantity:2)

Hegel uses ***Gesetzt (Posited)*** in the *Science of Logic* and in the *Phenomenology*. The term *gesetzt* (p.p. and adj.) means fixed, set, placed, established; *setzen* is the ordinary verb in German for to place, set, put. *Gesetzt* is traditionally translated as "posited," which has a very "philosophical" sound and preserves nothing of the very ordinary sense of *setzen*.[464] Gesetzt is equivalent to its etymological relative, 'to set'. gesetzt , is used for 'assuming, supposing (that)' or 'let us assume, suppose (that)'. The philosophical uses of setzen correspond to, and are influenced by, those of the Greek tithenai, tithesthai (1 'to place', 2 'to affirm, posit, assume'), but the common translation, 'to posit', comes from the past participle, positus , of the Latin ponere ('to put', etc.). It indicates, primarily (1) the assumption or supposition of a proposition ( Satz ); (2) the assertion or affirmation of a proposition, in contrast to its denial; (3) the affirmation or postulation of (the existence of) an entity. Fichte (and, under his influence, Schelling) uses setzen very frequently in a sense that combines the ideas of the assertion of propositions and the affirmation or positing of entities, and thus of intellectual assent and volitional affirmation or (self-)assertion. What is posited is not simply affirmed to be real, but is thereby made real.

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[464] Donald Phillip Verene, 2007, Hegel's Absolure: An Introduction to Reading the Phenomenology of Spirit,

Hegel calls the *Phenomenology* the first part of his "system of science." In the *Phenomenology* Hegel refers to his philosophy as a circle (par. 18) and in his *Science of Logic* Hegel says his system is a circle and in fact **"*a circle of circles*."**

Hegel uses the terms ***an sich (in itself)***, and ***fur sich (for itself),*** in several combinations. His discussion of these terms is concentrated in his introduction, but they occur throughout the text. Another concentration of these terms is in the final chapter of *Phenomenology,* "Absolute Knowing." He makes *an sich* into a noun, *das Ansich* (the in-itself), and *fьr sich* into *das Fьrsich* (the for-itself). He also connects them with being (*Sein*), thus: *das Ansichsein* (being-in-itself) and *das Fьrsichsein* (being-for-itself). Hegel makes both into the compound *das Anundfьrsichsein* (being-in-and-for-itself). The constructions of *An sich* and *fьr sich* are the two poles of Hegel's conception of the dialectical self-movement of consciousness. Through its presence in consciousness the object that initially has being-in-itself comes to have being-for-itself. It becomes something for consciousness.

Topological character implying homological transformation bears **Hegel's concept of sublation (*Aufheben).*** Aufheben a verb used in ordinary German that has no genuine equivalent in English. It has been rendered as "supersede" and by a combination of two English verbs, such as "cancel" and "preserve" (as well as by "transcend," "sublate"). *Aufheben* is the noun that describes such action; *aufgehoben* is the past tense of *aufheben*. Hegel also uses *das Aufgehobene*, what has been *aufgehoben*. *Aufheben* is Hegel's term for the way in which one stage of consciousness is transformed into a succeeding stage, the sense in which a preceding stage is replaced yet absorbed into and incorporated in a new way into a succeeding stage. A basic metaphor for this is the way in which within any human being the world of the child is transformed into the world of the adult and the relationship between any of the stages and substages along the way.

According to Verene, *Aufheben* is not a technical term and is used in ordinary German conversation. It has four basic English meanings and Hegel is exploiting all of them at once to convey his dialectical sense of development. *Aufheben* can mean: (1) to lift or raise something up, as in the simple sense of the verb *heben*, "to raise"; (2) to take something up, to pick it up or even seize it actively (a seeming intensification of the act of *heben*, "*auf*"-*heben*); (3) to keep or preserve something, to retain it; and (4) to abolish, annul, cancel, to put an end to something. Depending on the context, *aufheben* can be used in ordinary speech to emphasize one or more of these senses. *Aufhebung* or *aufheben* designates a theory Hegel has of the developmental and dialectical nature of consciousness.

Hegel's concept of sublation (aufheben) is essential to his philosophy. While aufheben is an ordinary German word, meaning both 'to negate' and 'to preserve', the peculiar and complex way Hegel combines these two senses has been widely misunderstood.

Topological character bears Hegel's use and meaning of ***Bildung*** as in ordinary German can mean, among other things: formation, shape, structure, development, growth, generation, education, training, cultivation, culture, civilization. *Bildung* links the Greek word *paideia.*
There are homological characteristics in the use of such words as ***Entsprechen, Sich Entsprechen (Correspond to, Meet).*** The term Entsprechen is a verb Hegel frequently uses regarding the relationship of concept and object. *Entsprechen* means answering, suiting, matching, being in accord with, meeting with, corresponding. Verene asserts that *Entsprechen* conveys only a notion of correspondence or accord and there is no sense of making identical with or unifying with.

Homological can be located in Hegel's use of ***Gleich (like).*** *Gleich means* the same, like, equal, equivalent, alike, similar, resembling, proportionate. This is a term Hegel uses to describe the relationship between appearance and essence (*Wesen*). NB that *gleich* cannot mean "identical" (*identisch*).

Another topological presence bears Hegel's concept of ***Die Bestimmte Negation (Determinate Negation).*** Hegel's concept of negation differs from that used in traditional logic, in that negation for Hegel does not represent a null class. Negation is never "empty" for Hegel. In the selfmovement of consciousness, when something is affirmed by consciousness to exist and be an object of knowledge for it, to have a truth for it, something is at the same time denied or negated. What is negated has content; there is something specific that is being negated even though it is not then explicit to consciousness in its act of knowing. But to know the truth of its object fully, consciousness must know it in relation to its opposite—what it has negated in order to affirm the object before it.

Topological notions exhibits Hegel's concept of ***Unendlichkeit (Infinity) - Schlecht-Unendliche (bad infinite)*** **and** ***Wahrhaft Unendliche (true infinity).*** In the *Science of Logic* Hegel explains his famous distinction between the "bad infinite" (*Schlecht-Unendliche*), the infinity that goes on and on and the "true infinite" (*wahrhaft Unendliche*), his special sense of an infinity in which the whole systematically recapitulates itself.

Topological is Hegel's ***Verdopplung (Doubling).*** In his account of the "unhappy consciousness" Hegel speaks of the *Verdopplung*, the "doubling" or self-duplication of consciousness.

The topological notions such as boundary, neighbourhood, openness and closseness correspond to the Hegel's term ***Vrmittlug (Mediation)*** **and** ***"die Mitte"*** (The German for ***'the middle'***). (Miller A. V., 1990, p.116) For Hegel both 'something' and 'other', therefore, are initially 'determinate beings' or 'somethings'. The "the mediation through which something and other each as well is, as is not," Hegel calls "limit." (Miller A. V., 1990, p.116)

As something is not its other, and the other (as it is also something) is not the something that its other is, each is equally determined to be what it is in not being its other. And this non-being of the other (which is also the nonbeing of itself) is the "ceasing-to-be" or the "limit" of something in its other. In his explanation, Hegel employs term "die Mitte" (The German for 'the middle'). (Miller A. V., 1990, p.116)

The German for '(the)' middle' is (die) Mitte. This generates an adjective mittel ('middle') and another noun (das) Mittel (originally '(the) middle, the thing in the middle', but now 'means, what serves the attainment of a purpose '). It also generates several verbs, especially mitteln ('to help someone to, to settle, mediate', e.g. a quarrel), which is now obsolete but has left mittelbar ('mediate, indirect') and unmittelbar ('immediate, direct'), and vermitteln ('to achieve union, mediate; to bring about', etc.). The past participle of vermitteln, vermittelt ('mediated, indirect') is used in contrast to unmittelbar. Both give rise to abstract nouns, Vermittlung ('mediation') and Unmittelbarkeit ('immediacy'). In non-Hegelian philosophy, unmittelbar is primarily an epistemological term. Immediate certainty is a certainty that is not mediated by inference or proof, or perhaps even by symbols or concepts.

In Hegel's use of "die Mitte" we could see the **topological notion of 'betweenness'**, 'in-between' in the pure meaning of Plato's 'methaxis' and Aristotle's '**metalepsis**' or exactly in accordance with Voegelin's concept of metalepsis as **'in-between-ness'**. The mediator is between the 'outside' and 'inside', between the 'quality' and 'quantity', between the 'time' and 'place', between the 'image' (bild) and the 'concept' (begriff). The tradition of the thinking 'in-between', culminated most generously in Hegel, could be expressed in the 'language' of mathematical models in epistemological term that emanate the notion of topological cobordism (Rene Thom's linguistics and topology). In Hegel's dialectical and logical 'cobordismä, something has its determinate being outside (or, as it is also put, on the inside) of its limit; similarly, the other, too, because it is a something, is outside [its limit]. Limit is the middle term [die Mitte] between the two of them in which they cease. They have their determinate being beyond each other and beyond their limit; the limit as the non-being of each is the other of both. (Miller A. V., 1990, p.127)

**Vermittlung (Mediation)** is a term associated with the dialectical action of consciousness on its object. *Vermitteln* = to mediate. *Mittel* is "means." *Unmittelbar* (adj.) is "immediate." To mediate something is to gain it or affect it through an intermediate agency or condition, to attain it not directly or "immediately" but through a means or medium that is not directly the thing itself. Consciousness always comes to the object as mediated, as existing in its own medium—in the medium of itself—because even when consciousness apprehends the object in its "immediacy" (*Unmittelbarkeit*), it is actually apprehending something already formed through an earlier stage of consciousness. Thus "immediate" and "mediate" become relative terms within the totality of the self-movement of consciousness. [465]

### 1.4. The topological notion of Hegel's "die Mitte"

Gadamer'*s concept of* the middle of language ***("die Mitte der Sprache")*** emphasizes topological notion of Hegel's ***Die Mitte.*** The topological notion of Hegel's "*die Mitte"* is subject of Rod Coltman's discussion on Gadamer's claim - "language is the middle *[die Mitte]*.

[465] Donald Phillip Verene, 2007, Hegel's Absolure: An Introduction to Reading the Phenomenology of Spirit, State University of New York Press, Appendix,

For Gadamer as for Heidegger, understanding, rather than being some mechanical activity of the mind, is *Dasein's* mode of being-in-the-world. Gadamer focuses not on being itself or human being per se, but on human *being-conscious* as an event, a process, a temporal and historical phenomenon that occurs in and through the "middle of language." For Gadamer, language is a "middle" – after Heidegger - the Middle of Language is The Middle of Being.

This claim is roughly analogous to the middle term in a logical syllogism in a Hegelian dialectical transition. On one hand, *Dasein* is essentially linguistic and temporal and, therefore, finite. On the other hand, being and understanding always occur within a linguistic milieu, from within or out of the middle of language *("die Mitte der Sprache"* ). Hence precisely at this moment of the most extreme distanciation from his famous mentor Gadamer, too, thinks the *Wesen* of both being and language in essentially **topological terms**."

The topological notion in Heidegger's works on language and poetry is pointed out by Coltman, who makes remarks how Heidegger provides us with repeated invocations and metaphors around particular words and phrases such as "path," "clearing," "dwelling," "the house of being," and even "the regions" of the fourfold) which evoke images of places, locations, sites. For Coltman "these **"topological" words** and phrases, however, are not to be construed as metaphorical constructs, but as various ways in which being reveals (and conceals) itself in language. They indicate, in other words, not the dialectical movement of concepts but various places in which we might discern the movement or the turning *(die Kehre)*onemight even say the counter-turning *(die Gegenwendung) in* being itself." [466]

In his book "The Language of Hermeneutics, Gadamer and Heidegger in Dialoque" [467], Rodney Coltman offers an excellent discussion on Gadamer's understanding of human finitude and Hegel's notion of interrelatedness of finitude and infinitude.

The idea that as humans we find ourselves within history, and that (in opposite to Hegel) "being historical means never being entirely exhausted in self-knowing", Gadamer develops in the middle third of his "Truth and Method". For Gadamer all human experience is at once

[466] Rod Coltman, The Language of Hermeneutics, Gadamer and Heidegger in Dialogue, p.120
[467] Rod Coltman, The Language of Hermeneutics, Gadamer and Heidegger in Dialogue, 1998, State University of New York, p.97-98

historically conditioned and linguistically constituted, thus "experience," according to him "is experience of human finitude."

According to Rodney Coltman [468] “at a particular moment in the "Logic of Being," in fact, we discover that finitude contains within it the infinite and vice versa. Having moved through the transitions from "being" to "nothing" to "becoming," the reader of the *Logic* finds him- or herself in the sphere of "determinate being" or *Dasein. Dasein* shows itself to be *a* determinate being or "something" insofar as its two qualities, "reality" and "negation," are reflected back into it. And yet something, as the negation of a negation, the mediation of itself with itself, is simply another form of *being* and, therefore, *in itself,* is also *becoming,* "which, however, no longer has only being and nothing for its moments." [469]

This first moment, the being of something, is now a *determinate* being and its second moment "is equally a *determinate* being, but determined as a negative of the somethingan *other."* [470]

For Hegel both something and other, therefore, are initially determinate beings or somethings. The "the mediation through which something and other each as well *is,* as *is not,"* Hegel calls "limit." [471]

As something *is not* its other, and the other (as it is also something) *is not* the something that *its* other is, each is equally determined to be what it *is* in *not being* its other. And this non-being of the other (which is also the nonbeing of itself) is the "ceasing-to-be" or the "limit" of something in its other.

In his explanation, Hegel employs term *"die Mitte"* (The German for ‘the middle'). [472] [473]

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[468] Rod Coltman, The Language of Hermeneutics, Gadamer and Heidegger in Dialogue, 1998, State University of New York, p.97-98

[469] Hegel's Science of Logic*,* trans. A. V. Miller, trans., Atlantic Highlands: Humanities, 1990.
Original German Text: Wissenschaft der Logik I, Frankfurt: Suhrkamp Verlag, 1970., 116.

[470] Rod Coltman, The Language of Hermeneutics, Gadamer and Heidegger in Dialogue, 1998, State University of New York, p.97-98

[471] Hegel's Science of Logic, trans. A. V. Miller, trans., Atlantic Highlands: Humanities, 1990. Original German Text: Wissenschaft der Logik I, Frankfurt: Suhrkamp Verlag, 1970., 124.

[472] Hegel's Science of Logic, trans. A. V. Miller, trans., Atlantic Highlands: Humanities, 1990. Original German Text: Wissenschaft der Logik I, Frankfurt: Suhrkamp Verlag, 1970., 127.

[473] The German for ‘(the)’ middle’ is ( die ) Mitte. This generates an adjective mittel (‘middle’) and another noun ( das ) Mittel (originally ‘(the) middle, the thing in the middle’, but now ‘means, what serves the attainment of a purpose ’). It also generates several verbs, especially mitteln (‘to help someone to, to settle, mediate’, e.g. a

In Hegel’s use of "die Mitte" we could see the topological notion of betweeness, the mediator between the outside and inside. In this epistemological term and it use by Hegel we could see and seek an interpretation of Hegel’s Logic toward cobordism. Here lies Hegel’s dialectical and logical cobordism - something has its determinate being outside (or, as it is also put, on the inside) of its limit; similarly, the other, too, because it is a something, is outside [its limit]. Limit is the middle term [die Mitte] between the two of them in which they cease. They have their determinate being beyond each other and beyond their limit; the limit as the non-being of each is the other of both. [474]

Rodney Coltman asserts that “the limit is itself only a preliminary moment here. For we quickly find ourselves involved in yet a further determination of "something": "Something with its immanent limit, posited as the contradiction of itself, through which it is directed and forced out of and beyond itself, is the *finite."* [475] Something, therefore, becomes determined as "finite" in that both its being and its ceasing-to-be are equally determinate and thereby come into contradiction. In the "unrest" brought about by its inability to contain the contradiction within itself *as a contradiction,* something is impelled out *beyond* itself. Hence the common notion of finitude, the conception of finitude that the understanding holds on to: A thing is finite insofar as it is limited and its very limitedness necessitates the thought of something beyond its limit.

For Coltman, “this way of thinking finitude, of course, appears to make perfect sense, and Gadamer's thinking of human finitude trades off this to a certain extent.” Here in underline note, Coltman poits out that “In fact, at one point Hegel seems to come rather close to Heidegger's thinking of Dasein's finitude in *Being and Time* as "being toward-death."

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quarrel), which is now obsolete but has left mittelbar (‘mediate, indirect’) and unmittelbar (‘immediate, direct’), and vermitteln (‘to achieve union, mediate; to bring about’, etc.). The past participle of vermitteln, vermittelt (‘mediated, indirect’) is used in contrast to unmittelbar. Both give rise to abstract nouns, Vermittlung (‘mediation’) and Unmittelbarkeit (‘immediacy’). In non-Hegelian philosophy, unmittelbar is primarily an epistemological term. Immediate certainty is a certainty that is not mediated by inference or proof, or perhaps even by symbols or concepts.

[474] Hegel's Science of Logic, trans. A. V. Miller, trans., Atlantic Highlands: Humanities, 1990. Original German Text: Wissenschaft der Logik I, Frankfurt: Suhrkamp Verlag, 1970., 127.

[475] Hegel's Science of Logic, trans. A. V. Miller, trans., Atlantic Highlands: Humanities, 1990. Original German Text: Wissenschaft der Logik I, Frankfurt: Suhrkamp Verlag, 1970., 129.

"Finite things are not merely limited," he writes, on the contrary, *non-being* constitutes their nature and being. Finite things *are,* but their relation to themselves is that they are *negatively* self-related, and in this very self-relation send themselves away beyond themselves, beyond their being. They *are,* but the truth of this being is their *end.* The finite not only alters, like something in general, but it *ceases to be,* and its ceasing to be is not merely a possibility, so that it could be without ceasing be, but the being as such of finite things is to have the germ of decease as their being-within-self: the hour of their birth is the hour of their death.[476]

Coltman makes an important note, here restating Hegel – "we should remain alert to Hegel's admonition that "Finitude is the most stubborn category of the understanding" [477]

In progress of his discussion Coltman establishes that:

"For Hegel, however, thinking finitude in this way is not "wrong" it just remains underdetermined, incomplete. It fixes the negation of something in itself, "and it therefore stands in abrupt contrast to its affirmative." The understanding, that is, while insisting on the determination of finite things as their inherent ceasing-to-be, reifies this determination, rendering it "imperishable and absolute." In other words, the understanding persists in viewing that which is inherently destined to die as nonetheless eternal. However, it also places the finite in irreconcilable opposition with its other, "the infinite," and thus "being, absolute being, is ascribed to the infinite". Thus, both the finite and the infinite are seen to be absolute. But, as we continue to think through the concept, this hypostatizing move shows itself to be untenable. [478]

…The infinite wants to be unlimited, but it is inherently limited insofar as it is not the finite. As Hegel points out, there are now "two determinatenesses; there are two worlds, one infinite

[476] Hegel's Science of Logic, trans. A. V. Miller, trans., Atlantic Highlands: Humanities, 1990. Original German Text: Wissenschaft der Logik I, Frankfurt: Suhrkamp Verlag, 1970., 129.
[477] 17 Hegel's Science of Logic, trans. A. V. Miller, trans., Atlantic Highlands: Humanities, 1990. Original German Text: Wissenschaft der Logik I, Frankfurt: Suhrkamp Verlag, 1970., 129.

[478] Rod Coltman, The Language of Hermeneutics, Gadamer and Heidegger in Dialogue, p.99-100

and one finite, and in their relationship the infinite is only the limit of the finite and is thus only a determinate infinite, an infinite which is itself finite." [479]

What the understanding forgets is that these alternating determinations of the finite and the infinite are ultimately (as indeed they have been all along) determinations of one unified something. And to view the finite and the infinite as two somethings (as it must inevitably do if it persists in this line of thought) is merely to reiterate, ad infinitum, the finitude of the infinite. The understanding overlooks the fact that "they are inseparable, but their unity is concealed in their qualitative otherness, it is the inner unity which only lies at their base." [480] It fails to realize that this very unity, "is itself the infinite which embraces both itself and finitudeand is therefore the infinite in a different sense from that in which the finite is regarded as separated and set apart from the infinite." [481]

This second determination of the infinite, in which both the finite and the infinite sublate themselves, the infinite in which the unified finite and the unified infinite are but moments, shows itself to be the "other side," as it were, of the "spurious" and one-sided "infinite progress" of the two prior unities that is carried forth in their simple alternation.

This is not to say, however, that this new unified, "affirmative" infinite will suffice on its own either. It is only in the very process that we have been trying to elaborate here that the so-called "true" infinite emerges. After rehearsing the "double meaning" that the finite had acquired in the foregoing explication (i.e., that it is both finite and infinite at the same time), Hegel explains that The infinite, too, has the double meaning of being [first] *one* of these two momentsas such it is the spurious infiniteand [second] also the infinite in which both, the finite and its other, are only moments. The infinite, therefore, as now before us is, in fact, the process in which it is deposed to being only *one* of its determinations, the opposite of the

[479] Hegel's Science of Logic, trans. A. V. Miller, trans., Atlantic Highlands: Humanities, 1990. Original German Text: Wissenschaft der Logik I, Frankfurt: Suhrkamp Verlag, 1970., 139-140.
[480] Hegel's Science of Logic, trans. A. V. Miller, trans., Atlantic Highlands: Humanities, 1990. Original German Text: Wissenschaft der Logik I, Frankfurt: Suhrkamp Verlag, 1970., 141.
[481] Hegel's Science of Logic, trans. A. V. Miller, trans., Atlantic Highlands: Humanities, 1990. Original German Text: Wissenschaft der Logik I, Frankfurt: Suhrkamp Verlag, 1970., 144.

finite, and so to being itself only one of the finites, and then raising this its difference from itself into the affirmation of itself and through this mediation becoming the *true* infinite.” [482]

Coltman concludes that „the true infinite, then, is neither the spurious infinite nor the affirmative infinite alone, but the *process* itself in which the former mediates its own transition into the latter. And, as such, the true infinite shows itself to be a further determination of *becoming.”* Coltman closes the circle of his discussion going back from Hegel to Gadamer:

“Even in such a brief sketch we can see that in the one-sided logic that constitutes the alternation of the finite and the infinite, back and forth "on to infinity," the understanding tries (and fails) to arrest the movement of the concept which, if allowed to unfold in terms of its own dialectical *Auslegung,* logically sublates itself in the infinite. Thus from the perspective of the "Logic of Being," Gadamer's insistence on a "simple limit" to human experience appears to be merely an artifact of the perspective of external reflection. But we must remember that, although (as with Hegel) reflexive thinking per se is problematic for Gadamer, it makes no sense to speak of it as "external." External to what? Reason? But insofar as language is at least the medium of both reflection and speculative reason, this seems to imply that language itself is "external" to reason. But reason, Gadamer would say, depends on language just as much as the understanding does. This is not to denigrate reason as such, but against the transcendental philosophy of the Enlightenmentwhich both Hegel and Gadamer criticize, and which claims a certain degree of independence from the words through which it is expressed Gadamer wants to reassert the universality of language..” (*Gadamer’s “Language is the language of reason itself*”) [483]

We should return to the issue of the use and the topological meaning of *"die Mitte"* in Hegel and Gadamer, following Coltman’s discussion and focusing this issue on the Gadamerien finitude and Hegelian infinity.

[482] Hegel's Science of Logic, trans. A. V. Miller, trans., Atlantic Highlands: Humanities, 1990. Original German Text: Wissenschaft der Logik I, Frankfurt: Suhrkamp Verlag, 1970., 148.
[483] Rod Coltman, The Language of Hermeneutics, Gadamer and Heidegger in Dialogue, p.99-101

For Gadamer the issue of human finitude is related with and embedded in the language. In the opening paragraphs of "The Middle of Language and its Speculative Structure," Gadamer reiterates two primary moments in the development of philosophical hermeneutics: namely, that "the alldetermining ground [of the hermeneutic phenomenon] is the *finitude of our historical experience"* and that "language is the mark of human finitude." Hegel's notion of *"die Mitte",* Gadamer develops in the language and dialoque. Coltman points out that:

"We see (here) a striking difference from Hegel's usual employment, in the *Logic,* of *"die Mitte"* as the logical "middle term" between two polar "extremes" that allows for the sublation of these opposites. Of course, as we have already pointed out, Gadamer is speaking here of what he will call "dialectic" as it occurs in living dialogue between two people (or between a person and a text) rather than of a strictly logical dialectic based on the grammar of the speculative proposition. But this is precisely the sharp divergence that will allow Gadamer to maintain a certain connection to Hegel. (And we should by no means assume that Hegel's formal use of the "middle term" will go unexploited by Gadamer.)

The first occurrence of *"die Mitte"* comes at the very end of the second division of *Truth and Method* as Gadamer makes the transition from "historicity" *Geschichtlichkeit)* to "linguisticality" *(Sprachlichkeit).* "Every conversation," he writes, "presupposes a common language, or better: it develops a common language from within. This is because something is laid down in the middle *[in die Mitte niedergelegt]...* in which both conversation partners participate and about which they exchange with one another."

At this point in the text we do not yet have a direct identity between language and this "middle" between interlocutors. In the next subsection, however, in fact on the very next page, we find that "Language is the middle *[die Mitte]* in which the coming-toanunderstanding of the partners *[die Verständigung der Partner]* and agreement about the subject matter *[die Sache]* carry them." [484]

For Coltman, "Gadamer is not pretending to identify Hegel's critique of finitude and the so-called spurious or bad infinite as a point where the "Logic of Being" breaks down. ..

[484] Rod Coltman, The Language of Hermeneutics, Gadamer and Heidegger in Dialogue, p.120

Gadamer's preference for the bad infinite indicates rather his own Heideggerian orientation toward the inescapable historicity and the radical finitude of human..”

**One of the strongest presence of topological cognition in Hegel is his use of *Vorstellung (Picture-thinking, Figurative thinking)***

The visual side of topological metaphorical thinking and reasoning in Hegel is presented in the therm Vorstellung (picture-thinking, figurative thinking)[485] Verene points out that “This is Hegel’s contrasting term to thought done in the true philosophical form of the concept (*Begriff*).”[486] Hegel also characterizes the stage of consciousness that he calls “Religion,” the stage immediately before that of “Absolute Knowing,” as a stage in which thought has the shape of *Vorstellungen* (picturethoughts). Verene asserts that “*Vorstellung* is not easily translated by any single English term that will suit all or most contexts. It is a flexible and easily used term in German.

The verb *vorstellen* literally means “place before” (*vor* = before; *stellen* = to place)[487] and directs to the topos (place), having a topological characteristic.*Vorstellen* is to present, represent, put forward, mean, signify; as a reflexive (*sich vorstellen*) it is to imagine, suppose, conceive. *Das Vorstellen* can be used to mean “the imagination”. *Vorstellung* is the term used for a performance, as a theater performance. It can mean: imagination, idea, notion, conception, mental image. In my view, the term strongly suggests the idea of topological theatre as the performance of imagination, idea, notion, conception, mental image. In the preface (par. 58) Hegel calls the habit of thinking in terms of *Vorstellungen* a kind of “material thinking,” a kind of contingent consciousness absorbed in material stuff. *Die Vorstellung* is placing something before the mind or consciousness as an image, a picture, a presentation, a vague idea. It is to be actually thinking a thought (as opposed to merely having a sensation or perception), but to be thinking the object in less than a true conceptual form (as understood in Hegel’s sense).

Topological notions bears Hegel’s ***Erinnerung (Recollection).*** *Erinnerung*, like recollection, is not simple remembering or mental retention but the power to recall what is already in

[485] Donald Phillip Verene, 2007, Hegel’s Absolure: An Introduction to Reading the Phenomenology of Spirit, State University of New York Press, Appendix, p. 117

[486] p. 118
[487] p. 118

consciousness and make it known again. “Recollection” does not contain in its construction as a word the sense of “inner” that is obvious in the German, *Erinnerung*. On the final page of the *Phenomenology*, Hegel uses *Erinnerung* four times in four different ways to summarize the standpoint of the *Phenomenology*, at one point hyphenating it as *Er-Innerung* to bring out the sense of the inner (*inner*, adj.; *das Innere*). Recollection is a way consciousness acts toward itself within itself, and in this “innerness” it is analogous to the dialectical selfmovement wherein consciousness brings forth a knowledge of its own being. From time to time Hegel uses the verbs *erinnern* and *sich erinnern* (reflexive) and at crucial points speaks of consciousness “forgetting” itself and the path of development it has traveled.

## 1.5. Topological notion of the triadic conception of Hegel’s dialectic and Hegel’s dialectical juxtapositions

Donald Phillip Verene, while discussing the so called “double ansich” in Hegel, states that Hegel has a dialectic of thesis-antithesis-synthesis, or something like this three-step movement. Related to this claim is the concern that Hegel uses the German verb “aufheben” as a doctrine of synthesis. Yet, Verene asserts, that “No first-rate Hegel scholar speaks of Hegel having a dialectic of thesis-antithesis-synthesis.” (Verene 2007:17).[488]

Verene reminds that, In 1958 Gustav Muller, in an article, “The Hegel Legend of Thesis, Antithesis, Synthesis,” pointed out that Hegel never uses these three terms together to describe the dialectic. Something more, Hegel attributes triadic form to Kant as a lifeless schema. Muller asks how this legend of the Hegelian triad came about, since it is not something one would naturally conclude directly from Hegel’s text. He traces the likely origin to some lectures given several times by Heinrich Moritz Chalyвдus, who shortly after Hegel’s death interpreted the “new philosophy” using the triadic formulation from Kantian and Fichtean philosophy. He gave these lectures on one occasion in Berlin, at which time, Mьller says, Karl Marx may have been present, as a student. But the myth persists, conveyed to students in survey courses in the history of philosophy. It is commonly held, both within and outside the field of philosophy, by those who have only a second-or third-hand awareness of Hegel.

[488] Donald Phillip Verene, 2007, Hegel’s Absolure: An Introduction to Reading the Phenomenology of Spirit, State University of New York Press, Preface 17

Verene states, that "Beyond this specific, historical origin, the triadic form of Hegel's dialectic is often embraced by those who have approached Hegel's system in the reverse of its own course of development. The most accessible work of Hegel is the *Encyclopaedia of Philosophical Sciences in Outline*. This has three parts: the logic, philosophy of nature, and philosophy of spirit, and the subdivisions of these parts are also triadic.

Many British Idealist interpreters have approached Hegel's system from the *Encyclopaedia*. The book of W. T. Stace, *The Philosophy of Hegel: A Systematic Exposition*, is rigidly based on this, making it a work of easy access to Hegel for generations of English-reading students. It must be kept in mind that Hegel wrote the *Encyclopaedia* as a handbook for teaching philosophy to young students. It presupposes the dialectic of consciousness in the *Phenomenology*, in which philosophical knowing is itself generated for the philosopher. (Verene 2007: 18)

The triadic conception of Hegel's dialectic has been closely interwoven with his use of *aufheben*. *Aufheben* has been a doctrine of great importance for those who have a more sophisticated version of the dialectic than the mechanism of thesis-antithesis-synthesis. *Aufheben* is an untranslatable German verb made from the ordinary verb in German, meaning "to raise," or "raise up," "lift up"—*heben*, with the prefix *auf*. *Aufheben* is often translated in Hegelian works as "to sublate"; in Miller's translation it is rendered as "supersede" (*Aufhebung*, "sublation"). *Aufheben* can mean two things at once: "to cancel or transcend" and "to preserve." It is in this double sense that Hegel most often uses it. How seriously are we to take Hegel's use of *aufheben*? (Verene 2007:18)

According to Verene, "Many commentators have taken it very seriously and have found in it a doctrine of how one stage of consciousness originates from the next on the pathway toward the absolute. This has produced what I will call the "snowball" theory of the absolute." (Verene 2007:18) Verene notes that "Some recent writers on Hegel, such as John H. Smith in *The Spirit and Its Letter*, have suggested that we have taken *aufheben* too seriously, and that it has an ironic connotation in the *Phenomenology*." For Verene, this is an interesting suggestion, and it fits with Hegel's general use of the trope of irony in the Hegel's *Phenomenology*. The ironic interpretation fits with the fact that in the *Phenomenology* one stage of consciousness seems just to collapse, and consciousness jumps straight to the next stage. There is no clear sense of progression or transformation, no sublation.

The *Phenomenology* is in essence a scene of ironic juxtapositions that lead up to absolute knowing. (Verene 2007:19) The snowball theory regards the *Phenomenology*, and Hegel's system as a whole, as a progression of forms in which each stage is taken up into the next and preserved in its differentiations. Like a snowball rolled across a lawn, it starts small and gets bigger and bigger until, in this view of Hegel's stages, we have a whole. The snowball theory is strong on the preservation or "taking up" meaning of *aufheben*. The ironic view does more justice to the sense of cancellation and transcendence. The ironic view is certainly more interesting, and likely closer to a true reading of Hegel's text. Me: What Verene calls – "Hegel's dialectical juxtapositions, actually is topological juxtaposition.

For Verene, "Hegel's dialectical juxtapositions is "like those in an ironic statement" that "are based on the power of human wit or *ingenium*, an ability well-known to the Renaissance humanists but lost to modern thinkers who, from Descartes on, approach thought in terms of method." (Verene 2007:19). For Verene, "*Ingenium* (ingenuity, wit) is the ability to see a connection between two seemingly diverse things. It lies behind the creation of hypotheses in the sciences and behind the tropes of metaphor and irony in the poetical and rhetorical use of language. Necessity is the mother of invention, and invention depends upon the power of *ingenium* to see new connections. Ingenium cannot be taught as such. It is unlike method, which can be taught. *Ingenium* may be grasped and developed through examples, but it is not subject to theoretical instruction. Hegel's dialectical method, like Socratic method, is not properly a method. Dialectic is a direct exercise of *ingenium* to connect diverse moments of experience. Hegel and Socrates show how this can be done. Once Hegel's dialectic is seen as an exercise of *ingenium* and not as a peculiar form of deduction, the reader can relax and set about to see what Hegel sees.

In *Hegel's Absolure: An Introduction to Reading the Phenomenology of Spirit*, Verene provides three suggestions for reading Hegel's Phenomenology yet these three suggestions are relevant in general for Hegel's text, in particular Hegel's Logic.

As Verene states "the first (suggestion) derives from principles of rhetoric, the second from Hegel's conception of language, and the third from what I believe to be the general nature of philosophical texts." (Verene 2007:xiv-xv).

First, Verene suggests that "Each section of the work should be read three times: first to grasp the meaning of the section as a whole, second to grasp the transitions from point to point within its thought, and third to grasp the particular phrases or modes of expression used to make its central ideas comprehensible and memorable." Verene states that "these three readings correspond to the three classical principles of composition in rhetoric as found in the Roman textbook on rhetoric of Quintilian: *inventio*, the discovery of materials; *dispositio*, their arrangement; *elocutio*, their formulation in language (III.3). Any work is composed in this way, whether or not its author is conscious of these principles. They are natural aspects of composition. No work can be complete that does not have these elements. Thus any work can be read so as to grasp each of them as a level within it. A reader approaching a work in this manner will come away from it with a good comprehension of its contents. . . . Philosophical works are like songs; their meaning is revealed in repetition. They must be heard over and over again." (Verene 2007:xiv-xv).

Second, Verence asserts that "Hegel's language has been thought to be dense, obscure, and full of special terms. Hegel wrote to Johann Heinrich Voss (May 1805) that Voss had made Homer speak German, Martin Luther had made the Bible, and now he, Hegel, intended to make philosophy speak German. Immanuel Kant writes a Latinized German. Hegel intends to write in German the way Dante chose to write in Florentine Italian instead of Latin.

None of Hegel's famous terms—*Geist, aufheben, Begriff*—is a technical term. They all are words of ordinary German. Some of them happen to be terms deeply grounded in the *Weltanschauung* of the German language and cannot be easily rendered into English. … Hegel's approach to philosophical speech is Socratic. Like Socrates, he takes words of ordinary language and begins to press their meanings toward further meanings by developing them into new contexts, all the while using the words any German speaker knows. How Hegel accomplishes this transformation of meanings can be seen by consulting the remarks on Hegel's terms in the appendix. Hegel always speaks in the *agora*, although what he is speaking about, as he says, will cause common sense to walk on its head.” (Verene 2007:xiv-xv).

The third suggestion offered by Verence is concerned with Hegel’s thriple sequence of expression. For Verene, “Hegel, like most great philosophers, says everything three ways. First he will make a point in purely intellectual ordiscursive terms, perhaps using terms such as “in-itself” and “for-itself,” or assertions such as “Substance becomes Subject.” Second, he will often use a common-sense example or analogy, such as saying that zoology is not the same as “all animals.” Third, he will use metaphor and irony, such as “the night in which all cows are black,” the “Bacchanalian revel,” or the “Golgotha of the Spirit.” His modes of expression will pass back and forth among discursive, commonsensical, and tropic.” (Verene 2007:xiv-xv).

**The first language of infinity is the image! The image is the form of recollection.** Verene emphasizes the role of the image in philosophical text, stating that “Any philosophical text depends upon images; they are always present. The reader can look first not for arguments in the work but for these root images. Once found, the reader can look for the questions that can be drawn forth from the images. The reader will then see how the image is directing and providing support for the question, which carries the reasoning process of the text forward. What are the images? What are the questions embedded in them?” (Verene 2007:xiv-xv).

### 1.6. The topological development of language, categories and concepts into the Qualitative quantity in Hegel’s “Encyclopedia of Philosophical Sciences”, Part One, referred to as The Lesser Logic

From the two circles of the *Pure Being* and *Pure Nothing*, Hegel constructs the triad of Becoming just to deconstruct it, deriving the Determinate Being (Quality), demonstrating the move from Becoming (the Determinate Being) to the Determinate Being (Quality).

Next step on the stage is the two fold interplay between *Quality* and *Negation*, where from their middle Something appear and we have the new triad constructed between Quality, Negation and Something, just to dissolve it, moving to Something/Other. Here again, the interplay is twofold entity between *Something/Other* and *Being for Other,* where Being for Other is also new posited as Being-for-other/Being-for-itself.

In order to move to the new triad, here Hegel includes the Determination between Something/Other and Being-for-itself. The triad of the Determination of the In-itself is done, just to be deconstructed again – excluding the middle intersection from the triad and moving to Constitution.

Then *Constitution* is opposed to *Determination* yet these two are in relation of the two fold again. From the relation between Constitution and Determination, the third is born – the Limit (Determinateness as Such).

The new reborn Determination as Determinateness as Such is the crown of the new triad, posited as Limit. The crown of the Limit is down in the emerged triad of Finitude.

Finitude is born from the toposes, the middle intersection between Constitution, Determination and Limit. Limit is in play with Limitation. The two spheres have something in common, their middle is the seed of Ought, and the new triad is here, constructed between the Finitude, Limitation and the Ought.

Again deconstruction is active and Hegel derives from this deconstructed triad the Enriched Finitude, born from the topos of the tree components. The new dual structure between *Enriched Finite* and *Another Finite* is established, just to lead us to Infinity.

Infinity is the crown of the interplay between Enriched Finite and Another Finite. The middle three intersections of this triad move to construct the Spurious Infinity.

Again the Spurious Infinity is involved in the twofold game with Another Finite. The posited two are enriched with the True Infinity and the triad of True Infinity is constructed between the Spurious Infinity, Another Infinity and True Infinity. Here the journey of Being-for-self begin from the transformed Spurious Infinity as part of the triad.

Again the twofold interplay is witnessed between the *Being-for-self* and *Being-for-other*. This double enclosure is named Being-for-one. From here, Hegel builds the new triad, the triad of One (Being-for-self,  Being-for-Other, The One). From this triad the shadow of new entity emerge – the One in its own self.

The dual structure of interplay between the *One* and the *Void* emerge and Repulsion is included. The very new triad of the One/Many Ones, the Void/Many Ones and the Repulsion is done.

The pathway of Attraction is derived from the middle intersection and relation between the One/Many, the Void/Many and Repulsion, and from the Repulsion itself. *Attraction* and *Repulsion* are here in the double action. From their interrelation the new triad is born, the triad of Quantity. The triad of Quantity is constructed by Quantity – Attraction/One – Repulsion/Many. This triad is the nest of Continuity.

Continuity is in move, implementing in its journey the middle again. The middle between the three circles from the triad of Quantity needs new face, new role and get this through the interplay between Continuity and Dicreteness, their middle. From the topos between the Continuity and Discreteness springs the Enriched Quantity, the head of the triad embedding the Continuity and Discreteness.

The very new face and role of Quantity here is the Magnitude. Magnitude is constructed from the toposes of Continuity and Discrteness and the Enriched Quantity. The name of this newborn Quantity is the Continuous Magnitude.

The Continuous Magnitude is free from the old triad of Enriched Quantity and now face his counterpart – Discrete Magnitude. The topos of the two Magnitudes is the seed of new triad, the triad of the Quantum, from which the Amount is derived, constructed from the toposes of the three. Amount is not left untouched and the twofold interplay between the Unit and Amount is here. Their topos introduces the new triad of Number – The Number – Amount – Unit. From the common intersection of these three circles, the Extensive Magnitude (Extensive Quantum) emerges.

The Extensive Magnitude meets its counterpart - the Intensive Magnitude, and Degree is here as their topos. The topos between the Extensive and Intensive Magnitude establishes the new triad, the triad of Quality of Quantum.

Intensive Magnitude (Degree) is born again from the next triad, from the toposes of the previous three. In contrast with the Intensive Magnitude (as twofold between Extensive and Intensive Magnitude), here is the new twofold of Extensive Magnitude which combine the Degree and Extensive Magnitude. The topos of the last two leads to the Qualitative Something.

The threefold of Qualitative Something give birth to the Quantitative Infinity. Quantitative Infinity will play with the Quantitative Infinity Progress and produce from their common new triad, where the head will be Infinitely Great/Small. With this new triad, Hegel presents the Direct Ratio.

And again, the Direct Ratio will be involved in twofold action with Inverse Ratio. The product from their between will be Ratio of Powers. These three ratio will constitute the new triad of Ratio of Powers. This triad will lead to the Immediate Measure.

Immediate Measure and Mediated Measure will form new relation, the triad of the Mediated Immediate Measure. Immediate Measure and Mediated Measure impose a topos, the new triad of Specifying  Measure. From Specifying Measure the move of the Rule is established.

The Rule as Limit and Specifying Measure as Amount construct the Rule Measuring its Other. From this double the new topos is born – The Ratio of Measures (Realized or Specifyed Measure).

The topos of the new triad will construct the Combination of Measures. Hegel sees here the new topological twofold of relations between the Externality of Measure and Measure as Series. From the intersection of these two the Series of Measure Relations appears. These two involve as third the Elective affinity and construct new triad from which Continuity of Affinity appears.

Again, the two - Continuity and Indifference Subtrate - presents the new double of Indifference of Affinity (Substrate). The topos of these two are the seed of the Nodal Line triad.

Hegel will deconstruct the triad of the Nodal Line, deriving from it the three circles of the Abstract Measureless moving from the dual interplay between Quality and Abstract Measureless. The topos of these two will introduce new triad, the triad of Infinity for Itself, from where Hegel will derive Absolute Indifference.

With the new born Hegel attracts again the ratio, this time as Inverse Ratio and will establish the relation between two – Inverse Ratio of Factors.

Finaly we arrive at the Essence constructed as triad containing the Absolute Indifference and Inverse Ratio.

The above presentation shows how Hegel construct true topological manifold from which the notion of multiplicity emerges.

In his speculative logic and semantic Hegel uses the pair of tropes, terms, categories in opposition within the twofold relation, where the emphasis is on the middle, the topos of the logical and semantological space between the tropes, terms, and categories. From this topos something new arises as meaning. This mode of developing both text and though can be recognized as a double-entendre. The **double-entendre** in Hegel's logic is recognized by Andrew Hass in his 'Hegel and the Problem of Multiplicity' (Haas, A. 2000. Hegel and the Problem of Multiplicity. SPEP Studies in Historical Philosophy. Evanston: Northwestern University Press).

As Haas suggests: "The particular quality of quantification, the reproduction and restoration is taken care of in the dialectic of qualitative-quantitative concept – and in Hegel, *multiplicity means the quantification of quality and the qualification of quantity, a multiplicity of the double-entendre, of the inevitable double-meaning.* (Haas, A. 2000: 113). [489]

[489] Haas, A. 2000. Hegel and the Problem of Multiplicity. SPEP Studies in Historical Philosophy. Evanston: Northwestern University Press

In the Science of Logic, Hegel develop his triads, deriving the new element for the each triad from the twofold pairs of tropes posited in opposition and distilating the new trope, term from the common place, the intersection between these two, from their topos. This rethorical mode used by Hegel represents not only the metonymy but double metonymy – two metonymies, one contained in the other, but only one expressed. For example from the twofold interplay between ***Quality*** and ***Negation***, from the middle of their relationship, from their topos, the semantological place between these, **Something** as the third trope and category emerge. When **Something** appears, we have the new triad constructed between **Quality**, **Negation** and **Something.** The third element emerges from the double meaning and the triads are constructed from the twofold relation between the two tropes, from their common space, from the intersection between the two circles.

**The figure of Metalepsis,** the trope Met-a-lep´sis is constructed from μετά (mĕta), behind, and λείπω (leipô), to leave, a leaving behind. The Figure is so called, because something more is deficient than in Metonymy, which has to be supplied entirely by the thought, rather than by the association or relation of ideas, as is the case in Metonymy. This something more that is deficient consists of another Metonymy, which the mind has to supply. Hence Metalepsis is a double or compound Metonymy, or a Metonymy in two stages, only one of which is expressed.[490]

The neigbourhood, the betweenness of Hegel's circles of logic, as developed from Understanding through Dialectical Reason to Speculative Reason, the twofold relation, both their double and their triple in the triads, can be represented with the model of Cobordism. The circle of circles morph and cobord each other.

Hegel is concerned not merely with the rules connecting the logical terms (i.e. syntax), but also the meanings of the terms themselves (i.e. semantics), conceptual syntax.

## 2. Picturing Hegel

[490] Bullinger, E. W., D.D. Entry for 'Metalepsis; or double metonymy'. Bullinger's Figures of Speech Used in the Bible. http://www.studylight.org/lexicons/fos/view.cgi?n=132.

Julie E. Maybee discusses the syntactic project of Hegel's logic. [491] In her describing of the construction of Hegel's logic, Maybee uses a metaphor of bricks or building blocks as one of two main components and as the other she offers the metaphor of the mortar that holds these bricks together. If the 'bricks' are the universal concepts as presuppositions of ordinary experimental consciousness on which empirical concepts depends, Maybee uses the metaphor of the mortar to address Hegel's syntax. As she explains "the logic also concerns itself with the development of the mortrar, or with the connective tishue of rational thought as well as the bricks."This connective tishue is syntax." (Maybee, 2009:34).[492] According to Maybee, "the syntax of Hegel's logic shows how reason can move from one concept to another concept that is connected or linked to it. It lays out the various syntactic devises that reason uses as it necessarily moves from one concept to the next in its development toward the unconditioned all-encompassing, universal or Absolute." (Maybee, 2009:34). In the Doctrine of Being, the primary dialectical relationship that concepts have with one another involves "passing over into another (§ 84). Concepts are defined by passing into their others.

If Carlson exhibits Hegel's logic in the form of pictorial triads of overlapping concepts (2003a: 93-101), Maybee in her *Picturing Hegel: An Illustrated Guide to Hegel's Encyclopaedia Logic* offers her own diagrams.

In Maybee's diagrams this "passing into" is represented by one-way arrows, but the logical relationships that emerge in the Doctrine of Being are much more complex, indeed topological. This topological nature of structure and syntax of Hegel's logic requires different type of visualizing through diagrams. My proposition here, is that the graphs and symplicial complexes would enhance the semantical side of Hegel' language and syntax of his Logic in accordance with topological notions implemented in categories.

For example, in Maybee's diagram and discussion of 'Being", Being (is the notion implicit only) where its special forms have the predicate 'is'; when these special forms are distinguished they are each of them an 'other': and the shape which dialectic takes in them, i.e. their further specialisation, is passing over into another. This further determination, or

[491] Julie E. Maybee. "Picturing Hegel: An Illustrated Guide to Hegel's Encyclopaedia Logic", Lexington Books,
[492] Julie E. Maybee. "Picturing Hegel: An Illustrated Guide to Hegel's Encyclopaedia Logic", Lexington Books, 2009, 34

specialisation, is at once a forth-putting and in that way a disengaging of the notion implicit in being; and at the same time the withdrawing of being inwards, its sinking deeper into itself. Thus the explication of the notion in the sphere of being does two things: it brings out the totality of being, and it abolishes the immediacy of being, or the form of being as such.

Being is the concept of bare presence. It includes no determinations of any kind, has no nature, or qualities of any sort. In her diagram of Being, Maybee represent Being with a dashed line because it has as of yet no content, quality or character, it is not really an 'empty' at all, but only a shadow. . . . figure 2.3 (in Maybee 2009: 34:35).[493]

Nothing – see p.51 – fig. 2.4. - - - me: interpreted as cobordism . . pair of pants…

Because Being has no character, determination, quality, nature, it is just as much Nothing as it is Being. The concept of Nothing is not a completely new idea, but rather a conclusion drawn about the concept of Being. Both Being and Nothing have the same empty content, since neither has any determination. The only difference between them is what is meant (§ 87). Being is meant as pure presence. Nothing is meant as pure absence. [494]

Hegel provides what he takes to be the proper logical definition of the philosophical concept of Being and Nothing. The basic logical definition of term Being is bare presence, and of Nothing is bare absence. Hegel credits the Parmenides for having introduced these concepts. Parmenides suggested that the world is One Being, with no motion or change in it. He distinquished the void or nothing, which as nothing, cannot exists.

With Being and Nothing, the speculative process of reason has generated its first contradiction or opposition.

There is only one way in which the logic can move forward from this point: it must characterize the two concepts together. Because neither Being nor Nothing has a content, their content cannot be combined into a new concept. Instead, they can be put together only as a

[493] Julie E. Maybee. "Picturing Hegel: An Illustrated Guide to Hegel's Encyclopaedia Logic", Lexington Books, 2009, 34:35
[494] Julie E. Maybee. "Picturing Hegel: An Illustrated Guide to Hegel's Encyclopaedia Logic", Lexington Books, 2009, 34:35

concept of the same, one, empty concept, first taken one way – as presence (Being) – then taken the other way – as absence (Nothing) – then taken the first way again – as presence – and so on.

Thus, the logic can proceed only by generating the thought of an endless flipping back and forth between an empty concept taken as presence, and the same empty concept taken as absence. This is precisely how Hegel will define Becoming in the next stage. [495]

When the logical elements in play have been exhausted under one strategy, then a new strategy must be found to move forward. Here, the logic began by defining Being and Nothing, each on their own. When the strategy was exhausted, however – where there was nothing further that could be said about Being and Nothing on their own – a logical impasse was reached. The new strategy involves character of Being and Nothing together, rather than each by itself. . . . . .(topological strategy of syntax – metalepsis.)

Becoming – page 53[496] in J Maybe – figure 2.5. – ((me: my diagram as torus with two arrows! Cobordism!!))

Following Maybee's discussion, Becoming is the concept that captures the only way of thinking about Being and Nothing taken together. Becoming is the concept of a flipping-back-and-forth process between the same empty thought taken as absence. It is the process of passing-away-and of bare absence (Nothing) into bare presence (Being). Hegel credits the Presocratic philosopher Heraclitus with having introduced the concept of Becoming. Hegel's concept of Becoming is a concept of constant flux without statis: to Become is to be constantly flipping back and forth between Being and Nothing.

The concept of Becoming is the first concrete concept in the logic, as Hegel apparently suggested. (§88). Being and Nothing are abstracted concepts. Becoming is the first concrete concept because unlike Being and Nothing has a determination or character. It is defined by the concepts of Being and Nothing – they give it the conceptual character that it has.

[495] Julie E. Maybee. "Picturing Hegel: An Illustrated Guide to Hegel's Encyclopaedia Logic", Lexington Books,
[496] Julie E. Maybee. "Picturing Hegel: An Illustrated Guide to Hegel's Encyclopaedia Logic", Lexington Books, 2009, 53

The strategy of Hegel in developing concepts of his logic, moving from abstract to concrete concepts, is topological – metalepsis – changing the strategy, using inclusion within the logical syntax, implementing topological syntax and rhetorical power of metalepsis.

Hegel himself cautions that Becoming is not a genuine unity (§88R4), by which he means that Becoming does not sublate Being and Nothing – it does not shallow them up or replace them as a higher-order concept. Being and Nothing remain independent of Becoming. Becoming as a concept is a process and never absorb Being and Nothing. According to **Julie E. Maybee, as she asserts, in her diagram drawing the Becoming is represented with arrows.**

Becoming is an endless process-dynamics which is instable, it defines the process but never the end of the process. The process is defined by both is (Being) and is not (Nothing). This is the moment of stasis. Becoming itself dies out or disappears (§89A). What is important is that the Becoming connects and relates the Being and Nothing, Being and Nothing are connected with one another.[497]

There are three logical elements in the logical development of the Doctrine of Being. There are three logical elements in the Doctrine of Quality: Being (the side of presence), Nothing (the side of absence), and Becoming (their relationship). The Doctrine of Quality explores these three elements step-by-step. First, each element is introduced and explored by itself, then the side of presence and absence are explored in step-by-step way corresponding to each move in the process of Becoming until the element have been exhausted. In the Doctrine of Quantity, the logical movement from Quantum through Degree exhaust the quantitative labels 'one' and 'many'.

In these stages (the One from the Doctrine of Quality) and two initial bits (the 'many' from the Doctrine of Quality) . . p.114-115 in Julie E. Maybee[498] . . . . .

**Me:to include Maybee's diagrams!! – figure 2.36**

[497] Julie E. Maybee. "Picturing Hegel: An Illustrated Guide to Hegel's Encyclopaedia Logic", Lexington Books, 2009, 34:35

[498] Julie E. Maybee. "Picturing Hegel: An Illustrated Guide to Hegel's Encyclopaedia Logic", Lexington Books, 2009, 114-115.

One ---------- Quantum: ‘one’ refers to outer boundary, ‘many’ as within.

1. One ---------- Neumber: ‘One’ on all sides.
2. Many --- Extensive and Intensive Magnitude: ‘many’ on outher, boundary, ‘one’ within.
3. Many **(within) ---- Degree: ‘many’ on all sides.**

Immediate Measure – Hegel must introduce a new element namely “quality”, which is not realy new, but is only new to this context. Here Hegel implements the idea that what is brought in must either be implied in or have been used in earlier stages.[499]

Maybee asserts that “a number of other syntactic devises have emerged” in the step-by-step process by which Hegel developes his logic, namely:

- a logical element first come into view in an indeterminate way (Being and Nothing) and then Becoming.
- a concept is defined at first as a set of other concepts (Becoming – Being and Nothing) and later comes to have a separated character of its own.
- Spurious Infinities keep eropping up, and are resolved by roughly the same move, namely, by a new “for-itself” concept that includes or grasps and hence negates or halts the passing – over or negation process in the spurious infinity (Being-for-itself) – Ratio – the Transition to Essence, for example). In each case, the new ‘genuine infinity” is a negation-of-the negation and a higher level. Or universality.
- a category is pushed as far as it can go before another category is brought in or reassearted, as in the stage of Ratio, where quality (the exponent) is tried out first as a way of  capturing the nature of the connection between two quantities. Only later is the category of quality brought in instead.

[499] Julie E. Maybee. "Picturing Hegel: An Illustrated Guide to Hegel's Encyclopaedia Logic", Lexington Books, 2009, ….

- a unity between two elements is at firt kind of gluing together but later the elements create their own unity (me: metalepsis), so to speak or unite themselves together by being defined in connection with one another. This is what happens to quality and quantity in the Doctrine of Measure. At first their inity is at immediate or given (in Immediate Measure). Later on, however, they fulfill their unity by being defined in relation to one another (in the Measurless). [500]

Maybee's diagrams from figure 2.36 (p.114-115) - stages (the One from the Doctrine of Quality) and two initial bits (the 'many' from the Doctrine of Quality) - can be presented through simplicial complexes. See figure . . . . . (to be included!)

Therefore, Hegel's logic has begun to reveal, repeating syntactic devices, which helps to undermine two criticisms of Hegel's logic. [501]

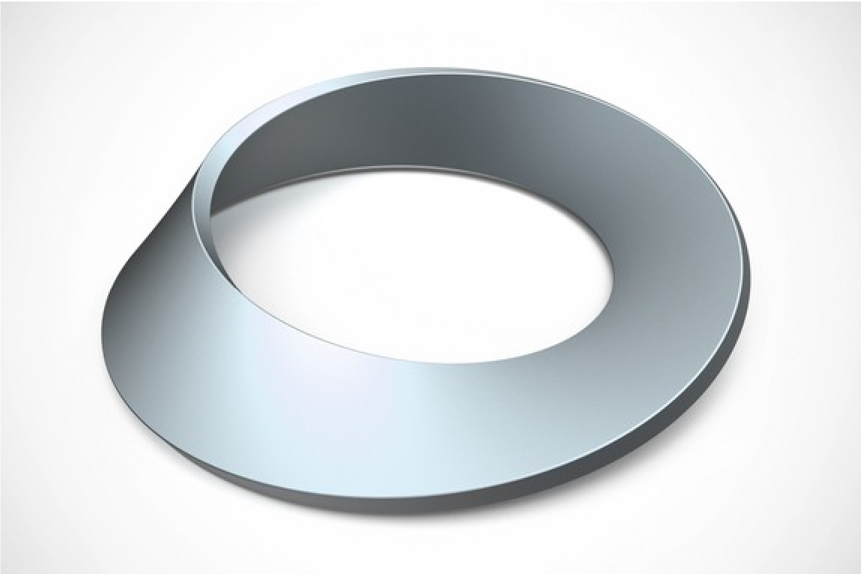
Hegel's logic has often been portraged as having one synctactic device, namely, the patern - thesis, antithesis, synthesis, according to which, same positive concept – the thesis – and its opposite – the antithesis – are 'resolved' into a synthesis that includes both. Our examination of the logic so far shows this old story to be both too simple and also somewhat false.

First, Hegel's logic reveals a variety of synctatic paterns, so that, even if this one could be applied to his work, if wanted at best be one syntactical devise among many. Second, this old story seems to misrepresent Hegel's syntactic patterns, even in places where it might at first seem to apply. Becoming, for example is not really a syntesis we are supposed to understand that two concepts have been implied, or mushed together, under a new concept. Becoming does not synthesize or mush together Being and Nothing. Being and Nothing remain independent of one another in Becoming, since Becoming is the thought of flipping back and forth between concept of Being and the concept of Nothing. In fact, Being is an independent element in the very new stage of Being-there, thought it remains connected to Nothing.

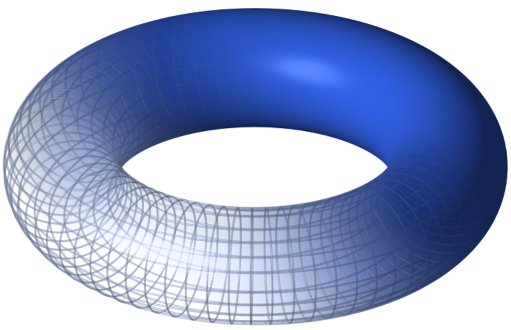
[500] Julie E. Maybee. "Picturing Hegel: An Illustrated Guide to Hegel's Encyclopaedia Logic", Lexington Books, 2009, 114-116.

[501] Julie E. Maybee. "Picturing Hegel: An Illustrated Guide to Hegel's Encyclopaedia Logic", Lexington Books, 2009, 114-116.

If Measure as the unity of Quality and Quantity, is supposed to be a ‘synthesis’, then the old story  misrepresent Hegel’s text too by suggesting that quality and quantity are opposites, such that one is the thesis, and one is the anti-thesis. There is no suggestion in Hegel’s text, in my opinion, that we should see quality and quantity as opposites. They are certainly different categories, and have different work to do in describing the world of Being. But they are not contradictory. Because the old ‘thesis-anti-thesis-synthesis’ story does, at best, a poor job of describing the syntactic patterns of Hegel’s logic.



A Mobius strip is the simplest example of a non-orientable surface.
Credit: Esben Oxholm | Shutterstock



A torus has one connected component ($b_0$), two circular holes ($b_1$,the one in the center and the one in the middle of the "donut"), and one two-dimensional void ($b_2$, the inside of the "donut") yielding Betti numbers of 1 ($b_0$),2 ($b_1$),1 ($b_2$).

## 2.1. Topological metalepsis (metonymy of metonymy) of Hegel

Hegel's language, syntax, concepts and notions have topological nature. The topological is presented in Hegel both as mathema (in his philosophy of mathemats) and as rethorics. If topological nature of Hegel's mathesis is presented within his fourfold of infinities (multiplicities) and Hegel's four type of judgments, Hegel's rethorics is bearing the four basic tropes of rhetoric: metaphor, metonymy, synecdoche, ironi as equally presented in his manifold (Mannigfaltigkeit) of infinities, quality and quantity, time and space.
In Hegel's philosophical narratives and semiology, the emphasis is on the metonymy seen as 'metonymy of metonymy' or 'metalepsis', presented in 'topological' notion of Qualitative quantity (The Quality of quantity).

David G. Carlson, in his Hegel's Theory of Measure, in particular in his discussion of (b) Measure as a Series of Measure Relations), states that "**Metonymy** is the theme of this new section's tongue. Metonymy is the inability to name the thing directly, but only the context of the thing. In metonymy, if the entire context is described, the unnameable thing becomes a ghostly space the existence of which is simply inferred from context." (David G. Carlson, 2003, Hegel's Theory of Measure:47)

## 2.2. Vorstellung (Picture-thinking, Figurative thinking) and Topology: Topological Seeing and Topological Vorstellung

Topology has conceptualized the thinking of human perception and our concepts of time and space, qualitative and quantitative, in different mode. Topology and Topological thinking has conceptualized pictorial thoughts and pictorial thinking.

Our visual system is sensitive to global topological properties. The extraction of global topological properties is a basic factor in perceptual organization. The perception of topological properties has the potential to serve as a unifying principle for visual functional lateralization thus topological properties are in fact primitive properties of object perception. Back in 1982 Lin Chen established that the topological property of the visual image represents the primary element and is the first to be perceived in form perception. (Chen, L. 1982), (Chen, L. 2000). Evidence for topological perception has long been supported by human visual psychophysical studies of visual sensitivity. (Chen, L. 1982), also (Lewin, K. 1936), (Warren McCulloch, W. 1945), (Piaget, J. & Inhelder, B. 1956), (Piaget J., Inhalder, B. 1958), (Piaget, J., Inhalder, B. 1967), (Smith, B 1994), (Rees, J.M. 2010).

Chen's study on topological perception illustrates the challenge facing many cognitive neuroscientists. The topological perception constitutes an important element in visual perception and further brain-imaging studies. Lin Chen's claim is about the importance of extracting global topological properties as primitives in object perception. The emphasis on global properties in perception is not new as it is known from the Gestalt theory of perception, but the topological perception theory specifically defines the global properties as topological invariants.

In addition, Lin Chen's theory states that the primitives of visual form perception are geometric invariants at different levels of structural stability under transformations.

Thus, a more stable property would be more primitive and more important to extract early in the process. Topological properties are the most stable in relation to other geometrical properties such as projective, affine, and Euclidean properties.

In the recent research, Lin Chen and colleagues reported schematic depiction of the left hemisphere's superiority in topological discrimination. (Chen, L. 1982), (Chen, L. 2005), (Chen, L., Zhang, S.W., Srinivasan, M. V. (2003), (Casati, R. 2000), (Donnelly, N. 2005), (Smith, P. 2011).

The deepest core of the human imagination is topological. Human beings see, think and feel 'topologically'. The topological kind of seeing, thinking and feeling forms a sub-set of imaginative perception. Topological modes of seeing, thinking and feeling are one of the ways that we intuit relations that are described as homeomorphism. Topology is a source of meaning and the way in which the most profound thoughts about the world are generated. Topology is the medium of human creation. Topology allows us to melt distinct figures of forms, shapes, images and thought. (Murphy, P. (2014).[502]

[502] Peter Murphy, Topeme: Truth. Topology. Cartography, Analogy., The Hydra Dialogues, May 22-23 2014, The Royal Danish Academy of Fine Arts, School of Architecture, Design and Conversation.

Topology as a mathematical representation of continuity, is the study of constancy in change, the study of the intensive identity of change and not-change. Topology is the study of shapes and place(s) that change and yet through change remain continuous with each other. These shapes and spaces remain connected to each other without breach.Even though they look different, under the surface of appearance they maintain unbroaken an identity with each other. (Murphy, P. (2014), Topeme: Truth. Topology. Cartography, Analogy., The Hydra Dialogues, May 22-23 2014)

Topology equates transformation and invariance, alteration and permanence, renovation and solidity, stability, longevity and immovability are indistinguishable from modification, adjustment and variation. (Murphy, P. (2014), Topeme: Truth. Topology. Cartography, Analogy., The Hydra Dialogues, May 22-23 2014)

John Willats assert that the "drawing systems are systems that map spatial relations between features of scene into corresponding relations on the picture surface". (Willats, J. 1997). Within the known type of drawing systems in evolution of culture and art, the drawing system described as 'primary geometrical', such as the 'perspective'; 'oblique projection' (commonly used in Hellenistic art, Mediavel art, Persian miniature painting and Chinese art); 'otragonal projection' (the basis for most Greek vase paintings, and now utilized in engineering and architectural drawings); 'horizontal oblique projection' (typical for naïve American landscape and icon painting); 'vertical oblique projection' (Indian painting, Cubist still life paintings), there are drawing systems can also be defined in terms of "secondary geometries". (Willats, J. 1997).

In addition to these known two dimensional geometry of the picture surface - the geometry of the 'ortagonals of the rectangular objects' and the 'oblique projections of the ortagonals', there are also two further systems that can be defined in terms of secondary geometry. The first, Wllats call the 'inverted or diverted perspective' (found in icon painting and some Cubist paintings), in which the ortagonals diverge, and the second, and "in some way the more important, is based on topological geometry" found in Children early drawings. (Willats, J. 1997).

In *Art and representation in the Analysis of pictures*, Chapter III, Topological transformation, (Willats, J. 1997, p.71) , Willats state that “In addition to the defining the spatial relations in pictures in term of projective geometry, the other main way of defining the drawing systems is in term of topological transformations. Topology is often described as “rubber sheet” geometry if a figure printed on the sheet is stretched or twisted, basic spatial relations such as proximity and enclosure will remain unchanged, although the distances between the marks may change and strait lines may not remain straight. These very basic spatial reflections form the subject of topological geometry. Figures or shapes are said to be topologically equivalent (‘homeomorfic’) if they share the same topological properties. For example a circle and a square are topologically equivalent because in both figures the outline is closed and separates the inside of the figures from the outside in two-dimensional space. Singularly, a closed box and a halbow rubber ball are topologically equivalent because the surface of each separates the inside of the sphere from the outside in three-dimensional space.” (Willats, J. 1997, p.71)

The relevance of the proposed with the present research project and works - ‘topological approach’ to drawing is highlighted by Willats’s statement – “There is an extensive literature on projective geometry as a basis from depiction, but although same pictures, such as the map of the London underground clearly preserve topological properties rather than projective properties. I know of no formal account of topology as applied to pictural representation. In pictures of this kind, spatial relations in the the seize sich as as spatial order, proximity, and interconnectedness are preserved in the picture, but not true shapes or true lengths.” (Willats, J. 1997, p.71).

In addition, Willats recall that “Piaget and Inhelder suggested that the spatial relations in drawing of young children are based on topological rather than projective geometry, and I shall suggests that the spatial relations in some artists’ pictures, such as same of Klee’s drawings, as well as many cartoons and caricatures, can also be described in term of topological geometry. In pictures topological relationships in the scene are represented by topological equivalent relations of the picture surface.” (Willats, J. 1997, p.71).

Piaget and Inhalder (Piaget, J. & Inhalder, B. 1967, 1958) state that the most basic cognition of the child is topological and the young child's first geometry is topological and projective, as the development of perception and notion of Euclidean geometries come later.

### 2.3. The notion of Metaxy and Metalepsis in Plato and Aristotel and Eric Voegelin's philosophy of consciousness or 'in-between' the infinite (Apeiron) and the finite (the divine mind or Nous) reality of existence, between the beginning of existence (Apeiron) and the Beyond existence (Epekeina).

The ontological as well as rhetorical notions of metalepsis, extends back to antiquity. The etymology of metalepsis usually is grasped from the Latin equivalent word of transumptio: "assuming one thing for another," but the true genesis of metalepsis shall be traced back to its origin in the philosophy of Plato and Aristotle.

The suffix 'lepsis' in Ancient Greek λῆψις (lêpsis) means "seizure", "taking action", and comes from 'lambanein' – 'to take hold of'. The suffix 'μετά' means 'with', 'across', or 'after', and 'beyond'. In rhetoric 'metalepsis' is a figure of speech and a form of metonymy of an indirect kind in which the substitution is of a word that is already being used figuratively. The notions of metalepsis are complex. Metalepsis is seen either as a variety of metonymy, metonymy of metonymy, a particular form of synonymy, or both. As metonymy, metalepsis has been identified in simple form, or expression of the consequent understood as the precedent or vice versa, also as a chain of associations. Metalepsis can also be understood in Quintilian's sense as the intermediate step or transition between a term which is transferred and the thing to which it is transferred, resulting in an inappropriate synonym.

The etymology of metalepsis links to the Metaxy (Greek: μεταξύ), defined in Plato's Symposium via the character of the priestess Diotima as the "in-between" or "middle ground". Diotima, tutoring Socrates, uses the term to show how oral tradition can be perceived by different people in different ways. It is the very same with the visual tradition, with an image, a picture, a drawing. The idea delivered to Plato by Diotima, points that reality (as well as work of art) is perceptible only through one's character (which includes one's desires and prejudices and one's limited understanding of logic). Man moves through the world of Becoming, the ever changing world of sensory perception, into the world of Being—the world of forms, absolutes and transcendence. Man transcends his place in ‘Becoming’ by Eros, where man reaches the Highest Good, an intuitive and mystical state of consciousness.

The concept of Metaxy is used also by the Neoplatonists like Plotinus to express an ontological placement of Man ‘between’ the Gods and animals. It is the same true for the artist, who is placed in the position of the mediator between the spectators and the highest state of his/her subjective reality, driven by Eros, reflecting the highest Being, the Cosmos by creating the media of art work as medium. The artist is much like the daemon of Eros expressed by Diotima as being in-between the Gods and mankind.

The concepts of ‘metaxy’ and ‘metalepsis’ are used in modern times by the political philosopher Eric Voegelin to designate the permanent place where man is in-between two poles of existence: such as the infinite (Apeiron) and the finite (the divine mind or Nous) reality of existence or between the beginning of existence (Apeiron) and the Beyond existence (ἐπέκεινα epekeina). Voegelin defined Metaxy as the connection of the mind or nous to the material world and the reverse of the material world's connection to the mind as "consciousness of being". Under Voegelin it can also be interpreted to mean a form of perception in contrast to consciousness a template of the mind (or nous) in contrast to the dynamic and unordered flow of experiential consciousness. As a form of reflectiveness in-between two poles of experience (finite and the infinite or immanent and transcendent). Metaxy is “the oscillation” between the finite and the infinite, between the real and the transcendent, between the ground and the sky. (Voegelin, E, 1990/2000).

While discussing the problem of ‘event’ as fundamental category, addressing the complexity, Voegelin points out that “…a word emerges from Plato’s vocabulary, the in-between, which I use in my analysis.” Plato’s ‘in-between’ is his approach to reality, and Voegelin asserts that “reality is not the human reality, nor the godly reality, but rather what takes place ‘in-between’ these two realities”. (Voegelin, E., 2004, p.390)

Form the philosophy of consciousness established by Eric Voegelin and presented in his Anamnesis (1966), emerges a new ontology, and vision of ‘reality’, founded in the reality of experience in the ‘in-between’ of ‘participatory consciousness’. For Voegelin ‘reality’ is 'no closed rational system'.

The notion of ‘metalepsis’ seen as ‘participation’ is derived from Eric Voegelin’s philosophy of consciousness. In term of consciousness, for Voegelin ‘participation’ is the key that describes best the ‘metalepsis’. Voegelin uses the concept of ‘participation’ to indicate the status of something finite in relation to the ontological perfection of the same in infinity. Metalepsis as participation is fundamental category in Plato (methexis) and Aristotle, the category that through the Neoplatonist became a primary category in the thought of the Church Fathers, and was elaborated by Aquinas in the medieval philosophy as the main principle of metaphysics. Voegelin clearly considers metalepsis as ‘participation’ as one of the most important concept and explanation of the structure of reality. According to Voegelin philosophers used the term metalepsis as communion and participation in attempt to explain how ‘nous’ is related to the intelligibilities that metalepsis apprehends. If for Plato, participation ‘in-between’ is ‘methexis’ and ‘metalepsis’, for Aristotle, ‘participation’ (metalepsis) is neither merely a metaphor nor merely a means of designating parallel attributes in man and the divine, but a ‘noetic’ expansion of the mythic insight that man's participation in the divine is constitutive of man's being in its specific essence, i.e., in the rational dimension. (Sandoz, 1981).

According to Ellis Sandoz, "the principle of participation is central to noetic science. It forms the existential basis of man's self-understanding insofar as from earliest times onward men are aware of participating in a structured reality of which they are but a part, one ontologically articulated by the symbolisms man, God, world, and society--the primordial quaternarian structure of being reflected in the earliest cosmological myths. Participation forms, therefore, as both the essence of the knower and the knowable and the inevitable perspective of the inquiry into reality. There is no Archimedean point outside of reality-as-participation available to men. Accordingly, it supplants the subject-object categories of cognition and ontology." (Sandoz, 1981, p.204).

The 'thought (nous) thinks itself through participation (metalepsis) in the object of thought (noeton), and for it becomes the object of thought (noetos) through being touched and thought, so that thought (nous) and that which is thought (noeton) are the same.'

Voegelin's interpretation is that the relationship between 'knower' and 'known', thought and 'being', is neither a meeting of completely different realities, nor a merging into absolutely identical reality, but something 'in-between' the two: the knower and what is known are, mysteriously, both the same and distinct.

The relarion between the 'knower' and the 'known', between the 'nous' and 'noetton', 'noetos' and 'noeton', the simultaneity of 'sameness' and 'difference' are the core explanation of metalepsis and what participation means. For Voegelin is clear that Aristotle, in spite of his criticisms (in Metaphysics) of Plato's use of the notion of 'participation' (methexis), doesn't hesitate to use metalepsis to explain the relationship between 'thought' and 'being', because for Aristotle there is a need for a concept that conveys a simultaneous sameness and difference. Voegelin's philosophy of consciousness is the philosophy of metalepsis, and the consciousness is conscious of reality and consciousness of being reality; the conscious of the ground of being and conscious of being the ground of being. Human nous and divine Nous are the same and yet not the same. (Hughes 1993:27-28)

According to Eugene Webb, metalepsis (also methexis) represents 'participation' and refers to "sharing the qualities of a supreme exemplar, in which they are present in their perfection. In 'participation in being' being is an analogical term with varying degrees of applicability; it describes existence in the metaxy as a condition between higher and lower degrees of reality." (Webb 1981:285)

**2.4. The notion of Metalepsis in modern narratology: Gérard Genette and Metaleptical transgression of boundaries between**

Within the history of poetics and narraology, the 'concept of metalepsis' is borrowed from the ancient rhetoric and represents a rhetorical figure originated in ancient legal discourse. The concept of 'narrative metalepsis' is a result from the convergence of rhetoric - placing it alongside metaphor and metonymy as tropes of transformation, substitution and succession, and the principle of narrative levels.

In modern narratology the term and concept of 'metalepsis' (literally 'sharing') was given a new narratological meaning by Gérard Genette in his Discours du récit (1972). (Genette, G.

2004). Genette introduces the term ‘metalepsis’ in order to describe a situation where the boundaries between narrative universes are violated or the hierarchy between narratological levels subverted. Normally a narrator belongs to a different narrative universe than the characters, since his narration takes place at a different time and often a different place than those of the events recounted. In such situations, the relationships between the author (narrator) and characters are hierarchical. The used of metalepsis, the term translated as ‘sharing’ and ‘participation’, changes the hierarchical structure of narrative, transforming it to heterarchical and the two universes of the author and characters merge in heteronomy as the narrator enters (‘shares’) the world of his character.

Gérard Genette defines metalepsis as a paradoxical combination between the world of the telling and the world of the told: “any intrusion by the extradiegetic narrator or narratee into the diegetic universe (or by diegetic characters into a metadiegetic universe, etc.), or the inverse […]” (Genette, G. 1972, 1980: 234-35)., or “deliberate transgression of the threshold of embedding” resulting in “intrusions [that] disturb, to say the least, the distinction between levels.” (Genette, G. 1972, 1980: 234-35). “Extradiegetic” and “diegetic” are generally accepted narratological terms defined by Gérard Genette in his Narrative Discourse: An Essay in Method (228). Genette calls the story-within-a-story the “metadiegetic” level. According to Genette, “It produces an effect of “humor” or of “the fantastic” or “some mixture of the two […], unless it functions as a figure of the creative imagination […]”[…]”(Genette, G. 1983, 1988-88). For Genette, metalepsis is an illusion of contemporaneousness between the time of the telling and the time of the told. It is doubling of the narrator/narrative axis with the author/reader axis. Metalepsis is seen as a “strange loop”, In the terms of mathematician Douglass Hofstadter (Hofstadter, D. (1979)., or as a short circuit’ between the fictional world and the ontological level occupied by the author, a sudden collapse of the narration system. (Wolf, W. 1993, p. 356–58)

In the the wake of the movement of French structuralism and new rhetoric, Genette’s rhetorical theory of metalepsis highlights the relation between “figural” and “fictional” metalepsis, where both “figure” and “fiction” derive from the Latin ‘fingere’ (to fashion, represent, feign, invent), such that a figure of substitution (a trope such as metaphor, metonymy, litote, etc.) forms the “embryo” or “outline” (esquisse) of a fiction (Genette 2004: 16–8). Genette extended the conception of stylistic figures from the lexical and syntactical to the textual level. Figure has become a textual rhetoric in narrative, as Genette's Métalepse (2004) exemplifies.

With emphasis on authorial metalepsis as a particular type of metonymy in which cause is expressed for effect or effect for cause and on the figural and fictional transgressions this entails, a fiction, taking form in the passage between figure as a formal but semantically weak verbal schema and figure as a transfer of meaning, is defined as "a figure taken literally and treated as an actual event". (Genette 2004: 16–8). In contrast to narrative considered as the "expansion" of a verb, fiction can be regarded as a figure taken à la lettre, and in the case of metalepsis "fictively literalized," it introduces into narratology the problem of ontological transgression in representation. The focus falls no longer on metalepsis as a narrative category forming a system with other describable categories (prolepsis, analepsis, etc.), but on the functioning of representation and the intersection of narrative and fiction.

The phenomena of metalepsis, was cultivated by the baroque, by romanticism and by certain types of modernism than by mimetically inclined classicism or realism, much as it shows a greater propensity for the comic and the ironic than it does for the tragic or the lyric.

Since Genette coined the term 'metalepsis' in his Narrative Discourse, the topic has gradually developed from a minor remark into a central notion in narratological theories.

Presently, there are various studies, out of different perspectives, devoted to the subject. The phenomenon of metalepsis drew critical attention during the days of postmodernist literature and has also contributed significantly to the establishment and development of postclassical narratology.

Over the past decade, several scholars have stressed that metalepsis is not an exclusively literary device, but can be found in almost every form of representation. With that knowledge, different research projects have begun to focus on metalepsis in painting, movies and comics. During the 3rd Conference 'Emerging Vectors of Narratology', of The European Narratology Network (ENN), held in Paris, March 29, 2013, there was a Panel – Metalepsis out of Bounds.

The statement of Roland Barthes that the narrative is a universal anthropological phenomenon, to the extent that is constitutive of culture, is proved today by the acknowledged ‘narrative turn’ in contemporart narratology, the turn that is touching also the visual narratology and visual art research. The present days narratologists expands their research beyond the literary corpus and take the“narrative turn,” embracing fields as diverse as psychology, anthropology, sociology, social science, history, law,  AI experts, digital technology, and visual art.

Metalepsis imply participation and is based on participation and heteronomy between narrator (author) and spectators or audiences. The art forms implementing such participation or dynamical and interactive visual narratives are metalepsis-friendly.

The phenomena of metalepsis is present in various ways and to different degrees in the theater arts, due to the possibilities of audience participation. It is typical for the cinema, with its technical capacity for hypotyposis (what is presented is depicted as though it were before one’s very eyes). The films can be highly metaleptic, contrary to music. The presence of metalepsis in visual art is demonstrated by the works of Escher and Magritte. Digital media, with their capacity for generating virtual realities, are fertile terrain for ontological transgressions triggered by metalepsis. The social network are highly metaleptical, build on interaction and participation. The model of relations within the social network is topological network not hierarchical.

Ernst Curtius, in his study *European Literature and the Latin Middle Ages* (1948) introduced the concept of ‘literary topos” as scholarly and critical discussion of literary commonplaces, claiming that much of Renaissance and later European literature cannot be fully understood without knowledge of that literature's relation to Medieval Latin rhetoric in the use of commonplaces, metaphors, turns of phrase, or, to employ the term Curtius prefers, *topoi*". [503] [504]

[503] Lind, L.R. (1951), "Rev. of Curtius, Europäische Literatur und lateinisches Mittelalter". The Classical Weekly 44 (14): 220–21.)

[504] Ostheeren, Klaus (1998). Ernst Robert Curtius (1886 – 1956). In Helen Damico Ed., *Medieval Scholarship: Literature and Philology*. Taylor & Francis. pp. 365–80.

Klaus Ostheeren in his study on Curtius (Ostheeren, K., 1998), establishes that "Topological studies in various branches of knowledge, such as law, go back to this original meaning…but literary topology is firmly based on Curtius's metonymic use of "topos", wich some scholars traced back to Aristotle. Inaugurating modern topology as a method for historical, cultural and literary research, Curtius transformed the inherited conception of from technique of finding arguments in rhetorical persuasion into the patterns of thought and expression originally found by applying this technique, but now established as inherited and aquitened constituents of literary competence – the indispensable cognitive units and literary production, reception, and interpretation." (Ostheeren, K., 199, p.373).

Curtius rhetorical modes of thought and expression crystalize into patterns or models wich Curtius called topoi, their study being "topological research" or "topology" (toposforshung). Curtius practiced this branch of topology, synchronic or syntagmatic topology, with brilliant results in studies of Divine Comedy. The links between the Curtius's 'literary topology' and mathematical concepts, similar to the modern topology as mathematical discipline, are not disputable.

The importance of Curtius's rediscovery of topoi and topics extended beyond the medieval studies. In philosophy, sociology, political science, and jurisprudence topology has come to be regarded as a bridge over the historical gap that opened in the eighteenth century Europe, when – for not yet fully understood – topic gave way to logical thinking. (Gelley, A. 1974).[505]

## 2.5. Transformative power of Metalepsis: Breaking the Frame - Metalepsis and the Construction of the Subject

Metalepsis is a vehicle for mediating 'in-between'. The effect of metalepsis is not only rhetorical, but ontological and epistemological transformative power. The transformative effect of metalepsis is discussed by Debra Malina in her Breaking the Frame: Metalepsis and the Construction of the Subject (2002). There are personal, behavioral aspects of such transformation and also sociopolitical parallels. The transformative power of metalepsis

[505] Gelley, Alexander. (1974), Ernst Robert Curtius: Topology and Critical Method. In *Velocities of Change.* Ed. Richard Mackey. Baltimore: The Johns Hopkins University Press

involves the active participation of the reader or spectator "momentarily, to experience new way of being…to construct our subjectivity to some degree. (Malina, 2002:.9) Metalepsis deconstruct and reconstruct our mental maps so, in some way ourselves. (Malina, 2002:19).

For Malina, "Metalepsis is practice present itself as a collapsing, a blurring, or a dissolution of boundaries." (Malina, 2002:.4). Part of the shock value of metalepsis derives from the fact that these universes are originally conceived as hierarchical ordered. From the inherent hierarchy comes the narratological concept of 'levels'. Because it traverses an ontological hierarchy, Metalepsis has the power to endow subject with greater or lesser degrees of 'reality' – in effect to, promote them in to subjectivity and demote them from it." (Malina, 2002:.9).

Metaleptical transgression of boundaries between narrative level, Malina define simply as 'breaking the frame'. Malina asserts that:

"Metalepsis dramatizes the problematization of boundaries between fiction (in my term – visual art work) and reality endemic to the postmodern condition. More specifically, because it disturb narrative hierarchy in order either to reinforce or to undermine the ontological states of the fictional subject or selves, it provides a model of the dynamics of subject construction in an age that has witnessed both the deconstruction of the essential self in favor of a subject constituted in and by narrative and the complication of simple, teleological model of narrative with an emphasis on the form's repetitive self-undermining, and even violent aspect." (Malina, 2002:4).

Metalepsis is a narrative and visual narrative devices of transition. Metalepsis can be regarded as a kind of "recursion," forcing readers or spectators to lose their bearings, sense of time and place.

Katharina Lorenz in her *The anatomy of metalepsis: visuality around on the late fifth century pots*, (Lorenz K., 2007:118 – Visualising Metalepsis), asks the questions: "What state is defined by metalepsis? And in what ways can a category developed to characterize texts be used to do the same for puctures?" Lorenz preferred to provide explanation by explaining that "When Gerard Genet explains its functioning, he used a short story by Julio Cortazar in which the protagonist is killed by a character from the book he is reading: a fictional character takes control of the realm of its (equally fictional recipient). "(Lorenz K., 2007:118). Writers as James Joyce, Samuel Beckett, Franz Kafka, Julio Cortazar, Vladimir Nabokov, Umberto Eco, Thomas Pynchon are famous practitioners of the art of metalepsis. Cervantes's novel Don Quixote is is among the texts most frequently cited in theoretical work on metalepsis.

In "Speculative philosophy"[506], Verene focuses on four principle authors – Vico, Hegel, Cassirer and Joyce. Adding James Joyce to these three philosophers, Verene explains that "Joyce is the poet whose ghost haunts his discussions. Joyce based "Finnegans Wake" on Vico's "New Science" in the way he based "Ulysses" on Homer's "Odisey". If Vico is the hero of the universal fantastic, of "imaginative universals", the primar sense of metaphors that he sees as the master key to his New Science", Joyce is a hero of the epiphany and the trope of irony, his work "Finnegans Wake" in which language is manipulated to allow a vision of language itself.

According to Verene, Vico has arranged the source of his mature thought as a tetrad. Tetralogy is Greek originally is a term used in oratory to refer the group of speeches or logoi, that are delivered in a lawsuit. In certain cases in the law courts in Athens, the accused and the accuser were each allowed two speeches. The writing of tetralogies is based on the sophistic art of arguing not from evidence of witnesses but from the probabilities of the case. Vico's intention is to propose his four authors as his logographers, the writers of his own speech. In Vico's speech of the "New Science", Plato and Tacitus present the speech of the accused, as both are the speackers of the civic wisdom of ancients. Bacon and Grotius are the moderns. If Plato and Tacitus are the two pagans, Bacon and Grotius are the two heretics.

Each of this four parts mirrors Vico's distinction between the philosophical and philological. Examining Vico' speech of the "New Science", Verene establishes that this speech combines all four corners of the tetrad – the vertical movement between philosophical and philological, and the horizontal movement between ancient and modern. According to Verene, Vico's New Science may be called the speech of the tetrad.

[506] Donald Phillip Verene, "Speculative philosophy", Lexington Books, 2009

In the “Speculative philosophy”, Verene proposed the so called “Vico’s tetrad” and intimated his own tetrad build upon on the Vico’s one. Vico’s tetrad is constructed by Plato, Tacitus and Bacon and Grotius. In Verene’s tetrad, there are Hegel, Cassirer, Vico and Joyce.

Each of the famous representatives in these tetrads are ordered in pairs, from general to particular. In Verene’s tetrad, Hegel’s Begriff is made more specific by Cassirer, in his notion of the symbol, where the symbol is an embodiement of the Begriff as a cultural form. Vico’s universale fantastic is formulated by Vico as an idea, a way to take thought back to its origins in the language of the body. Joyce’s ironic and ephiphanic use of language is an attempt to write the reader back to the language of universale fantastic, to allow the reader to experience the breaks of language. What governs all four authors is that “the time is the whole” (Hegel) and that “the whole is really the flower of wisdom” (Vico), that “self-knowing is the highest aim of philosophical” (Cassirer), that the key to selfknowledge is what is “fabled by the daughters of memory” (Joyce).

### 2.6. Metalepsis in Heidegger

Andrew Haas in his study Gewalt and Metalepsis: On Heidegger and Greeks ( Haas 2008.), while discussing Heidegger’s The Origin of the work of Art, sees the ‘artist’, the author, Heidegger himself, as ‘translator’ of the true - aletheia. Haas asserts that “if the question of truth (of art and translation, like that of being) is as old as Western philosophy, as old as the Greeks, we must then eventually return to them, and to Greek, to that which Heidegger calls the ‘Greek sense’, in order to think of the work of art in that which makes the translation, metalepsis, of aletheia and its truth first possible”. (Haas,A. 2008).

In "The Origin of the Work of Art" Heidegger explains the essence of art in terms of the concepts of being and truth. For Heidegger this ‘translation’ (of the origin of art) has to do with the ‘essense of being’. The essence of being is not the form, but that which is continuous, what allows the interpretation or translation of the thing qua form and matter, the constancy, consistency or continuity of the thing. If the role of the philosopher and artist is to mediate and translate ‘between’ the two poles, this task of ‘translation’ could be defined also as ‘participation’, and this is also one of the important meaning of the concept of metalepsis.

We can resume that metalepsis is ‘participation’, and ‘participation’ is metalepsis. Hass concludes that “so participation is translation, metalepsis – but what is metalepsis. It is the movement (trans-) of lepsis, taking, accepting, seizing: on the one hand, actively, violently, carrying-off as booty, grasping, with the hand or mind, perceiving, apprehending, comprehending; on the other hand, passively receiving, being griped, possessed, violated, had, echo, Schein. So metalepsis is (always inadequate, impossible – or rather uncertain), sub-stitution, movement over to another insofar as it is moved – for in seeking to size, we are seized, taken over in the take-over, translated by translation, transformed in the transformation, violated in the violation. Thus metalepsis is a metalepsis; and the truth is translation, the violence of taking one for other, not one for one.”  (Haas,A. 2008, p.13).

According to Haas, “As a happening of alētheia then, poiēsis shows itself as the illumination of metalēpsis—for concealment-unconcealment is original translation. Representative art (and its negation, non-representative art) is a translation of truth—but translation itself is poietic. So the question of the origin of the work of art, of the onto-heno-chrono-phenomenology of artworks, becomes far more the question of the violence of translation as translation. Andrew Haas in his study Gewalt and Metalepsis: On Heidegger and Greeks (Haas,A. 2008, p.13).

How then, did the Greeks think translation? Not simply as hermēneia, but as metalēpsis. The Platonic doctrine of the forms, for example, is not simply participation; rather things are the translations of ideas, and art partakes of the eidos insofar as it transforms the language of originals to that of the copy, or the copy of the copy.

Hass established:

“Heidegger takes pains to clarify the difference between, on the one hand, the posi-ting, placing, putting of an object over and against a subject, or for consciousness, as in German idealism; and on the other hand, the thesis of the Greeks as letting-stand-up, letting-be-set-up, Erstehenlassen, let be brought forth into immediate unconceal-ment, let lie forth in its presence (PLT, 82; HW, 68). Two questions remain however, with respect to standing. First, where does being, or a being, stand, hic et nunc? Can it be said to be here or there, especially if it is in transition, motion, becoming, or on a threshold, transgressing a border, crossing a frontier, at the limit of a Ge-stell, or horizon of its unity? Do we then stand here or there—or perhaps rather both and neither? And second, can being or a being stand on or in an abyss, particularly if truth is abyssal? Can we stand in continuous discontinuity? Or must discontinuity rather be thought as discontinuous? And if the essence of the thing lies with constancy, what is the essence of constancy? Or if this search for essence—like that for meaning, ground, origin, truth—is infinite, is it not perhaps because truth, as Heidegger writes, is an abyss? (Haas,A. 2008, p.13).

Hegel's language, syntax, concepts and notions have topological nature. The topological is presented in Hegel both as mathema (in his philosophy of mathemats) and as rethorics. If topological nature of Hegel's mathesis is presented within his fourfold of infinities (multiplicities) and Hegel's four type of judgments, Hegel's rethorics is bearing the four basic tropes of rhetoric: metaphor, metonymy, synecdoche, ironi as equally presented in his manifold (Mannigfaltigkeit) of infinities, quality and quantity, time and space.

In Hegel's philosophical narratives and semiology, the emphasis is on the metonymy seen as 'metonymy of metonymy' or 'metalepsis', presented in 'topological' notion of Qualitative quantity (The Quality of quantity).

David G. Carlson, in his Hegel's Theory of Measure, in particular in his discussion of (b) Measure as a Series of Measure Relations), states that "**Metonymy** is the theme of this new section's tongue. Metonymy is the inability to name the thing directly, but only the context of the thing. In metonymy, if the entire context is described, the unnameable thing becomes a ghostly space the existence of which is simply inferred from context." (David G. Carlson, 2003, Hegel's Theory of Measure:47)

### 2.7. Hegel's notion of 'die Mitte' : Metalepsis in Hegel

There are relations between Eric Vœgelin concept of 'metalepsis' (seen as 'participation' in the 'in-between') and Hegel's notion of 'die Mitte'. This relation could be unfolded through the topological language of hermeneutics in Gadamer's the 'middle of language' ("die Mitte der Sprache"). In his book "The Language of Hermeneutics, Gadamer and Heidegger in Dialoque" (Coltman, R.,1998) Rodney Coltman offers an excellent discussion on Gadamer's understanding of human finitude and Hegel's notion of interrelatedness of finitude and infinitude. The idea that as humans we find ourselves within history, and that (in opposite to Hegel) "being historical means never being entirely exhausted in self-knowing", Gadamer develops in the middle third of his “Truth and Method”. For Gadamer all human experience is at once historically conditioned and linguistically constituted, thus "experience," according to him "is experience of human finitude."

According to Rodney Coltman, at a particular moment in the ‘Logic of Being’, in fact, we discover that finitude contains within it the infinite and vice versa. Having moved through the transitions from "being" to "nothing" to "becoming," the reader of the Logic finds him- or herself in the sphere of "determinate being" or Dasein.

Dasein shows itself to be a determinate being or "something" insofar as its two qualities, "reality" and "negation," are reflected back into it. And yet something, as the negation of a negation, the mediation of itself with itself, is simply another form of being and, therefore, in itself,is also becoming, "which, however, no longer has only being and nothing for its moments." (Coltman, R.,1998)

This first moment, the being of something, is now a determinate being and its second moment "is equally a determinate being, but determined as a negative of the somethingan other." (Miller A. V., 1990, p.116)

For Hegel both ‘something’ and ‘other’, therefore, are initially ‘determinate beings’ or ‘somethings’. The "the mediation through which something and other each as well is, as is not," Hegel calls "limit." (Miller A. V., 1990, p.116)

As something is not its other, and the other (as it is also something) is not the something that its other is, each is equally determined to be what it is in not being its other. And this non-being of the other (which is also the nonbeing of itself) is the "ceasing-to-be" or the "limit" of something in its other. In his explanation, Hegel employs term "die Mitte" (The German for ‘the middle'). (Miller A. V., 1990, p.116)

The German for ‘(the)’ middle’ is (die) Mitte. This generates an adjective mittel (‘middle’) and another noun (das) Mittel (originally ‘(the) middle, the thing in the middle’, but now ‘means, what serves the attainment of a purpose ’). It also generates several verbs, especially mitteln (‘to help someone to, to settle, mediate’, e.g. a quarrel), which is now obsolete but has left mittelbar (‘mediate, indirect’) and unmittelbar (‘immediate, direct’), and vermitteln (‘to achieve union, mediate; to bring about’, etc.). The past participle of vermitteln, vermittelt (‘mediated, indirect’) is used in contrast to unmittelbar. Both give rise to abstract nouns, Vermittlung (‘mediation’) and Unmittelbarkeit (‘immediacy’). In non-Hegelian philosophy, unmittelbar is primarily an epistemological term. Immediate certainty is a certainty that is not mediated by inference or proof, or perhaps even by symbols or concepts.

In Hegel’s use of "die Mitte" we could see the topological notion of ‘in-between’ in the pure meaning of Plato’s ‘methaxis’ and Aristotle’s ‘metalepsis’ or exactly in accordance with Voegelin’s concept of metalepsis as ‘in-between-ness’. The mediator is between the ‘outside’ and ‘inside’, between the ‘quality’ and ‘quantity’, between the ‘time’ and ‘place’, between the ‘image’ (bild) and the ‘concept’ (begriff). The tradition of the thinking ‘in-between’, culminated most generously in Hegel, could be expressed in the ‘language’ of mathematical models in epistemological term that emanate the notion of topological cobordism (Rene Thom’s linguistics and topology). In Hegel’s dialectical and logical ‘cobordismä, something has its determinate being outside (or, as it is also put, on the inside) of its limit; similarly, the other, too, because it is a something, is outside [its limit]. Limit is the middle term [die Mitte] between the two of them in which they cease. They have their determinate being beyond each other and beyond their limit; the limit as the non-being of each is the other of both. (Miller A. V., 1990, p.127)

### 2.8. Topological notion of Metalepsis in Giambatista Vico and Hegel

The philosophical system created by Hegel is the first systematic inquiry which deals with the logic of autopoiesis. In his Science of Logic, Hegel starts with Being, Nothing and Becoming which is nothing but autopoietic relation of Being itself, as being Thought, up to the Notion, as the explicit materiality of thought (or, ideal self-relation of externality of nature).

Donald Kunze discusses metalepsis in relation with “autopoiesis”, emphasizing that this is not the ‘autopoiesis’ of Humberto Maturana, Francisco Varela, and Niklas Luhman, but “the autopoiesis grounding the architecture of dissensus has to do with the uniquely human discomfort within nature; the speaking subject's unstable relationship to appearance and placement." (Kunze 2013)

Through Slavoi Žižek, Kunze links the idea of this type of autopoiesis with G.W.F. Hegel ("the idea that self-regulation arises out of discomfort comes from G. W. F. Hegel.), recalling Slavoi Žižek's statement in Less than Nothing (2012).

Žižek explains: "Hegel is — to use today's terms — the ultimate thinker of autopoiesis, of the process of the emergence of necessary features out of chaotic contingency, the thinker of contingency's gradual self-organization, of the gradual rise of order out of chaos." (Kunze 2013), also (Žižek 2012, p.467, p.157-58).

In addition, Kunze directly links 'metalepsis' as 'autopoiesis' with Giambatista Vico, claiming that "the theme of autopoiesis reveals Giambattista Vico to be Hegel's unrecognized predecessor, the thinker who, in comparatively obscure conditions, carried out the Hegelian project with an even greater autopoietic effectiveness." According to Kunze, "the Vichian motto verum ipsum factum (est), the made is itself exchangeable with the true, sums up self-generation in the term ipsum, "itself." (Kunze 2013) For Kunze, Vico's thesis of ipsum into a logic of the first human master signifier, the imaginative universal, the basis of mythic thought, where irony's invisibility affords a non-ironic visibility. The world appears with the greatest force as a signifying presence of authority. As Kunze puts in, "Vico's paradigm was the thunder, demonic and traumatic. Retreat from this traumatic Real, askesis, constructs the historical civil world where political relations replace religious ones through a gradual secularization. But, we must not forget that the original trauma was self-constructed, and in this autopoiesis was formed a logical kernel — a remainder — able to survive all successive transformations: a "permanent uncanny" that, as self-fear, remains radically alien but essentially our own — our only true — possession, our ipsum, our autopoiesis, not the autopoiesis of the cells of plants and animals." (Kunze 2013), also (Kunze, 1990). Kunze admits that Vico is the first thinker to realize the importance of Vico's imaginative universal as the first theoretical recognition of reversed predication and Hegel is the second.

Vico's imaginative universal is the first theoretical recognition of reversed predication, and Vico is the first thinker to realize the importance of this phenomenon, Hegel the second. (For

Vico's account of the imaginative universal see The New Science (1744), §381, §460, §809, §1033.)

Kunze recast the relationship of vertical ideology versus horizontal dissensus in terms of the "ironic topology" that connects Hegel's autopoiesis with (Kunze) project. For Kunze "verticality and horizontality are qualities of spaces and times that must be completed by the imagination, "virtually." (Kunze 2013).

According to Kunze, "Hegelian autopoiesis creates a unique program for the "distribution of the sensible," a detached virtuality that regulates human symbolic systems by means of a "lock" binding the real and imaginary." (Kunze 2013)

By lock, Kunze explains, he mean the "Borromeo knot" of three interlocking rings used by Jacques Lacan as description of the relations of the Symbolic, the Imaginary, and the Real. According to Kunze, "subjectivity can be explained by the topology of the knot". For his explanation of "knot", Kunze stress on the requirement to view the knot as seen from above as "a series of rings lying on top of one another in a cloverleaf. This projection, however, will conceal the topological logic, which will appear as a "surprise" when we look closely into the connections." The projection view, states Kunze "invites us to use such terms as a "first ring" and "last ring," but first and last do not exist in the topology of the knot." (Kunze 2013)

Kunze assert:

"Although each ring seems initially to rest on top of the preceding one, the "final" ring slips under the "first," making a lock. This lock not only binds the system together; it creates a new dimensionality by distributing the "boundary conditions" (the arbitrarily defined first and third rings) into all of the parts, creating a Möbius band effect that permeates the whole system as an invisible glue. There is really no top or bottom ring topologically speaking; the stack gives way to the topology of the knot. The boundary converted into the bounded, a "reversed predication," is the autopoiesis of what is uniquely human. This is the irony that Hegel constructed as dialectic, as we will see later in the example of the parable of Lordship and Bondage. But, nearly one hundred years before, Giambattista Vico had uncovered this connection to autopoiesis when he sought to uncover the origins of mythic thought. And, one

hundred years later, Freud located autonomy within the automaton of the unconscious; and, half a century later, Lacan, in the process of rescuing Freud from his misconstruers, described this automaton as a dark, internal subjectivity active in externalities, everyday experiences: an ability to be inside and outside at the same time, later refined in the idea of extimity (extimité)."  (Kunze 2013)
Kunze states that:

"Metalepsis is a magic trick based on the division between the frame and the framed, diegesis and mimesis. It allows the audience to encounter some form of itself, installed within the mimetic particulars of the work. Equally, it allows characters and elements to escape from their mimetic prisons, to jump off the stage or hop off the screen, as does the film actor Tom Baxter in Woody Allen's The Purple Rose of Cairo (1985). The absurd exception to framing rules forces the audience to recognize that which it itself had done originally in establishing the conditions of diegesis and mimesis. ((Kunze 2013, 6).

Metalepsis is not simply an abstract condition; it is a staged collapse of spatial and temporal protocols that normally hold the audience on one side of a line and the show on the other. Violating this boundary requires the special engineering supplied by the literary forms of the fantastic, the genre peculiar to violations of normal causality, as a means of developing its specific architecture of detached virtuality.

Let me define the genre of the fantastic in a limited but architecturally pertinent way. The Argentine master of the fantasy, Jorge Luis Borges, condensed his genre into four motifs: the double, the story in the story, travel in time, and contamination of the horizontal reality of mimesis by the dream or fiction. All motifs are, in fact, a contamination that develops along the vertical line established by diegesis. And, all are variations on diegesis's career as a "story in a story." (Kunze 2013, 23).

The proposition of availability of the notion of 'metalepsis' in Hegel, is supported also by Cyril O'Regan in his Hegel on the Modern World (O'Regan, C, 1994). O'Regan asserts that

"One may see in the text of Hegel`s mature period the presence of a trope that distinguishes Romantic discourse in general, i.e. the trope of metalepsis." (O'Regan, C, 1994, p.47).[507]

Metalepsis is intrinsic to the philosophical text and can be experienced in Hegel's notion of multiplicity, as interplay of the categories of quantity and quality, time and space, in particular within Hegel's fourfold of infinities. The category and method of Qualitative quantity is metaleptic category.

## 3. The Shape of Hegel's Logic: Applying Topological Data Analysis

### 3.1. Topological Qualitative quantity as Prospective Research Method and Methodology

Research, applied research and empirical research survey methods can be used for the purpose of prospective research implementing the topological approach to Hegel's Logic as applied philosophy.[508] Both qualitative and quantitative research methods can be used to collect data

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507 O'Regan, (1994), The Heterodox Hegel. SUNY Series in Hegelian Studies. State University of New York Press, Albany, p. 47.

508 See:Borislav Dimitrov:

- 2014, Auditor Independence within An Auditing Analysis Situs: Topology of Places as factor for enhancing auditor independence, competence, and audit quality, between the global and local: Topological Approach to Audit Dynamics, Focused on Auditor Independence, Competence and Audit Quality through Qualitative quantity methodology and Topological Data Analysis, Philosophy of Science for Social Science, Lund University, Faculty of Social Science.
- 2014, The Struggle of Cultural Identity between the Dichotomies of Society and Community, between Liberalism and Communitarianism: Dialogue or becoming Topological? Philosophical topology of intercultural (identity) relationships, Study presented at the International Conference 'The Individual and Society: Challenges of Social Change", April 5th, 2014, Sofia, Bulgaria (Bulgarian Academy of Science and Arts, Serbian Royal Academy of Science and Arts, European Center of Business, Education and Science, published in the Conference edition collection 'The Individual and Society: Challenges of Social Change' (ISBN 978-954-411-151-9), 2014, p. 266-296.
- 2014, Hegel's Analysis Situs: Topological Notions of Multiplicity in Hegel's Fourfold of Infinities, approved and pending publication, Sophia Philosophical Review, 2014
- 2014, Philosophical topology and Topological philosophy as the mode of thinking of Evolution of Hierarchical Systems: The Role of Heterarchy and Heteronomy in Evolution, Conference Edition: Evolution of Hierarchical Systems, Sofia, Faber Publishing House, September 2014, p.285-318
- 2013 "Topological Ontology and Logic of Qualitative quantity", https://www.academia.edu/3237237/Topological_Ontology_and_Logic_of_Qualitative_quantity
- 2012 A Topological Approach to 'The Hospital of the Future': Topological Model based on the qualitative quantity research method, Amazon.

and information through field research. As a complimentary to the traditional qualitative and quantitative methods, the novel method of Topological Qualitative quantity can be tested for the ability to capture and provide insights on the processes exhibiting gradualness and continuous and transformation (persistence), emphasizing on the relation and relationships in space seen as topological space (topological notions and concept such as ‘homeomorphism’, ‘homology’, ‘boundaries’, ‘closeness’, ‘part and whole’, ‘inclusion’ and exclusion’) .

The language elements, categories and notions in Hegel’s syntax in Science of Logic can be accepted as the data collected that can be mapped and analyzed also by using Topological Data Analysis, a recent mathematical method for analyzing data that has had new, dramatic, and unexpected applications to statistics among other areas.

Topological data analysis uses a branch of mathematics called algebraic topology to capture the shape of a point-cloud data set that "persists" in a dynamical setting. For this purpose the data, themes or issues will be represented as shape of a point-cloud data set that "persists" in a dynamical setting.

The main problems subject of Topological Data analysis could be recognized in Hegel’s Science of Logic, and these are the following:

- how one infers high-dimensional structure from low-dimensional representations; and

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- 2012 “The Topological Approach of Qualitative quantity Implemented in Autopoietic Law and Audit: The Cultural Phenomenology of Qualitative quantity and … The Cultural Phenomenology of Law and Auditing as Autopoiesis”, „Ariadne – Topology and Cultural Dynamics – Institute for Cultural Phenomenology of Qualitative quantity”, http://ariadnetopology.org/3.html;
- 2012 “Cultural Phenomenology of Law and Topological Approach to Law”, A Series of papers presenting the essentials of Topological Approach to Law : Qualitative quantity - The Cultural Phenomenology of Literature and … The Cultural Phenomenology of Law; Law and Literature Movement; Cognitive Science and The Law - Topological Approach To Law; Phenomenology of Law; Law and Social Choice: Qualitative quantity, Topological Social Choice and Topological Approaches to Law; The proposition of Qualitative quantity mode of Inquiry in the Classic Debate – Qualitative vs Quantitative research; The Topological approach of Charles Sanders Peirce’s qualitative-ness and The Topological Qualitative quantity; The philosophy of Émile Boutroux – a profound influence on Henri Poincaré and Charles Peirce. „Ariadne – Topology and Cultural Dynamics – Institute for Cultural Phenomenology of Qualitative quantity”, http://ariadnetopology.org/3.html;
- 2011 “The Relevance of Topological Approach, based on Qualitative quantity research method, to Audit Dynamics and Auditing Research – Cultural Phenomenology of Audit and Auditing research”, „Ariadne – Topology and Cultural Dynamics – Institute for Cultural Phenomenology of Qualitative quantity”, http://ariadnetopology.org/Cultural_Phenomenology_of_Audit_Dynamics_and_Auditing_Research_web.pdf

- how one assembles discrete points into global structure.

Both of the problems are very much related with the role of the philosopher and applied philosophy - to assemble discrete point into global structure and to enhance or improve the low-dimensional representations into the high dimensional structure.

Topological Data Analysis focusses on continuous flow, recognizing that the human brain can easily extract global structure from representations in a strictly lower dimension, i.e. we infer a 3D environment from a 2D image from each eye. The inference of global structure also occurs when converting discrete data into continuous images, e.g. dot-matrix printers and televisions communicate images via arrays of discrete points.

The main method used by topological data analysis is:

1. Replace a set of data points with a family of simplicial complexes, indexed by a proximity parameter. This converts the data set into global topological objects.

2. Analyse these topological complexes via algebraic topology — specifically, via the new theory of **persistent homology**.

3. Encode the persistent homology of a data set in the form of a parameterized version of a Betti number which will be called a **barcode**.

In Topological Data Analysis, the primary mathematical tool considered is a homology theory for point-cloud data sets—**persistent homology**—and a novel representation of this algebraic characterization— **barcodes**. Topological Data Analysis considered the shape of data.

Robert Ghrist in his article "Barcodes: The Persistent Topology of Data" [509] concluded:

[509] Robert Ghrist in his article "Barcodes: The Persistent Topology of Data", 2007, Bulletin of American Mathematical Society, Volume 45, Number 1, January 2008, Pages 61–75

“When a topologist is asked, ‘How do you visualize a four-dimensional object?’ the appropriate response is a Socratic rejoinder: “How do you visualize a threedimensional object?” We do not see in three spatial dimensions directly, but rather via sequences of planar projections integrated in a manner that is sensed if not comprehended.

We spend a significant portion of our first year of life learning how to infer three-dimensional spatial data from paired planar projections. Years of practice have tuned a remarkable ability to extract global structure from representations in a strictly lower dimension.

The inference of global structure occurs on much finer scales as well, with regard to converting discrete data into continuous images. Dot-matrix printers, scrolling LED tickers, televisions, and computer displays all communicate images via arrays of discrete points which are integrated into coherent, global objects. This also is a skill we have practiced from childhood. No adult does a dot-to-dot puzzle with anything approaching anticipation.” [510]

Similar discussion on the discrete presentation of data /information/ but our ability to perceive gradualness of information and the continuous nature of the shape of data, offers Magnus Bakke Botnan in his work “Three Approaches in Computational Geometry and Topology - Persistent Homology, Discrete Differential Geometry and Discrete Morse Theory” [511] :

“One of the most remarkable properties of the human brain is the ability to infer the world as a three-dimensional space. We do not see three spatial dimensions directly, but from experience we know how to visualise three dimensions via sequences of paired planar projections. In other words, we know how to extract global structures by studying representations from a strictly lower dimension. Another skill developed is how to infer a continuum from discrete data. As an example, consider the painting The Seine at La Grande Jatte by the French artist Georges Seurat. This painting consists of discrete data points and is obviously noisy. Nonetheless, we have no problems perceiving the tree by the waterline, the

[510] Robert Ghrist in his article “Barcodes: The Persistent Topology of Data”, 2007, Bulletin of American Mathematical Society, Volume 45, Number 1, January 2008, Pages 61–75

[511] Magnus Bakke Botnan in his work “Three Approaches in Computational Geometry and Topology - Persistent Homology, Discrete Differential Geometry and Discrete Morse Theory”, 2011

person in the kayak or the sailboat. Rather than altering out noise qualitatively it is favourable to have a quantitative measure." [512]

The paintings of Georges Seurat, as his "Seine at La Grande Jatte", are probably one of the best representation of the interplay of quality and quantity, in particular the gradual and continuous notion of qualitative quantity. Pixel-based raster graphics, as in television and flat-panel screens, mimics the pointillist method are associated with the artist Georges Seurat, who covered the canvas with tiny spots of paint.

Shmuel Weinberger in his work "Persistent Homology" also emphasized on the human ability to take sense data of individual points and assemble them into a coherent image of a continuum. Weinberger illustrating his explanation of persistent homology again with Seurat's painting: "Consider the art of Seurat or a piece of old newsprint. The eye, or the brain, performs the marvelous task of taking the sense data of individual points and assembling them into a coherent image of a continuum—it infers the continuous from the discrete. Difficult issues of a similar sort occur in many problems of data analysis.Onemighthave samples that are chosen nonuniformly (e.g., not filling a grid), and, moreover, one is constantly plagued by problems of noise—the data can be corrupted in various ways. Puremathematicians have problems of this sort as well. One is often interested in inferring properties of an enveloping space from a discrete object within it or, in reverse, seeking commonalities of all the discrete subobjects of a given continuous one. To give one example, this theme is a central one in geometric group theory, in which a typical problem, going back to Furstenberg and Mostow, asks to reconstruct a connected Lie group from a lattice in it." And Weinberger concluded again with the approach of topology: "Because topology is essentially a qualitative field, it is perhaps not surprising that there has been a development of some common topological technology for these problems." [513]

Speaking of "clouds of data", Ghrist claimed that: "Very often, data is represented as an unordered sequence of points in a Euclidean $n$-dimensional space E$n$. Data coming from an array of sensor readings in an engineering testbed, from questionnaire responses in a

[512] Magnus Bakke Botnan in his work "Three Approaches in Computational Geometry and Topology - Persistent Homology, Discrete Differential Geometry and Discrete Morse Theory", 2011

[513] Shmuel Weinberger, "Persistent Homology"

psychology experiment, or from population sizes in a complex ecosystem all reside in a space of potentially high dimension. The global 'shape' of the data may often provide important information about the underlying phenomena that the data represent. One type of data set for which global features are present and significant is the so-called **point cloud data** coming from physical objects in 3-d. Touch probes, point lasers, or line lasers sweep a suspended body and sample the surface, recording coordinates of anchor points on the surface of the body. The cloud of such points can be quickly obtained and used in a computer representation of the object. A temporal version of this situation is to be found in motion-capture data, where geometric points are recorded as time series. In both of these settings, it is important to identify and recognize global features: where is the index finger, the keyhole, the fracture? Following common usage, we denote by point cloud data any collection of points in E*n*, though the connotation is that of a (perhaps noisy) sample of points on a lower-dimensional subset." [514]

Vin de Silva and Robert Ghrist in their work "Homological Sensor Networks", directed to topology as "the rapidly evolving area of applied computational topology":

"The need to move from local to global is one which a large spectrum of engineers and scientists are finding to be prevalent. Very few of the calculus-based tools with which they are most familiar prove sufficient. Recently, it has been demonstrated that homology theory is useful for problems in data analysis and shape reconstruction, computer vision, robotics, rigorous dynamics from experimental data, and control theory…Topology is especially keen at giving criteria for when one can or cannot find a particular global object (a homeomorphism, a nonzero section, an isotopy, etc.): this falls under the rubric of *obstruction theory*. This perspective is one which has not yet permeated the applied sciences, in which the question, *"What is possible?"* is usually approached from the top-down, *"Here's something we can build,"* as opposed to the bottom-up approach that topological methods yield. A brilliant example of this obstruction-theoretic viewpoint in an applied context is Farber's *topological complexity* for robot motion planning. In this article, we use homology theory to give coverage criteria for networked sensors which are 'nearly senseless.' It seems counterintuitive that one can provide rigorous answers for a network with neither localization

[514] Robert Ghrist in his article "Barcodes: The Persistent Topology of Data", 2007, Bulletin of American Mathematical Society, Volume 45, Number 1, January 2008, Pages 61–75

capabilities nor distance measurements. A topologist is not surprised that such coarse data can be integrated into a global picture. Some engineers are. Homological methods have the pleasant consequence that they may allow engineers to focus on designing simpler sensors which are nevertheless useful in a security network. Why bother miniaturizing GPS for "smart dust" if you can solve the problem without it? If topological methods can determine the minimal sensing needed to solve a global problem, then such methods may have significant impact on the way systems and sensors are developed and deployed."[515]

The nodes from the structure of Hegel's Science of Logic can be represented as a network and graph.

For the purpose of the demonstration and simplification in the following lines and illustration I will use four elements, corresponding to the four syllogisms in Hegel.

Graphs are among the most ubiquitous models of both natural and human-made structures. They can be used to model many types of relations and process dynamics in physical, biological and social systems. Many problems of practical interest, such as *data*, issues, themes and clusters, can be represented by graphs. Gpaphs are used to represent networks of communication, data organization, computational devices, the flow of computation, etc.

The edges may be directed (asymmetric) or undirected (symmetric).

For example, if the vertices represents two of the four conditional factors /indicators/ of particular data or issue, let say these are the "being" and the "nothing" or "quality" and "quantity", and there is an edge between these two /like these two are in mutual interaction/, then this is an undirected graph, because if "quality" A interact with "quantity" B, then "quantity" B also interact with "quality" A.

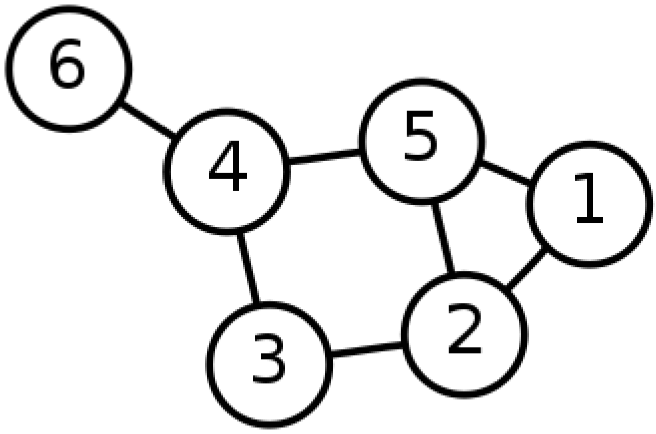
On the other hand, if the vertices represent the four conditional factors /indicators/ such as the four syllogisms in Hegel, and there is an edge from "quality" A to "quantity" B, when A act on B, then this graph is directed, because acting on some vertice is not necessarily a symetric

[515] Vin de Silva and Robert Ghrist in their work "Homological Sensor Networks", http://www.math.upenn.edu/~ghrist/preprints/noticesdraft.pdf

relation (that is, is “quality” act on the quantity does not necessarily imply the reverse. This latter type of graph is called a *directed* graph and the edges are called *directed edges* or *arcs*.

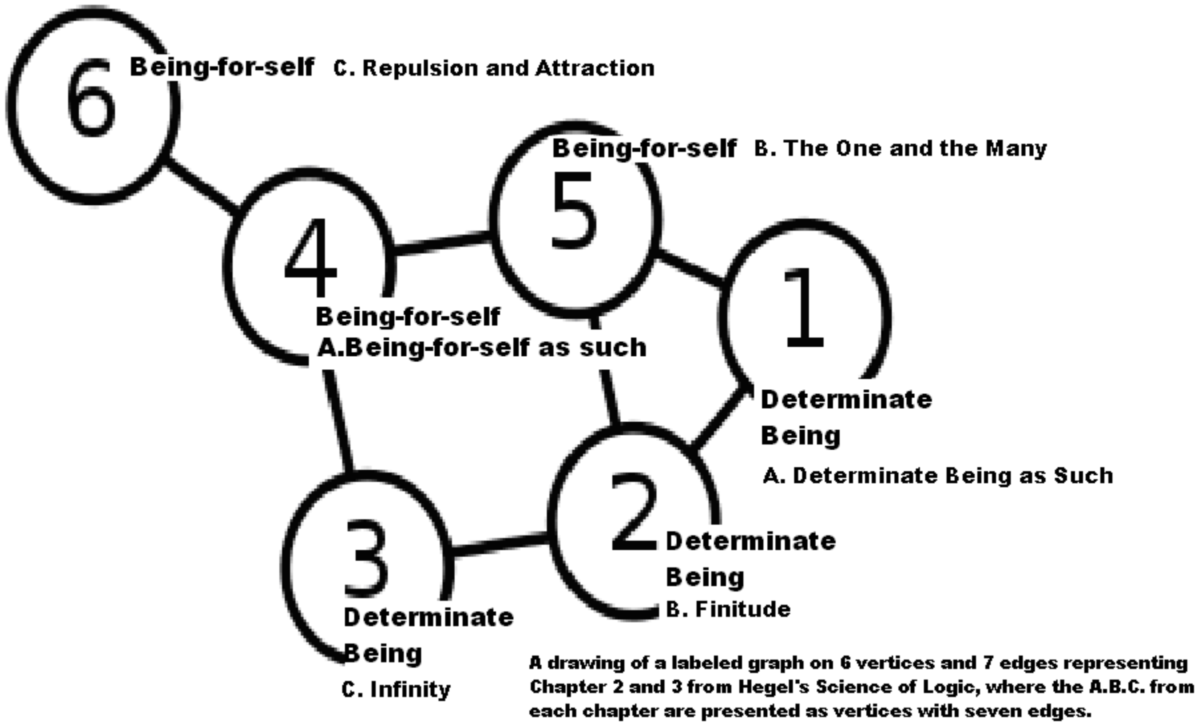
In mathematical term a network is represented by a graph. In the most common sense of the term, a graph is an ordered pair $G = (V, E)$ comprising a set $V$ of vertices or nodes or point together with a set $E$ of edges or lines, which are 2-element subsets of $V$ (i.e., an edge is related with two vertices, and the relation is represented as unordered pair of the vertices with respect to the particular edge).

The paper written by Leonhard Euler on the Seven Bridges of Koningsberg and published in 1736 is regarded as the first paper in the history of graph theory. This paper, as well as the one written by Vandermonde on the knight problem, carried on with the *analysis situs* initiated by Leibniz. Euler's formula relating the number of edges, vertices, and faces of a convex polyhedron is at the origin of topology.

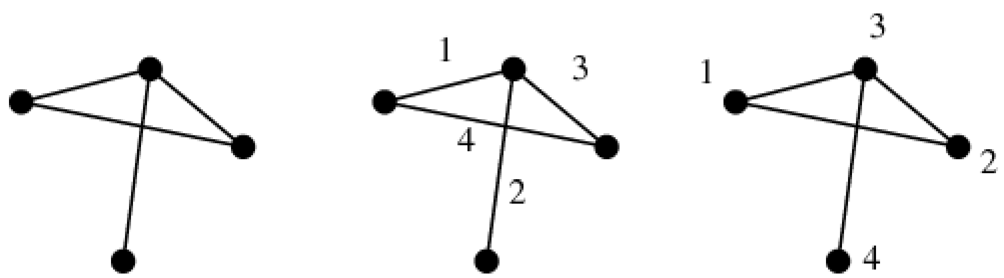


A drawing of a labeled graph on 6 vertices and 7 edges

We can draw a labeled graph on Hegel’s chapter 2 from Science of Logic **Determinate Being – with the three subsection of** A. Determinate Being as Such; B. Finitude; C. Infinity, **and chapter 3** from Science of Logic **Being-for-self -** A.Being-for-self as such - B. The One and the Many - C. Repulsion and Attraction , where the capital numbers of the subsections from two chapters forms six vertices and seven edges.



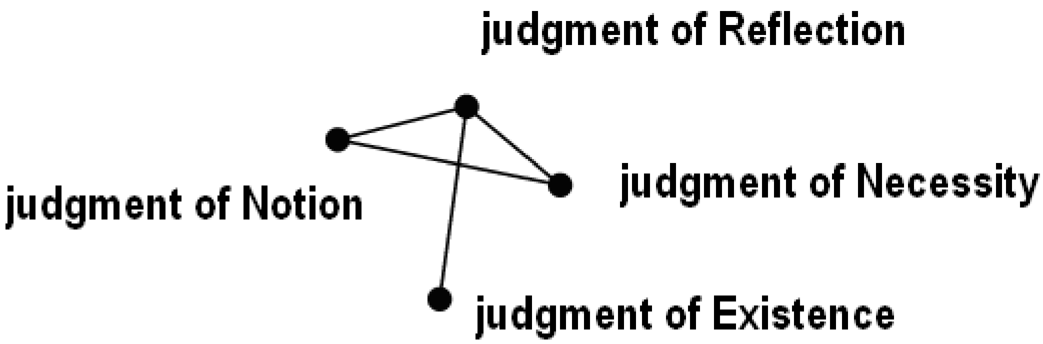

Here is the graph diagram of Chapter 2 (**Determinate Being**) and Chapter 3 (**Being-for-self**) from Hegel's Science of Logic with two sets. The first set is the set of vertices presenting **the three subsections from Chapter 2 -** A. Determinate Being as Such; B. Finitude; C. Infinity, **and the subsections from chapter 3 -** A.Being-for-self as such - B. The One and the Many - C. Repulsion and Attraction. The second set is the set of edges. The vertex set is just a collection of the labels for the vertices, a way to tell one vertex from another. The edge set is made up of unordered pairs of vertex labels from the vertex set.



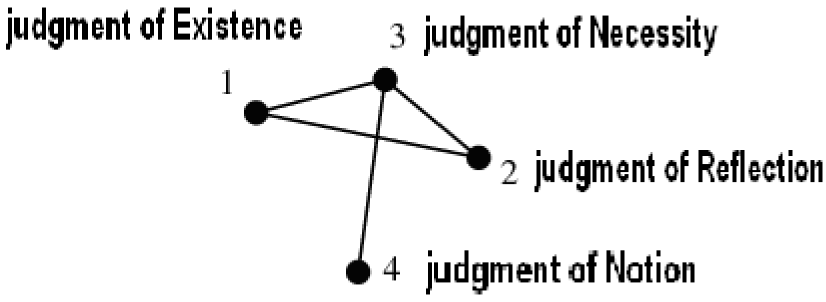

*unlabeled graph* *edge-labeled graph* *vertex-labeled graph*

- **Judgment of Existence.**
- **Judgment of Reflection.**
- **Judgment of Necessity.**
- **Judgment of the Notion.**

In the graph of Hegel's judgments, the four judgments in Hegel – judgments of Existence – Reflection – Necessity - Notion[516] are presented as a collection of dots, with some pair of dots connected by lines. The four dots, the four judgments are called vertices, and their lines are called edges.

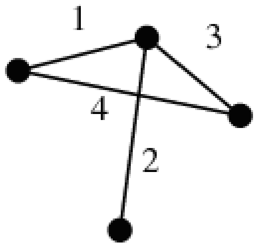



*unlabeled graph*



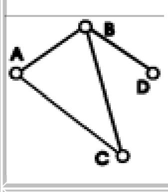

*vertex-labeled graph*

[516] David Gray Carlson, 2005, Why Are There Four Hegelian Judgments?, p.114:125, in Hegel's theory of the subjects, David G. Carlson, ed. 2005, Palgrave Macmillan 2005

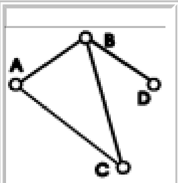



*edge-labeled graph*

Here is a diagram of a graph of the four judgments in Hegel – judgments of Existence – Reflection – Necessity - Notion, and the sets that the graph is made from:

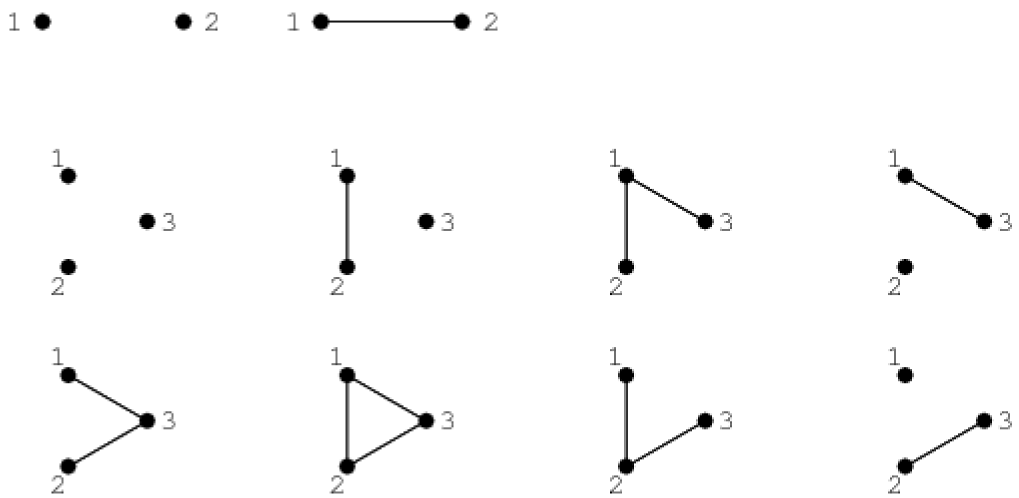
| | |
|---|---|
|   | V={A,B,C,D}<br>--The vertex set.<br>E={(A,B),(A,C),(B,C),(B,D)}<br>--The edge set. |
| A graph diagram. | The sets that make up a graph. |

Where:

| | |
|---|---|
|   | V={A=judgment of Existence, B=_judgment of Reflection, C=judgment of Necessity, D=judgment of Notion}<br>--The vertex set.<br>E={(A,B),(A,C),(B,C),(B,D)}<br>--The edge set. |

A labeled graph of Hegel's four judgment $G=(V,E)$ is a finite series of graph vertices $V$ with a set of graph edges $E$ of 2-subsets of $V$. Given a graph vertex set $V_n=\{1,2,\ldots,n\}$, (in our graph vertex set it will be V={1, 2, 3, 4} ) the number of vertex-labeled graphs is given by $2^{n(n-1)/2}$.

Two graphs $G$ and $H$ with graph vertices $V_n=\{1,2,\ldots,n\}$ are said to be isomorphic if there is a permutation $p$ of $V_n$ such that $\{u,v\}$ is in the set of graph edges $E(G)$ iff $\{p(u),p(v)\}$ is in the set of graph edges $E(H)$.



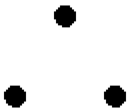
The term "labeled graph" when used without qualification means a graph with each node labeled differently (but arbitrarily), so that all nodes are considered distinct for purposes of enumeration. The *total* number of (not necessarily connected) labeled $n$-node graphs for $n = 1$, 2, ... is given by 1, 2, 8, 64, 1024, 32768, ... (Sloane's A006125; illustrated above), and the numbers of connected labeled graphs on $n$-nodes are given by the logarithmic transform of the preceding sequence, 1, 1, 4, 38, 728, 26704, ... (Sloane's A001187; Sloane and Plouffe 1995, p. 19).

The numbers of graph vertices in all labeled graphs of orders $n = 1$, 2, ... are 1, 4, 24, 256, 5120, 196608, ... (Sloane's A095340), which the numbers of edges are 0, 1, 12, 192, 5120, 245760, ... (Sloane's A095351), the latter of which has closed-form

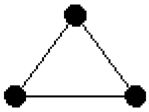
$$e(n) = n(n-1)\, 2^{n(n-1)/2-2}.$$

Similar graph diagram of the four judgments in Hegel – judgments of Existence – Reflection – Necessity - Notion could be build and represented for the sub areas /issues/ of each one of the four types of judgment:_

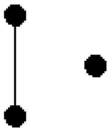
## $3K_1$ = co-triangleB?



**triangle = $K_3$ = $C_3$Bw**

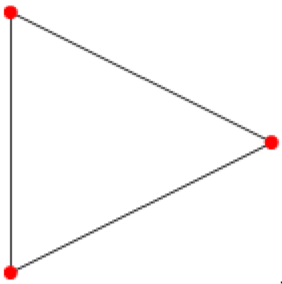


**$P_3$BO**



**$P_3$Bg**

## Triangle Graph



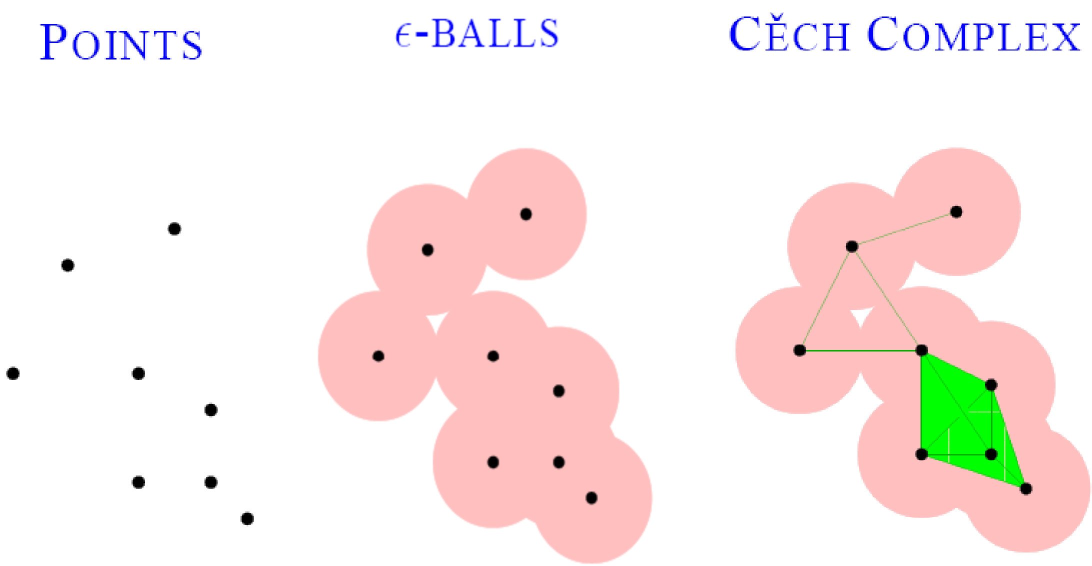
http://www.graphclasses.org/smallgraphs.html#nodes3

**3 vertices - Graphs are ordered by increasing number of edges in the left column.**

Leaving the representation of Hegel's four judgments and the example of Chapter 2 (**Determinate Being**) and Chapter 3 (**Being-for-self**) from Hegel's Science of Logic with two sets (where the first set is the set of vertices presenting **the three subsections from Chapter 2 -** A. Determinate Being as Such; B. Finitude; C. Infinity, **and the subsections from chapter 3 -** A.Being-for-self as such - B. The One and the Many - C. Repulsion and Attraction) through the theory of graphs, we are moving to another topological venture – **the concept of topological persistence**.

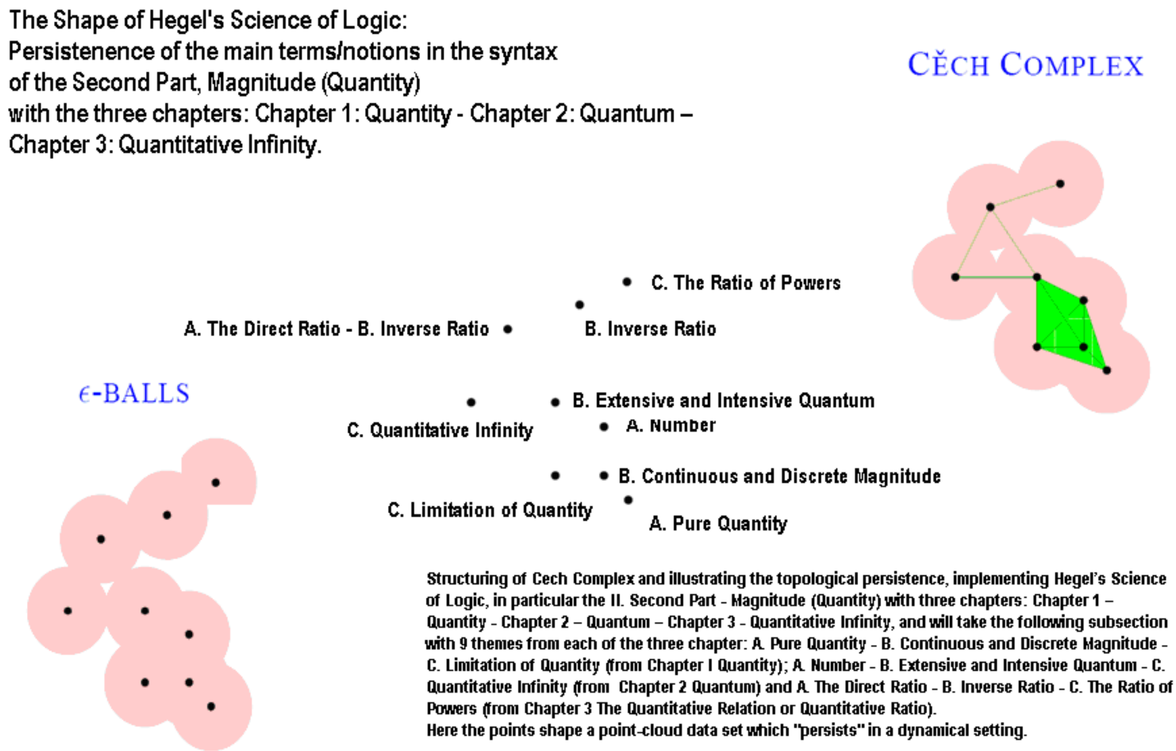
Now we will go back to the issue of structure of Hegel's Science of Logic, in particular the II. Second Part - Magnitude (Quantity) with three chapters: Chapter 1 – Quantity - Chapter 2 – Quantum – Chapter 3 - Quantitative Infinity, and will take the following subsection with 9 themes from each of the three chapter: A. Pure Quantity - B. Continuous and Discrete Magnitude - C. Limitation of Quantity (from Chapter I Quantity); A. Number - B. Extensive and Intensive Quantum - C. Quantitative Infinity (from Chapter 2 Quantum) and A. The Direct Ratio - B. Inverse Ratio - C. The Ratio of Powers (from Chapter 3 The Quantitative Relation or Quantitative Ratio). These are 9 themes. Our intention is to bring these 9 issues into the Topological Data Analysis, and to demonstrate how these 9 issues could be represented according to the Topological Data Analysis. This will demonstrate how the transition from typology to topology is possible.

The 9 issues in Hegel's chapter 1-2-3 (part II of SL and their sub /issues/ representing the Typology of Themes and Dynamics in Hegel's logic, will be treated as points in the sense of topological persistence, thus the points shapes a point-cloud data set which "persists" in a dynamical setting.



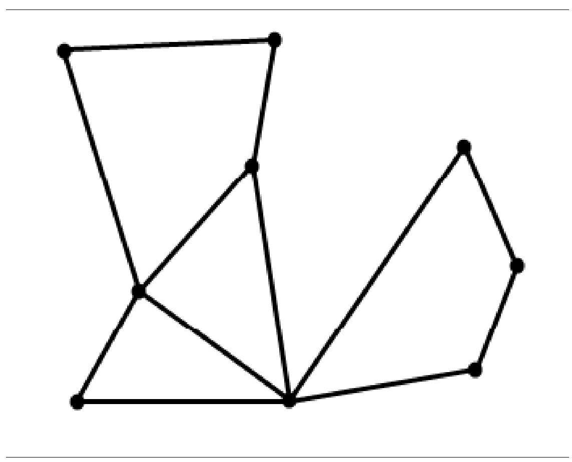

The ideas of topological persistence

Let us take out for example the following 9 .....




Let us take out a sheet of paper and draw any 9 number of dots on the paper, the dots represents the typology of 9 issues from Part II, chapter 1, 2 and 3. Next is to connect the dots with lines. The lines may not cross each other as they move from dot to dot. Every dot on our page must be connected to every other dot through a sequence of lines. If we are using 9 dots, our picture should look like in the illustration above.

If we are using 10 dots, our picture should look something like this:

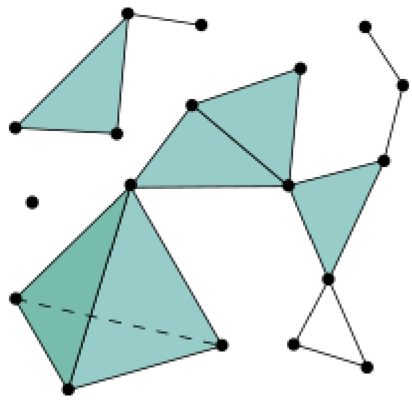


We have to count the number of dots (D), lines (L), and regions separated by lines (R). We should not forget to count the outside as a region too.  When we compute D - L + R, we get -

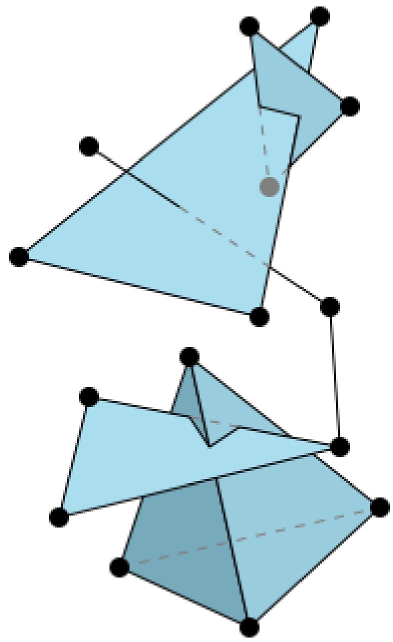
2! (not to be confused with 2! = 2 x 1). This number D - L + R is the **Euler Characteristic of the shape**. The Euler Characteristic classifies a large collection of shapes.[517]

In our demonstration we are moving to the next issue of Topological data analysis – **the simplicial complexes.**

In mathematics, a simplicial complex is a topological space of a certain kind, constructed by "gluing together" points, line segments, triangles, and their n-dimensional counterparts. Simplicial complexes should not be confused with the more abstract notion of a simplicial set appearing in modern simplicial homotopy theory.



A simplicial 3-complex.

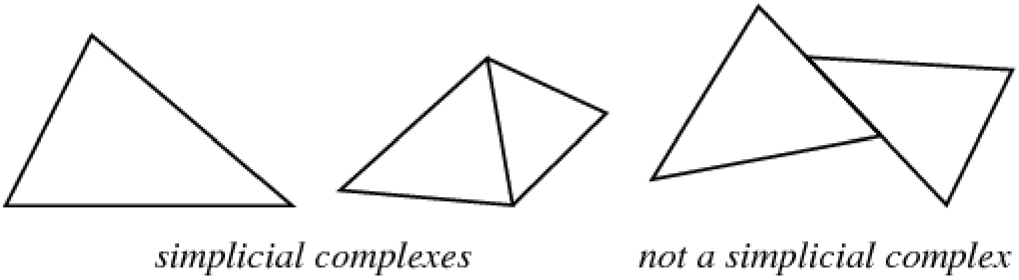


Not a Simplicial Complex

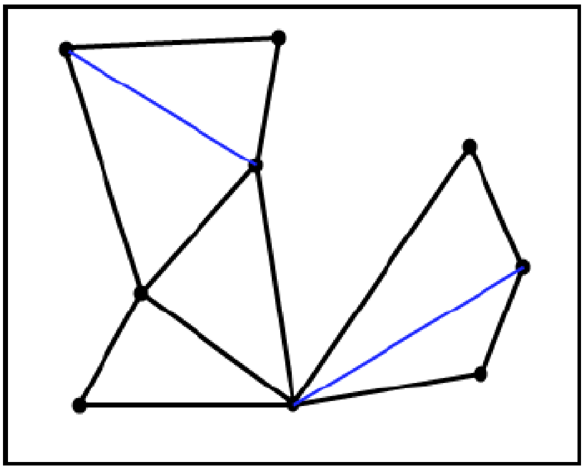
[517] see: http://en.wikipedia.org/wiki/Euler_characteristic

and

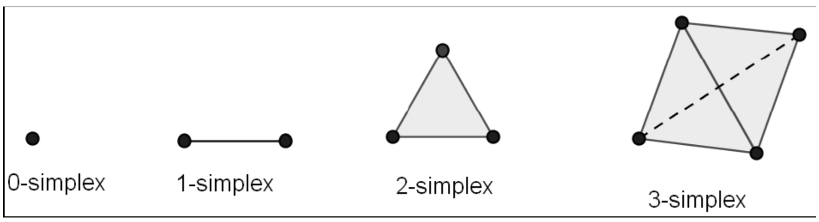
An arrangement of simplices that is not a valid simplicial complex.




Looking at the picture we drew before, we see the collection of points (vertices) and connected by lines (edges), than add more lines in such a way that each quadrilateral is built up by triangles. The figure will be this:

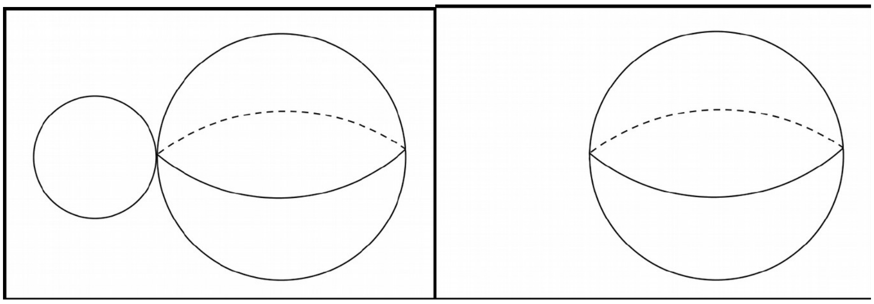


It is Important to know that adding these /blue/ lines doesn't change the Euler Characteristic! And we should know why!? The reason is that our drawing is an example of a 2-dimensional simplicial complex. We can build an n-dimensional version of a simplicial complex! The building blocks will be points, lines, triangles, tetrahedra, n-simplicies.



An n-simplex is an n-dimensional generalization of a triangle.

The Euler characteristic is very useful (and an active subject of research), but sometimes fails at distinguishing topological spaces. Let us see the two figures below. Both have the same Euler characteristic!



The topological differences between these two shapes are "visible" due to the nth Betti number, which we denote by Bn, of a simplicial complex is the number of n-dimensional holes in the complex. The Betti numbers record the significant topological features of the shapes. Looking at these two shapes we can find their Betti numbers.

**We can use topology to analyze the data, including data. The points of the 9 terms/ notions in part II of Hegel's Science of Logic with their sub issues, are fundamental building blocks of simplicial complexes.**

**We can build a sequence of related simplicial complexes from the point-cloud and examine the Betti numbers [518] of the complex at each sequence.**

[518] The term "Betti numbers" was coined by Henri Poincare after the Italian mathematician **Enrico Betti** (1823 – 1892), now remembered mostly for his 1871 paper on topology that led to the later naming after him of the Betti number. Betti discovered Betti's theoreme, a result in the theory of elasticity.

In algebraic topology, the **Betti numbers** are used to distinguish topological spaces based on the connectivity of *n*-dimensional simplicial somplexes. For the most reasonable finite-dimensional spaces (such as compact manifolds, finite simplicial complexes or CW complexes), the sequence of Betti numbers is 0 from some points onward (Betti numbers vanish above the dimension of a space), and they are all finite.

A torus has one connected component ($b_0$), two circular holes ($b_1$,the one in the center and the one in the middle of the "donut"), and one two-dimensional void ($b_2$, the inside of the "donut") yielding Betti numbers of 1 ($b_0$),2 ($b_1$),1 ($b_2$). The $n^{th}$ Betti number represents the rank of the $n^{th}$ homology group, denoted $H_n$, which tells us the maximum amount of cuts that must be made before separating a surface into two pieces or 0-cycles, 1-cycles, etc. These numbers are used today in fields such as simplicial homology, computer science, digital images, etc.

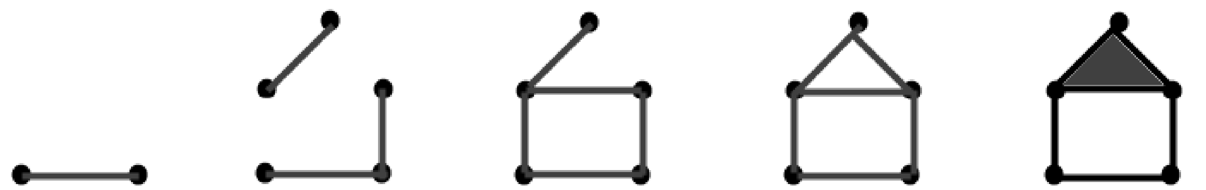
**Since we are assuming the data cloud is sampled from an underlying topological space, we recover the space by looking for topological features that persist as we pass from complex to complex in the sequence.**

Reminding that the main method used by topological data analysis is to replace a set of data points with a family of simplicial complexes, indexed by a proximity parameter. This converts the data set into global topological objects.

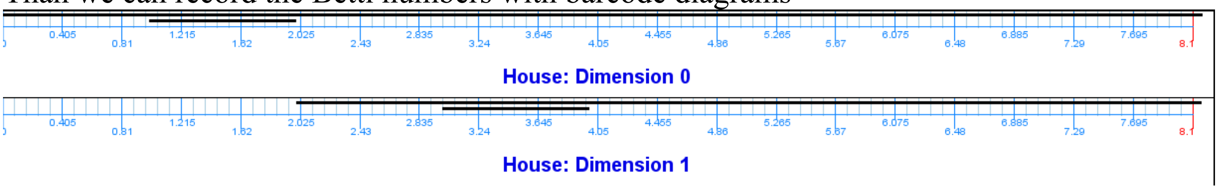
The second step is to analyse these topological complexes via algebraic topology — specifically, via the new theory of persistent homology.

The third step is to encode the persistent homology of a data set in the form of a parameterized version of a Betti number which will be called a barcode.

The illustration bellow represents a sequence of simplicial complexes that build up to a house.
:



Than we can record the Betti numbers with barcode diagrams

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The first few Betti numbers have the following definitions for 0-dimensional, 1-dimensional, and 2-dimensional simplicial complexes:

- $b_0$ is the number of connected components
- $b_1$ is the number of one-dimensional or "circular" holes
- $b_2$ is the number of two-dimensional "voids" or "cavities"

The two-dimensional Betti numbers are easier to understand because we see the world in 0, 1, 2, and 3-dimensions, however. The following Betti numbers are higher-dimensional than apparent physical space.

**The logical structure of concepts and syntax in Hegel's logic can be presented as topological space - simplicial complex and series of simplicial complexes.**

**Triadic structure of Hegel's logic - The triadic conception of Hegel's dialectic -corresponds to the simplex (plural simplexes) in geometry, where the notion of a triangle or tetrahedron is generalized to arbitrary dimensions.**

**The Triadic structure of Hegel's Logic allow us to build a family of simplicial complexes as representation model of Hegel's master terms (notions) from Science of Logic. Each triple can be constructed as a simplicial complex and the whole structure of Hegel's Logic can be presented as a family of simplicial complexes.**

In mathematics, a **simplicial complex** is a topological space of a certain kind, constructed by "gluing together" points, line segments, triangles, and their *n*-dimensional counterparts (see illustration). The purely combinatorial counterpart to a simplicial complex is an abstract simplicial complex.

In geometry, a **simplex** (plural: *simplexes* or *simplices*) is a generalization of the notion of a triangle or tetrahedron to arbitrary dimensions. Specifically, a ***k*-simplex** is a *k*-dimensional polytope which is the convex hull of its $k + 1$ vertices. More formally, suppose the $k + 1$ points $u_0, \ldots, u_k \in \mathbb{R}^n$ are affinely independent, which means $u_1 - u_0, \ldots, u_k - u_0$ are linearly independent. Then, the simplex determined by them is the set of points

A regular 3-simplex or tetrahedron

In topology and combinatorics, it is common to “glue together” simplices to form a simplicial complex. The associated combinatorial structure is called an abstract simplicial complex, in which context the word “simplex” simply means any finite set of vertices.

The total number of faces is always a power of two minus one. This figure (a projection of the tesseract) shows the centroids of the 15 faces of the tetrahedron.

**Why simplices can be used in interpretation of element of Hegel’s syntax in his Logic?**

Because **there is homology between the elements (terms, categories and notions) presented in the Logic, and the simplicial complexes are used to define a certain kind of homology[519] called simplicial homology. There is simplicial homology between the speculative elements of Hegel’s logic. Hegel’s language and syntax.**

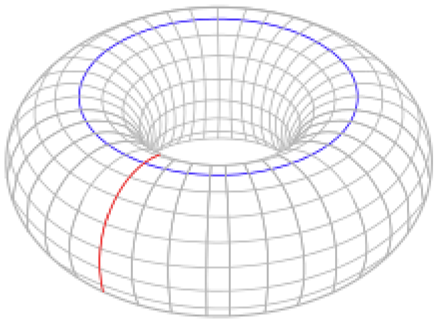
[519] **Homology** refer to various of science and disciplines such as anthropology (analogy between human beliefs, practices or artifacts owing to genetic or historical connections); biology (any characteristic of biological organisms that is derived from a common ancestor); chemistry (the relationship between compounds in a homologous series); mathematics (a procedure to associate a sequence of abelian groups or modules with a given mathematical object); homology modeling (a method of protein structure prediction); psychology (behavioral characteristics that have common origins in either evolution or development); sociology (a structural 'resonance' between the different elements making up a socio-cultural whole).

**The term Homological refer both to homological word and homological algebra** (as a branch of mathematics). An **autological word** (also called **homological word** or **autonym**) is a word that expresses a property that it also possesses (e.g. the word "short" is short, "noun" is a noun, "English" is English, "pentasyllabic" has five syllables, "word" is a word, "sesquipedalian" is a long word. The opposite is a **heterological** word, one that does not apply to itself (e.g. "long" is not long, "verb" is not typically a verb, "monosyllabic" has five syllables, "German" is not German, etc.).

**Hegel' syntax can be seen as 'topological syntax'** [520]

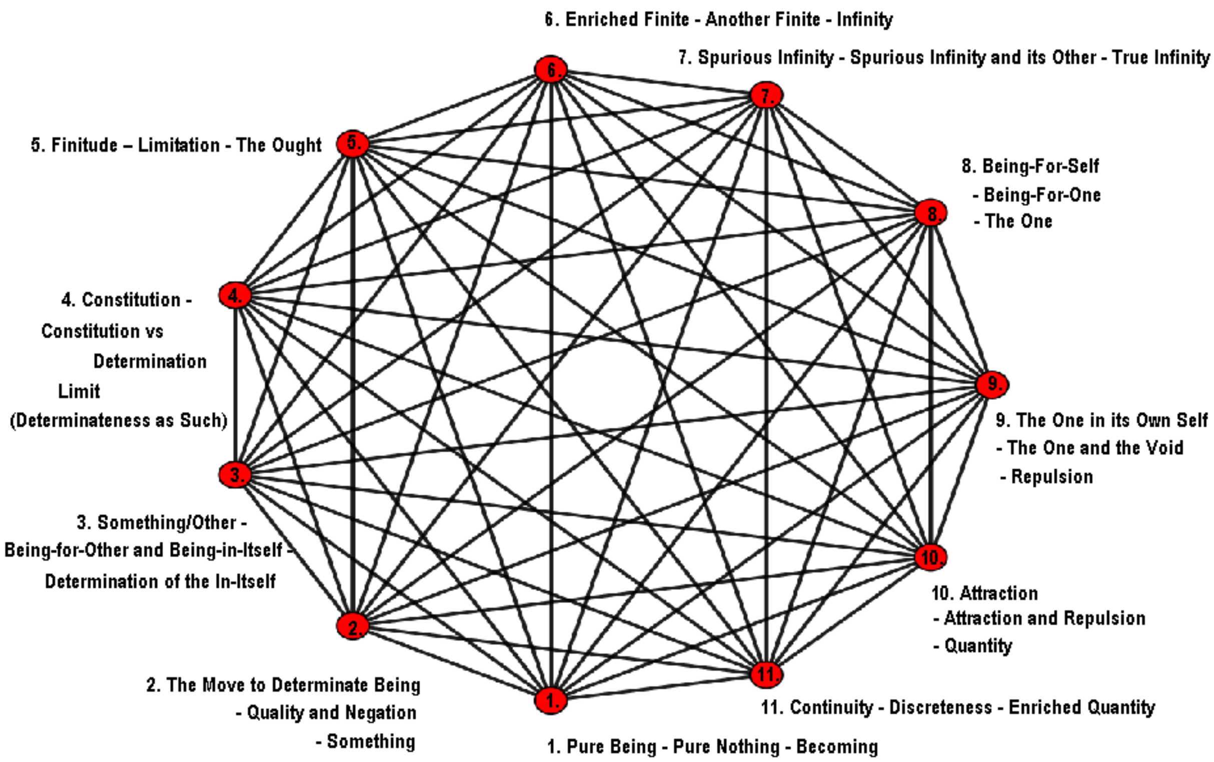
$$H_k(T) = \begin{cases} \mathbb{Z} & k = 0, 2 \\ \mathbb{Z} \times \mathbb{Z} & k = 1 \\ \{0\} & \text{otherwise} \end{cases}$$

The two independent 1D holes form independent generators in a finitely-generated abelian group, expressed as the Cartesian product group $\mathbb{Z} \times \mathbb{Z}$.



**Homology** theory starts with the Euler polyhedron formula, followed by Riemann's definition of genus and *n*-fold connectedness numerical invariants in 1857 and Betti's proof in 1871 of the independence of "homology numbers" from the choice of basis.

[520] See Wolfgang Wildgen, Per Aage Brandt, Rene Thom, Semiosis and Catastrophes: René Thom's Semiotic Heritage, page 56)

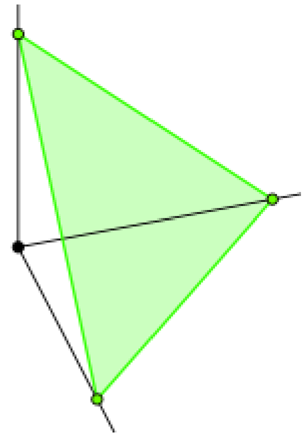



**Ten figures from Carlson's Hegel's Theory of Quality:**
**Represented as Regular hendecaxennon (10-simplex)**

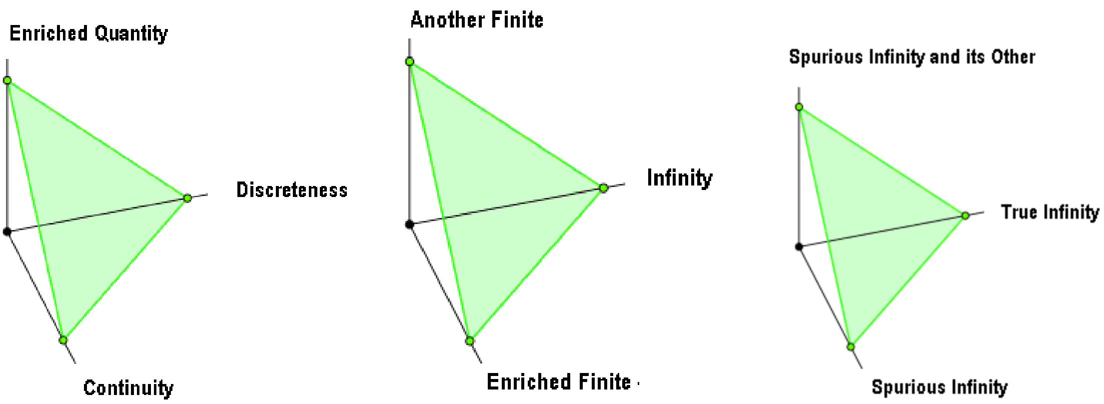
In geometry, a 10-simplex is a self-dual regular 10-polytope. It has 11 vertices, 55 edges, 165 triangle faces, 330 tetrahedral cells, 462 5-cell 4-faces, 462 5-simplexe 5-faces, 330 6-simplex 6-faces, 165 7 - simplex 7-faces, 55 8 - simplex 8-faces, and 11 9-simplex 9-faces. Its dihedral angle is $\cos^{-1}(1/10)$, or approximately 84.26°.

Geometric simplicial complexes were first formalized in the seminal work of Poincaré (H. Poincaré. Complément à l'analysis situs. Rendiconti del Circulo Matematico di Palermo 13:285–343).

Each of the 10 – simplexes represents the standard 2-simplex in $\mathbf{R}^3$ triangle with three vertices and three edges.



The standard 2-simplex in $\mathbf{R}^3$




**Demonstration model**

**on How the logical structure of concepts and syntax in Hegel's logic**

**can be presented as topological space**

**- simplicial complex and series of simplicial complexes.**

**Hegel intuitively constructed his Science of Logic, in particular his Objective Logic as topological space in the epistemological mode of his speculative thinking, cobording categories and notions, cobording understanding, dialectic and speculative logic in manifold of his Science.**

**Topological (in) Hegel or Hegel's Analysis Situs of his Science of Logic can be presented topo-logically as simplicial complex,** a topological space, constructed by "gluing together" any of the particular categories and relation between them within the emerging thriads – not only mathematically but topologically as points, line segments, triangles, and their n-dimensional counterparts. Hegel's logic developes and works as an **algebraic topology**, where the categories and notions are simplices used as building blocks to construct not only philosophical topology but an topological philosophy, where 'logical' or 'philosophical' in unfolded as 'topological' in the class of topological spaces called simplicial complexes. The space(s) of Hegel's categories in their multiplicity (manifold) built from simplices glued together in a combinatorial fashion of speculative thought. Hegel's simplicial complexes are used to define a certain kind of homology between the categories that can be defined as simplicial homology of quality and quantity, being, noting and becoming, as simplicial homology of judgement. The associated combinatorial structure of Hegel's Objective Logic can be called an abstract simplicial complex, in which context the word "simplex" simply means any finite set of vertices. Hegel's thriads works in the mode of geometry, where a **simplex** (plural: *simplexes* or *simplices*) is a generalization of the notion of a triangle or tetrahedron to arbitrary dimensions. Specifically, a ***k*-simplex** is a *k*-dimensional polytope which is the convex hull of its $k+1$ vertices. More formally, suppose the $k+1$ points $u_0, \ldots, u_k \in \mathbb{R}^n$ are affinely independent, which means $u_1 - u_0, \ldots, u_k - u_0$ are linearly independent. Then, the simplex determined by them is the set of points

A regular 3-simplex or tetrahedron

A 2-simplex is a triangle, a 3-simplex is a tetrahedron, and a 4-simplex is a 5-cell.

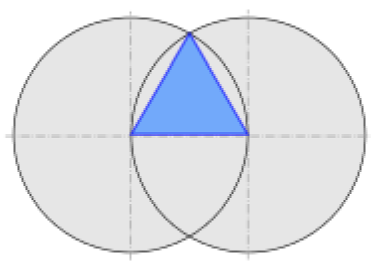
Triadic structure of Hegel's logic - The triadic conception of Hegel's dialectic -corresponds to the simplex (plural simplexes) in geometry, where the notion of a triangle or tetrahedron is generalized to arbitrary dimensions.

The Triadic structure of Hegel's Logic allow us to build a family of simplicial complexes as representation model of Hegel's master terms (notions) from Science of Logic  Each triple can be constructed as a simplicial complex and the whole structure of Science of Logic can be presented as a family of simplicial complexes.

Both Carlson and Maybee represent in their drawing the diagram of Hegel's main notions as circles, perhaps following Hegel's statement that the logic is circle – circle of circles .Yet from the circles we can derive the three partite structure of Hegel's logic – the triangle and move from the triangle to the simplex – or simplexes, just to end up with the proposition that the structure of Hegel's Logic can be presented and seen as simplicial complex.

The transformation from Carlson's and Maybee's diagrams to the the simplexes, involve an mirror answer to the question - How we can move from circle to triangle (simplex) and from simplex (triangle) to sphere?

A **triangle** is a polygon with three edges and three vertices. It is one of the basic shapes in geometry. A triangle with vertices *A*, *B*, and *C* is denoted $\triangle ABC$. In Geometry, we can construct equilateral triangle with compass and straightedge. The proof that the resulting figure is an equilateral triangle is the first proposition in Book I of Euclid's Elements - Construction of equilateral triangle:



## Prospective Research based on the present thesis

Prospective Research based on the present thesis will be applied research focused on the methodology of cultural and social dynamics[521] seen as topological dynamics[522], implementing on topological epistemology[523] and topological ontology[524], with an emphasis on 'topological

[521] The father of the term and the contemporary content of 'Cultural Dynamics' is Pitirim Aleksandrovich Sorokin (1889 – 1968). His magnum opus, the monumental Social and Cultural Dynamics /Social and Cultural Dynamics: A Study of Change in Major Systems of Art, Truth, Ethics, Law and Social Relationships (1957 (reprinted 1970) ed.)/ spanned 2,500 years and attempted to isolate the principles of social change as they were manifested in his studies of art, philosophy, science, law ethics, religion and psychology.

[522] The American mathematician George David Birkhoff is considered the founder of Topological Dynamics. In 1913, Birkhoff proved Poincare's "Last Geometric Theorem," a special case of the three-body problem, a result that made him world famous. In 1927, he published his Dynamical Systems his Dynamical Systems. See: Birkhoff, George David. 1913. "Proof of Poincaré's geometric theorem" Trans. Amer. Math. Soc. 14: 14–22.; and Birkhoff, George David. 1913. "Proof of Poincaré's geometric theorem" Trans. Amer. Math. Soc. 14: 14–22.; and Birkhoff, George David. 1917. "Dynamical Systems with Two Degrees of Freedom" Trans. Amer. Math. Soc. 18: 199–300

[523] Dimitrov, Borislav (2013), Philosophical Topology and Epistemology: The new frontier of Topology as Epistemology, unpublished.

[524] Dimitrov, Borislav (2013),"Topological Ontology and Logic of Qualitative quantity", https://www.academia.edu/3237237/Topological_Ontology_and_Logic_of_Qualitative_quantity;

turn' and 'topological approaches' to human geography, economics, law[525], auditing[526], in particular with the aim to introduce - How topological logic and dialectic of Hegel's Qualitative quantity can contribute to the the analytical foundations of research methodology, within the debate between the two paradigms in the research methods – qualitative vs. quantitative research.

In mathematics, topological dynamics is a branch of the theory of dynamical systems in which qualitative, asymptotic properties of dynamical systems are studied from the viewpoint of general topology. The central object of study in topological dynamics is a topological dynamical system, i.e. a topological space, together with a *continuous transformation*, a *continuous flow*, or more generally, a *semigroup of continuous transformations* of that space. My further aim will be to investigate Hegel's Topological Qualitative quantity both as category and research method within the interaction between cultural and social dynamics,

[525] Dimitrov, Borislav (2012) "Cultural Phenomenology of Law and Topological Approach to Law", A Series of papers presenting the essentials of Topological Approach to Law : Qualitative quantity - The Cultural Phenomenology of Literature and … The Cultural Phenomenology of Law; Law and Literature Movement; Cognitive Science and The Law - Topological Approach To Law; Phenomenology of Law; Law and Social Choice: Qualitative quantity, Topological Social Choice and Topological Approaches to Law; The proposition of Qualitative quantity mode of Inquiry in the Classic Debate – Qualitative vs Quantitative research; The Topological approach of Charles Sanders Peirce's qualitative-ness and The Topological Qualitative quantity; The philosophy of Émile Boutroux – a profound influence on Henri Poincaré and Charles Peirce. „Ariadne – Topology and Cultural Dynamics – Institute for Cultural Phenomenology of Qualitative quantity", http://ariadnetopology.org/3.html;

[526] Dimitrov, Borislav (2014), Auditor Independence within An Auditing Analysis Situs: Topology of Places as factor for enhancing auditor independence, competence, and audit quality, between the global and local: Topological Approach to Audit Dynamics, Focused on Auditor Independence, Competence and Audit Quality through Qualitative quantity methodology and Topological Data Analysis, Philosophy of Science for Social Science, Lund University, Faculty of Social Science;

Dimitrov, Borislav (2011) "The Relevance of Topological Approach, based on Qualitative quantity research method, to Audit Dynamics and Auditing Research – Cultural Phenomenology of Audit and Auditing research", „Ariadne – Topology and Cultural Dynamics – Institute for Cultural Phenomenology of Qualitative quantity", http://ariadnetopology.org/Cultural_Phenomenology_of_Audit_Dynamics_and_Auditing_Research_web.pdf and topological dynamics, due to the exhibit form of this category – the gradual changes and the continuous transformations.

The American mathematician George David Birkhoff is considered the founder of Topological Dynamics. In 1913, Birkhoff proved Poincare’s "Last Geometric Theorem," a special case of the three-body problem, a result that made him world famous.[527] In 1927, he published his Dynamical Systems.[528]

John WP Phillips, in his paper On Topology (2013)[529], critically examines the recent arguments asserting a topological turn in culture, the range of topologically informed interventions in social and cultural theory, and remarks that such contemporary fashionable notions of ‘topological approaches’ and ‘becoming topological of culture’ “demands a greater critical reflection than the notion of a ‘topological turn’ suggests.” Richard Ek establishes in Theorizing the Earth (2010), that “The spatial turn is actually a ‘philosophical turn’ repackaged and promoted as a fundamental concern with question of space, place and polity in the social sciences and humanities.” (Ek, Richard, Mekonnen, T. 2010:49-66)

## 1. Topology and Economics

Topology is not foreign discipline in the economic and behavioral economics. The presence of *topology* in economics and mathematical economic could be traced back to Leon Walras and Joseph Schumpeter. The concept of growth as economic growth and economic development was pioneered with *Schumpeter’s evolutionary economic*, build upon the understanding of continuous notion, thus topological.

*Schumpeter’s emphasis on the importance of qualitative change and qualitative approach* is well known and recognized. Schumpeter considered Leon Walras to be the "greatest of all economists". Schumpeter claims: Walras is … greatest of all economists. His system of economic equilibrium, uniting, as it does, the quality of ‘revolutionary" creativeness with the

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529 Phillips, John WP. (2013), On Topology, Theory, Culture and Society, 9/2013; 30(5):122-152: http://www.researchgate.net/profile/John_Phillips20/publications [accessed Mar 21, 2015].

quality of classic synthesis, is the only work by an economist that will stand comparison with the achievements of theoretical physics."[530] (Schumpeter, J. A., 1994 [1954])

Robert Solow, one of Schumpeter's outstanding students, developed his theory of general economic equilibrium, later enhanced by Kenneth Arrow and Gerard Debreu.

In 1954, Debreu published his breakthrough paper "Existence of an Equilibrium for a Competitive Economy", together with Kenneth Arrow. Debreu and Arrow provided a *definitive mathematical proof of the existence of a general equilibrium using topological methods.*

The *relationships between topology and economics* are firmly established by the Argentine American mathematical economist Graciela Chichilnisky, who earned her second Ph.D. under the supervision of Gerard Debreu. She is best known for her *topological social choice theory.* According to Graciela Chichilnisky, as she establishes in her article "Topology and Economics: The Contribution of Stephen Smale" (1990)[531], "Topology has long ago been connected with general equilibrium of market" and "All proofs of existence of an economic equilibrium use topological tools". (Graciela Chichilnisky, G., 1990)

The connection between topology and general equilibrium nodels in economics has been established in the works of John Von Numann, Keneth Arow and Gerard Debreu /1954/. Arow and Debreu formalized the Valrasian model of general equilibrium.

For Chichilnisky, *topology is intrinsically necessary for the understanding of the fundamental problem of conflict resolution in economics*. The attempt to find solution to conflicts among individual interests or individual preferences led to three different theories about how economies are organized and how they behave – these are: general equilibrium theorem, the theory of games and social choice theory. For Chichilinsky *each of these theories leads naturally to mathematical problems of topological nature*. Steve Smale contributed to the first

530 Schumpeter, J. A., 1994 [1954], History of economic analysis, Oxford University Press, p. 795

531 Graciela Chichilnisky, Topology and economics : the contribution of Stephen Smale : presented at the Colloquium in Honor of Stephen Smale, University of California at Berkeley, 1990

and the second approaches, but Chichilinsky argues that Smale's work is connected also with the social choice theory.

Influenced by the ideas of the Bourbaki group, in the field of behavioral economics, Gerard Debreu firmly established topology as a tool in the concepts of preferences, public or social choice.

In the book "*Topology and Markets*" (1998), edited by Graciela Chichilnisky[532], in the forewords summarizing the result of a workshop on "Geometry, Topology and Markets" (1994) at the Fields Institute of Research in Mathematical science in Waterlo, Ontario, Chichilnisky regarded topology as the new frontier of research:

"Today we face a new frontier, at the age of social evolution and the impact that humans have on nature. This frontier leads to new, mathematical questions and research. It includes economic issues about how human organize themselves and distribute resources, their impact on nature, and the tension between individual goals and social goals. Market emerge in this context as a powerful instrument of social organization and resource allocation. To explore this new frontier a workshop on "Geometry, Topology and Markets" took place from July 23 to 28 of 1994 at the Fields Institute of Research in Mathematical science in Waterlo, Ontario. The title of the workshop reflects the use of qualitative mathematics applied to the theory of markets.

Qualitative structural results are useful in the social sciences, such as economics, where measurment are often unreliable. Indeed the beginning of the mathematical theory of markets was closely related with topology: John Von Neuman, who formalized and produced one of the first proofs of existence of a market equilibrium, remarked that his result depended on topological tools such as fixed point theorems, and that there was no other way to obtain such results than by using topological tools. ….Market equilibrium, can be understood as the solutions to a system of simultaneous non-linear equations. Finding such solution requires typically topological tools."[533] (Chichilnisky, G.,1994)

[532] Graciela Chichilnisky, "Geometry, Topology and Markets" (1994), Fields Institute of Research in Mathematical science in Waterlo, Ontario
[533] Graciela Chichilnisky, "Geometry, Topology and Markets" (1994), Fields Institute of Research in Mathematical science in Waterlo, Ontario

**How topological logic and dialectic of Hegel's Qualitative quantity can contribute to the debate on the analytical foundations of the evolutionary approach to economics**

**Qualitative change is the central analytical issue of an evolutionary analysis.** The topological notion of qualitative quantity is embedded in the exhibit form of this category – gradual changes and discontinuity, thus topological homeomorphism. The gradualness of changes typical for the qualitative quantity made this category and method suitable for the so called evolutionary approach to economics.

Methodological debate has been accompanying modern evolutionary economics since its inception with Nelson and Winter [534](1982)." (Nelson, R.R., Winter**,** S.G. 1982)
There has been much concern in the analytical foundations of the evolutionary economics, with the relevance and applicability of biological analogy, in particular of neo-Darwinism and Darwinian concepts, and with the self-organization approach.

The Hungarian-born mathematician and economist Nicholas Georgescu-Roegen was the first to formally demonstrate the thermodynamic foundations of the economic process. He had a profound influence on leading alternative economic theorists in the field of ecological economics. Georgescu-Roegen's influence extends well beyond his well-known work on the thermodynamic foundations of economic systems and his career involved his ambitious attempt to reformulate economic process as "bioeconomics", a new style of dialectical economic thought.

Nicholas Georgescu-Roegen[535] has considered that the dialectical method is more suitable, than the pure quantitative approach regarded as the arithmomorphism, for social sciences and

[534] Nelson, R.R. and S.G. Winter (1982), *An Evolutionary Theory of Economic Change*. Cambridge, MA: Belknap Press.

[535] Georgescu-Roegen, N. (1975), Energy and Economic Myths. The Southern Economic Journal 41(3): 347-381; reprinted in: Georgescu-Roegen (1976: 3-36)
Georgescu-Roegen, N. (1976), Energy and Economic Myths. Institutional and AnalyticalEconomic Essays. New York: Pergamon Press
Georgescu-Roegen, N. (1978), De la science économique à la bioéconomie [From economic science to bioeconomics]. Revue d'économie politique 88(3): 337-382
Georgescu-Roegen, N. (1979), Methods in Economic Science. Journal of Economic Issues 8(2): 317-328
Georgescu-Roegen, N. (1992), Nicholas Georgescu-Roegen about Himself. In: Szenberg, M. (ed.) (1992), Eminent Economists: Their Life Philosophies. Cambridge: Cambridge University Press, 128-159

especially for economics. Roegen accepts the arithmomorphic model but strictly as a tool and only accompanied by dialectical argumentation which is the only one to highlight change, quality, and novelty.[536] (Georgescu-Roegen, N., 1975), Christoph Heinzel, established that "**Qualitative change is the central analytical issue of an evolutionary analysis".** [537] Heinzel investigates "the **importance of qualitative change in** Nicholas Georgescu-Roegen **and Schumpeter's analytical** system.

Georgescu-Roegen's fundamental concern was for qualitative change as the basic feature of (economic) evolution and its substantial analysis.

The **quality-quantity relationship and economic analysis are discussed by** Heinzel as follow:

"Given the overwhelming importance of qualitative change in the economic process and the mathematical character of economic analysis, Georgescu-Roegen was particularly concerned with the consistent treatment of the relationship between quality and quantity in economic analysis. In this section, first two general contributions of his are introduced which occupy an important place at the basis of his argument. The first one is his consideration of the possibility, and determination of different degrees, of measurability, the second his distinction between arithmomorphic and dialectical concepts as two categories of concepts upon which all sciences necessarily rely. Finally, the conclusions deriving from them for economic analysis as well as his methodical suggestions are considered."[538]

Georgescu-Roegen concern was on the measurability issue and arithmomorphic and 'dialectical' concepts. He investigated in a fundamental manner the possibilities of measurement and the nature of the concepts economics deals with, emphasizing that quality always precedes quantity. Georgescu-Roegen stress of the limits of arithmomorphic

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[536] Ion Pohoata, Nicholas Georgescu-Roegen. From Cause – Effect and Arithmomorphism to Dialectics: http://anale.feaa.uaic.ro/anale/resurse/33_Pohoata_I_-_NG_Roegen__De_la_cauza-efect_si_aritmomorfism_la_dialectica.pdf/.

[537] Christoph Heinzel, Schumpeter and Georgescu-Roegen on the foundationsof an evolutionary analysis - The problem of qualitative change, its methodical implications and analytical treatment, Dresden Discussion Paper in Economics No. 10/06, ISSN 0945-4829, p.4

[538] Ibid.

representation, however, went together with the expression of its necessity: “there is a limit to what we can do with numbers, as there is to what we can do without them”[539]

According to Heinzel, “the issue of measurability arises in any study that aims at a meaningful quantitative analysis. In this context, Georgescu-Roegen felt it necessary to precise the established notions of cardinal and ordinal, defining the three categories of *cardinal*, *weakly cardinal* and *purely ordinal* measurability. He describes *ordinal* measurability, which means the assignment of ranking numbers to things considered, as the most basic of these categories, for the possibility of ranking consitutes the precondition of any kind of measurement. *Cardinal* measurability presupposes furthermore, in a strict sense, that the entity to be measured is physically indifferently subsumable and subtractable.”[540]
Georgescu-Roegen’s general stance can be seen from the first lines of his 1964 article “Measure, Quality, and Optimum Scale”:

“It is difficult to contemplate the evolution of the economic science over the last hundred years without reaching the conclusion that its mathematization was a rather hurried job. [… M]any epistemological issues, which ought to have been clarified before any attempt at using the new tools for the old tasks were ignored or avowedly bypassed.

The most important of these issues is that of the relation between quality and quantity. [… Q]uality is our most basic concept. It is definitely prior to that of quantity, for before we can speak of a measure of A relative to B we must distinguish between A and B in some way or other. And as we do not yet have a measure of A this way can be but qualitative”[541]

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[539] Georgescu-Roegen [1958] 1966: 275, 1971: 94: Georgescu-Roegen, N. (1958), The Nature of Expectation and Uncertainty. In: Bowman, M.J. (ed.), Expectations, Uncertainty and Business Behavior. A publication of the Social Science Research Council, New York, 11-29; reprinted in: Georgescu-Roegen (1966: 241-275); Georgescu-Roegen, N. (1966), Analytical Economics. Issues and Problems. Cambridge, MA: Harvard University Press, p.275 and Georgescu-Roegen, N. (1971), The Entropy Law and the Economic Process. Cambridge, MA: Harvard University Press, p.94.

[540] Christoph Heinzel, Schumpeter and Georgescu-Roegen on the foundationsof an evolutionary analysis - The problem of qualitative change, its methodical implications and analytical treatment, Dresden Discussion Paper in Economics No. 10/06, ISSN 0945-4829.

[541] Georgescu-Roegen, N. (1976), Energy and Economic Myths. Institutional and Analytical Economic Essays. New York: Pergamon Press, p. 271.

According to Georgescu-Roegen, all concepts which relate to qualitative change are necessarily ‘dialectical’ (in his sense) in character, for neither qualitative change nor a quality itself can be fully represented by an arithmomorphic scheme.[542] He provides the example with the water: “[B]y a physical operation independent of any measure we can subsume a glass of water and a cup of water or take out a cup of water from a pitcher of water. In both cases the result is an instance of the same entity, ‘water.’[543]

Georgescu-Roegen in his work from 1971, “*The Entropy Law and the Economic Process”,* (Georgescu-Roegen, N., 1971). introduced a serious criticism – with his - the so called “anti-arithmomorphic” dialectic position. Georgescu-Roegen argued that all we are seeing in such models is discontinuous changes in variables or functions and not a true qualitative change. The latter would presumably be something beyond the ability of mathematics to describe. It would not be simply a change in function or values of existing state variables, but the emergence of a completely new variable or even a new function or set of functions and variables. But at a minimum such structural changes imply qualitatively different dynamics, even if the variables themselves are still the same, in some sense.

According to J. Barkley Rosser, “Among the deepest problems in political economy is that of the qualitative transformation of economic systems from one mode to another. A long tradition, based on Marx, argues that this can be explained by a materialist interpretation of the dialectical method of analysis as developed by Hegel. Rosser, argues “that nonlinear dynamics offers a way in which a mathematical analogue to certain aspects of the dialectical approach can be modelled, in particular, that of the difficult problem of qualitative transformation alluded to above.”[544] (Rosser, Barkley, 2000)

The topological qualitative quantity is the methodological answer of the problem of qualitative change within the nonlinear dynamics, catastrophe theory, chaos theory, and

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[542] Georgescu-Roegen, N. (1971), *The Entropy Law and the Economic Process*. Cambridge, MA: Harvard University Press, ch.3

[543] Georgescu-Roegen, N. (1971), *The Entropy Law and the Economic Process*. Cambridge, MA: Harvard University Press, p.98

[544] Barkley Rosser, “Aspects of Dialectics and Nonlinear Dynamics” - Barkley Rosser, “Aspects of Dialectics and Nonlinear Dynamics”- Cambridge Journal of Economics, May 2000, vol. 24, no. 3, pp. 311-324

complex emergent dynamics theory model. Qualitative quantity is the tool for methodical implications and analytical treatment of this new science.

Rosser discuss certain elements of catastrophe theory, chaos theory, and complex emergent dynamics theory models that allow for a mathematical modelling of "quantitative change leading to qualitative change", one of the widely claimed foundational concepts of the dialectical approach, and a key to its analysis of systemic political economic transformation."

These approaches are all special cases of nonlinear dynamics, and their special aspects which allow for this analogue depend on their nonlinearity. We note that there are some linear models that generate discontinuities and various exotic dynamics, e.g. models of coupled markets linked by incommensurate irrational frequencies. However, we shall not investigate these examples further.

Rosser acknowledges that "In most linear models, continuous changes in inputs do not lead to discontinuous changes in outputs, which will be our mathematical interpretation of the famous "quantitative change leading to qualitative change" formulation."[545]

Reviewing the basic dialectical concepts, Rosser discusses how catastrophe theory can imply dialectical results, considering chaos theory from a dialectical perspective. He examines some emergent complexity concepts along similar lines, culminating in a broader synthesis.

For Rosser, these phenomena of the new science are focused on the idea of wholes consisting of related parts implied by this formulation. For Levins and Lewontin [546] (Levins, R. and Lewontin, R., 1985) this is the most important aspect of dialectics and they use it to argue against the mindless reductionism they see in much of ecological and evolutionary theory. Levins [547] (Levins, R., 1968) in particular identifying holistic dialectics with his 'community matrix' idea. As Rosser states, this can be seen as working down from a whole to its interrelated parts, but also working up from the parts to a higher order whole, and this latter

[545] Ibid.

[546] Levins, R. and R. Lewontin. 1985. *The Dialectical Biologist*, Cambridge, Harvard University Press

[547] Levins, R. 1968. *Evolution in Changing Environments*, Princeton, Princeton University Press

concept can be identified with more recent complex emergent dynamics ideas of self-organization of Turing[548] (Turing, A.M., 1952) and Wiener[549] (Wiener, N. 1961), autopoesis of Maturana and Varela[550] (Maturana, H.R., Varela.F., 1975.) emergent order of Prigogine [551] (Nicolis, G., Prigogine, I., 1977) and Kauffman, [552] (Kauffman S.A., 1993) anagenesis Boulding[553] (Boulding, K.E., 1978) and Jantsch[554] (Jantsch, E., 1979), and emergent hierarchy. It is also consistent with the general social systems approach of the dialectically oriented post-Frankfurt School – the works of Niklas Luhmann[555] (Luhmann, N., 1982) (Luhmann, N., 1996) and Habermas[556] (Habermas, J.,1979).

Rosser highlights the works of Friedrich Hayek[557] (Hayek, F.A., 1952) (Hayek, F.A., 1967) emphasizing on self-organization and developing an emergent complexity theory based on an

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548 Turing, A.M. 1952. The chemical basis of morphogenesis, *Philosophical Transactions of the Royal Society B,* vol. 237, no. 1

549 Wiener, N. 1961. *Cybernetics: or Control and Communication in the Animal and the Machine*, 2nd edition, Cambridge, MIT Press

550 Maturana, H.R. and F. Varela. 1975. *Autopoietic Systems*, Report BCL 9.4, Urbana, Biological Computer Laboratory, University of Illinois

551 Nicolis, G. and Prigogine, I. 1977. *Self-organization in Nonequilibrium Systems: From Dissipative Structures to Order Through Fluctuations*, New York, Wiley-Interscience

552 Kauffman S.A. 1993. *The Origins of Order, Self-Organization and Selection in Evolution*, Oxford, Oxford University Press

553 Boulding, K.E. 1978. *Ecodynamics: A New Theory of Societal Evolution*, Beverly Hills, Sage

554 Jantsch, E. 1979. *The Self-Organizing Universe: Scientific and Human Implications of the Emerging Paradigm of Evolution*, Oxford, Pergamon Press

555 Luhmann, N. 1982. The world society as social system, *International Journal of General Systems*, vol. 8, 131-8; Luhmann, N. 1996. Membership and motives in social systems, *Systems Research*, vol. 13, 341-8

556 Habermas, J. 1979. *Communication and the Evolution of Society*, translated by T. McCarthy, Boston, Beacon Press; Habermas, J. 1987. *The Philosophical Discourse of Modernity: Twelve Lectures*, translated by F.G. Lawrence, Cambridge, MIT Press , 1979, 1987; Offe, 1997

557 Hayek, F.A. 1952. *The Sensory Order*, Chicago, University of Chicago Press; Hayek, F.A. 1967. The theory of complex phenomena, in *Studies in Philosophy, Politics, and Economics*, Chicago, University of Chicago Press

early version of neural networks models and eventually [558] explicitly acknowledging his link with Ilya Prigogine and with Hermann Haken.[559] (Haken, H., 1983)

## 2. Topology and Law

Prospective Research based on the present thesis will be applied research focused on the interdisciplinary relation between Topology and Law, presented as Applied Topological Philosophy of Law and related with the relatively new but already well established research area, lead by the works of Ugo Pagallo [560] [561]. A Topological Approaches to Law is based on the series of works of Ugo Pagallo, who is building on Hayek's classical distinction between cosmos and taxis, i.e., evolution vs. constructivism, spontaneous orders vs. human (political)

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[558] Hayek, F.A. 1988. *The Fatal Conceit: The Errors of Socialism*, Chicago, University of Chicago Press, p.9

[559] Haken, H. 1983. *Synergetics: Nonequilibrium Phase Transitions and Social Measurement*, 3rd edition, Berlin, Springer-Verlag/.

[560] Member of the National Bar Association of Italy since 1986, Ugo Pagallo is a full Professor in Philosophy of Law at the Faculty of Law, University of Turin, since 2000. Ugo Pagallo held PhD in Philosophy of Law at the University of Padua (1990). In the last years, he has also delivered lectures at the universities of Helsinki, New York (IVR), Madrid (Complutense), Caracas (UCV), Buenos Aires (UCA), Honolulu, Tokyo (Meiji), China (Kunming), Stanford CA., Washington D.C. (Georgetown), and at the European University of Fiesole, Florence. Pagallo's main research interests are Digital and Comparative Philosophy, Topology of Complex Social Networks, History of Ideas and General Theory of Law, especially applied to transformations in European and International Law.

[561] See: U. Pagallo, Aliquid Est Sine Ratione: On Some Philosophical Consequences of Chaitin's Quest for O, in Randomness and Complexity. From Leibniz to Chaitin, edited by C. Calude, World Scientific: Singapore 2007, pp. 287-300; U. Pagallo, "Small world" Paradigm and Empirical Research in Legal Ontologies: A Topological Approach, in The Multilanguage Complexity of European Law: Methodologies in Comparison, edited by G. Ajani, G. Peruginelli, G. Sartor, and D. Tiscornia, European Press Academic Publishing: Florence 2007, pp. 195-210; U. Pagallo, "Small World" Paradigm in Social Sciences: Problems and Perspectives, in Glocalisation: Bridging the Global Nature of Information and Communication Technology and the Local Nature of Human Beings, edited by T. Ward Bynum, S. Rogerson, and K. Murata, e-SCM Research Center and University of Meiji: Tokyo 2007, pp. 456-465; U. Pagallo and G. Ruffo, On the Growth of Collaborative and Competitive Networks: Opportunities and New Challenges, in Ethicomp Working Conference 2007, edited by S. Rogerson e H. Yang, Yunnan University 2007, pp. 92-97; U. Pagallo and G. Ruffo, P2P Systems in Legal Networks: Another "Small World" Case, in "Eleventh International Conference on Artificial Intelligence and Law", Acm, Stanford, CA. 2007, pp. 287-288.

planning. According to Pagallo, "Law is indeed a good field in which to test that interchange we need among different theoretical outlooks that deal with complex social networks."[562]

For Pagallo, recent empirical evidence confirms that the informational complexity of the law is not reducible to taxis alone and, furthermore, orders spontaneously emerge from the complexity of the environment through specific laws of evolution. AI and Law, legal theory, etc., focuses on the taxis-side of the law. What Pagallo illustrated is the informational nature of complex social systems via a theory of spontaneous orders and an evolutionary theory of complex social networks. By distinguishing three levels of analysis, namely information as reality, for reality, and on reality, **a topological approach shows how information is produced and distributed in current legal systems**, how it is possible to harness these properties and obtain useful applications in the legal domain, while shedding further light on some aspects of current legal research, e.g., AI work on legal ontologies.

In his paper '**As Law Goes By: Topology, Ontology, Evolution',** Pagallo deals with Hayek's classical distinction between cosmos and taxis, i.e., evolution vs. constructivism, spontaneous orders vs. human (political) planning. According to Pagallo, recent empirical evidence confirms that the informational complexity of the law is not reducible to taxis alone and, furthermore, orders spontaneously emerge from the complexity of the environment through specific laws of evolution. Whereas, most of the time, today's research on AI & Law focuses on the taxis-side of the law, Pagallo's aim is to illustrate the informational nature of complex social systems via a theory of spontaneous orders and an evolutionary theory of complex social networks. By distinguishing three levels of analysis, namely information as reality, for reality, and on reality, a topological approach shows how information is produced and distributed in current legal systems, how it is possible to harness these properties and obtain useful applications in the legal domain, while shedding further light on some aspects of current AI research.

Pagallo discusses 'legal topologies' and 'legal ontology' in relation with the main applications of the "small world" concept and - topological approach to complex legal systems. Pagallo asserts that first, it is possible to deepen the analysis of some fields of artificial intelligence

[562] U.Pagallo, The Paradox of Elegance – A Very Short Introduction to the Topology of Complex Social Systems and the "Small World"-Paradigm in the Realm of Law - http://www.aisb.org.uk/convention/aisb09/Proceedings/SNAMAS09/FILES/RuffoG.pdf

and the law concerned with the study of legal reasoning and argumentation through computational methods as in the example of case-based legal reasoning, of knowledge discovery in legal databases, or of legal information retrieval [see Pagallo, Ruffo 2007a].

In addition, Topological approached to law are presented in the works of S. Muller-Mall, Legal Spaces - Towards a Topological Thinking of Law, Humboldt University Berlin, Germany, also Andreas Philippopoulos-Mihalopoulos's works as Law's Spatial Turn: Geography, Justice and a Certain Fear of Space, School of Law, University of Westminster, London.

Laurence H. Tribe, is one of the greatest living intradisciplinary scholar of constitutional law, who was interested in topology before he was interested in constitutional law. As he once compared the two disciplines: 'If you see some wonderful connection between a multidimensional homotopy space in fact, my senior thesis in mathematics related to the equivalence between two different definitions under which a multidimensional, closed loop in space would be equivalent to another seemingly different multidimensional, closed loop in space when you see a connection like that, the attempt to translate it into ordinary conversation is bound to be futile.' /c/o *Kathleen M. Sullivan, Law and Topology, 42 Tulsa L. Rev. 949 (2006). Available at: http://digitalcommons.law.utulsa.edu/tlr/vol42/iss4/11*

**3. How topological logic and dialectic of Hegel's Qualitative quantity can contribute to the the analytical foundations of research methodology, in particular within the debate between the two paradigms in the research methods – qualitative vs. quantitative research**

The relevance of these two paradigm is widely recognized in the research culture as the fact that "the two dominant research paradigms have resulted in two research cultures, "one professing the superiority of 'deep, rich observational data' and the other the virtues of 'hard, generalizable' . . . data". [563]

[563] R. Burke Johnson and Anthony J. Onwuegbuzie, "Mixed Methods Research: A Research Paradigm Whose Time Has Come", Educational Researcher, Vol. 33, No. 7, pp. 14–26: Reff. Sieber, S. D. (1973). The integration of fieldwork and survey methods. American Journal of Sociology*, 73*, 1335–1359. About the incompatability thesis – See: R. Burke Johnson and Anthony J. Onwuegbuzie Reff: "Both sets of purists – qualitative and quantitative - view their paradigms as the ideal for research, and, implicitly if not explicitly, they advocate the *incompatibility thesis* (Howe, K. R. (1988). Against the quantitative-qualitative

I will just make a note on some of the authors and their publication supporting the qualitative[564] and the quantitative[565] side of this debate, focusing on the proposed recently [566] "mixed methods research"[567] as "the third research paradigm" (by the researchers in the field of the educational and social research).

The aim of my prospective research based on the present thesis is to focus on the existence of the third approach or the third paradigm and to establish the relevance of "qualitative quantity" method research as the possible development of this third paradigm of integrative research. The proposition of qualitative quantity method of research is the foundation of the topological approach to dynamics of research culture. Although both qualitative and

incompatability thesis, or, Dogmas die hard. *Educational Researcher, 17*, 10–16. and Howe, K. R. (1992). Getting over the quantitative-qualitative debate.*American Journal of Education, 100*, 236–256.), which posits that qualitative and quantitative research paradigms, including their associated methods, cannot and should not be mixed."

564 **Qualitative mode in the debates is represented by:** - Campbell, D. T., & Stanley, J. C. (1963). *Experimental and quasiexperimental designs for research*. Chicago, IL: Rand McNally.; - Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills, CA: Sage) - Schwandt, T. A. (2000). Three epistemological stances for qualitative inquiry: Interpretivism, hermeneutics, and social constructionism. In N. K. Denzin & Y. S. Lincoln, *Handbook of qualitative research* (pp.189–213). Thousand Oaks, CA: Sage. - Smith, J. K. (1983). Quantitative versus qualitative research: An attempt to clarify the issue. *Educational Researcher, 12*, 6–13. - Smith, J. K. (1984). The problem of criteria for judging interpretive inquiry. *Educational Evaluation and Policy Analysis, 6*, 379–391.
Qualitative purists (also called *constructivists* and *interpretivists*) reject what they call positivism. They argue for the superiority of constructivism, idealism, relativism, humanism, hermeneutics, and, sometimes, postmodernism

565 **Quantitative mode in the debate is represented by:** - Ayer, A. J. (1959). *Logical positivism*. New York: The Free Press; - Maxwell, S. E., & Delaney, H. D. (2004). *Designing experiments and analyzing data*. Mahwah, NJ: Lawrence Erlbaum; - Popper, K. R. (1959). *The logic of scientific discovery*. New York: Routledge; - Schrag, F. (1992). In defense of positivist research paradigms, *Educational Researcher, 21*(5), 5–8.
These authors representing quantitative mode articulate assumptions that are consistent with what is commonly called a positivist philosophy. The quantitative claim is based on believe that social observations should be treated as entities in much the same way that physical scientists treat physical phenomena. Further, they contend that the observer is separate from the entities that are subject to observation. Quantitative purists maintain that social science inquiry should be objective.

566 R. Burke Johnson and Anthony J. Onwuegbuzie, "Mixed Methods Research: A Research Paradigm Whose Time Has Come", Educational Researcher, Vol. 33, No. 7, pp. 14–26; http://www.aera.net/uploadedFiles/Journals_and_Publications/Journals/Educational_Researcher/Volume_33_No_7/03ERv33n7_Johnson.pdf

567 As it is noted by R. Burke Johnson and Anthony J. Onwuegbuzie in their article, both of the authors of the article prefer the label *mixed research* or *integrative research* rather than *mixed methods research*. The alternative labels are broader, more inclusive, and more clearly paradigmatic. The authors chose to use the term *mixed methods* in the article Mixed Methods Research: A Research Paradigm Whose Time Has Come" because of its current popularity.

quantitative philosophies continue to be highly useful the advantage of the philosophy of qualitative quantity is laid by the topological notion of this category and method.[568]

The mixed methods research as the natural complement to traditional qualitative and quantitative research, is introduced by R. Burke Johnson and Anthony J. Onwuegbuzie in their article "Mixed Methods Research: A Research Paradigm Whose Time Has Come". The aim of the authors with this mixed mode research is to "to present pragmatism as offering an attractive philosophical partner for mixed methods research, and to provide a framework for designing and conducting mixed methods research. Providing an excellent review on the paradigm "wars" - qualitative vs quantitative - and incompatibility thesis /which posits that qualitative and quantitative research paradigms, including their associated methods, cannot and should not be mixed/, Johnson and Onwuegbuzie claimed that there are some commonalities between quantitative and qualitative research. The authors's thesis is based on the tenets of pragmatism in their explanation of the fundamental principle of mixed research and how to apply it. Johnson and Onwuegbuzie offered their own understanding and definition of the term "research paradigm" coined by Thomas Kuhn [569] who popularized the idea of a "paradigm". Khun pointed out that "paradigm" is a general concept including a group of researchers having a common education and an agreement on "exemplars" of high quality research or thinking. [570]
According to Johnson and Onwuegbuzie, the "research paradigm" as it is used in their article, is to "set of beliefs, values, and assumptions that a community of researchers has in common regarding the nature and conduct of research." There are ontological, epistemological, axiological and methododlogical levels in the research paradigm: "the beliefs include, but are not limited to, ontological beliefs, epistemological beliefs, axiological beliefs, aesthetic beliefs, and methodological beliefs." According to the the authors their use of the term "research paradigm" refers to a research culture. The argument of Johnson and Onwuegbuzie is that "there is now a trilogy of major research paradigms: qualitative research, quantitative research, and mixed methods research."

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[568] In Borislav Dimitrov, "The Topological notion of Qualitative quantity", 2009-2011
[569] Kuhn, T. S. (1962). *The structure of scientific revolutions*. Chicago, IL: University of Chicago Press.
[570] Kuhn, T. S. (1977). The essential tension: Selected studies in scientific tradition and change. Chicago, IL: University of Chicago Press.

In “Mixed Methods Research: A Research Paradigm Whose Time Has Come”, R. Burke Johnson and Anthony J. Onwuegbuzie offered the Development of a Mixed Methods Research **Typology.**

I will approach the issue of the third paradigm – the mixed methods research – with the topological “paradigm shift” illustrated by the “mixed-mode Semiosis” and the transition from Typological Mode to the Topological Mode proposed by Jay L. Lemke. [571] Jay L. Lemke introduced the mixed-mode Semiosis - Typological vs. Topological Semiosis. [572]

According to Lemke there are two fundamentally different kinds of meaning-making: "typological" meaning-making - meaning-by-kind /natural language/ and “topological” meaning-making - meaning-by-degree /visual language/, “which is more easily presented by means of motor gestures or visual figures -- the meaning of continuous variation or "topological" meaning.” Meaning-by-kind is qualitative and meaning-by-degree is quantitative.

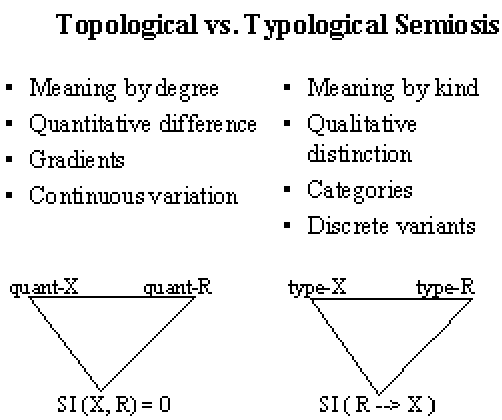


Figure by Jay L. Lemke, Typological vs. Topological Semiosis – .http://www-personal.umich.edu/~jaylemke/webs/wess/tsld002.htm

[571] Jay L. Lemke, “Topological Semiosis and the Evolution of Meaning” – http://www-personal.umich.edu/~jaylemke/webs/wess/index.htm

[572] Jay L. Lemke, Typological vs. Topological Semiosis – .http://www-personal.umich.edu/~jaylemke/webs/wess/tsld002.htmSee: Jay L. Lemke, “Mathematics in the middle: measure, picture, gesture, sign, and word, and Opening Up Closure: Semiotics Across Scales”.

For Lemke Typological semiosis is qualitative semiosis and Topological semiosis is quantitative semiosis. Qualitative character of typological semiosis determines the discrete variant and Quantitative character of topological semiosis and the continuous variation. I support the idea that qualitative quantity is possible to appear as categorical gradiant /mixture from categories in typological semiosis and gradients in topological semiosis/. The topological and continuous notion of qualitative quantity as gradualness and continuous variation or "topological" meaning" can be seen in Figure above given in Lemke's work " Mathematics in the middle: measure, picture, gesture, sign, and word". In "Opening Up Closure: Semiotics Across Scales", Lemke proposes the "Mixed-mode Semiosis".

In this mixed-mode of typological and topological semiosis, the domain of qualitative quantity could be seen, as "categorical gradiant" /Lemke/ of the "unified system for meaning-making" /Lemke/.

The Qualitative quantity Method Research due to the gradual and nondiscursive notion of the category /qualitative quantity/ and the ability to exhibit topological homeomorphism is the method /within the system of meaning-making/ with the ability to transform the form or the structure from typology to topology. The aim of the The Qualitative quantity Method Research is to create "Topology of meaning".

Typology of research topics in a specific area of research in research culture are necessary very useful to understand the relationships between the research topics. Since Aristotle, the organization of knowledge is based on the conceptualization, classification, typology or taxonomy. Typology is instrument of hierarchical system.

The semiotic bridge in qualitative vs quantitative debate is Qualitative quantity – the categorical gradiant which unite in one these two fundamentally different kinds of meaning-making - meaning-by-kind and meaning-by-degree in the continuous variation of "topological" meaning.

The proposed qualitative quantity method of research is not opposing the pragmatism of Charles Sanders Peirce, William James, and John Dewey or postpragmatism school of though.

As to Johnson and Onwuegbuzie established in their work the Pragmatism is the Philosophical Partner for Mixed Methods Research. Something more, the category of Qualitative quantity and the exhibit form of this category - topological notion - could be seen and unfold in the works of Charles Sanders Peirce.

### 3.1. The Topological approach of Charles Sanders Peirce's qualitative-ness

There are recent research works[573] [574] [575] giving us the ground to speak about The Topological Peirce.

The roots of Hegel's Qualitative quantity and the presence of the modern category of Qualitative quantity can be unfolded in the works of Charles Sanders Peirce. Peirce arrived at his own system of three categories after a thoroughgoing study of his predecessors, with special reference to the categories of Aristotle, Kant, and Hegel. The names that Peirce used for his own categories varied with context and occasion, but ranged from reasonably intuitive terms like *quality*, *reaction*, and *representation* to maximally abstract terms like *firstness*, *secondness*, and *thirdness*, respectively.

The influence of German tradition and especially of Hegel in Peirce categorical system is widly recognized. [576] The prevalence in Peirce's thinking of Kantian and Hegelian resources are well illustrated in his early essay "On a New List of Categories". Peirce's argument here patently draws on Kantian concepts of categorial analysis as a means of distinguishing ontological realms, although the categories he proposes, here denominated Quality, Relation, and Representation, are of very different character from Kant's. Instead, this triadic scheme of categories seems clearly, if roughly, correlate with the three movements of the Hegelian dialectic (considered as Being, Negation, and Mediation); in later formulations, Peirce even

[573] Robert W. Burch, "A Peircean Reduction Thesis: The Foundations of Topological Logic". Texas Tech University Press,Lubbock, TX, 1991.

[574] Helmut Pape, "Abduction and the Topology of Human Cognition" – http://user.uni-frankfurt.de/~wirth/texte/pape.html

[575] Arnold Johanson, "Modern Topology and Peirce's Theory of the Continuum", Transactions of the Charles S. Peirce Society, Vol.37, No..1, Indiana University Press, 2001,pp.1-12
http://www.jstor.org/pss/40320822

[576] See: Richard S. Beth, "On the Recent German reception of Peirce, with emphasis on Habermas", *Congressional Research Service (U.S.A.),* paper prepared for delivery at the Sixth Pan-European International Relations Conference of the Standing Group on International Relations of the European Consortium for Political Research, Turin, Italy, September 12-15, 2007

identifies his third category (in one of its aspects) with mediation (e.g., EP 2.183, in the fourth Harvard Lecture [1903]).

The category of Qualitative quantity is a sign, a sign of gradualness, a topological sign. The aspects of the Qualitative quantity involve not the dyadic relations typical for the physical sciences, but the triadic relations of the semiotic sciences. The interplay of qualitative andquantitative in the qualitative quantity is the interplay of the predicates of predicates.

As Peirce asserted:

“I will now say a few words about what you have called Categories, but for which I prefer the designation Predicaments, and which you have explained as predicates of predicates.”[577]
I will direct to the Qualitative quantity with the known Peirce’s expression – “That wonderful operation of hypostatic abstraction”:

“That wonderful operation of hypostatic abstraction by which we seem to create *entia rationis* that are, nevertheless, sometimes real, furnishes us the means of turning predicates from being signs that we think or think *through*, into being subjects thought of. We thus think of the thought-sign itself, making it the object of another thought-sign.”[578]
The notion of Qualitative quantity and the presence of the Topological Peirce could be discovered it Peirce’s Abduction [579], Peirce’s Theory of the Continuum [580] and his concept of Hypostatic abstraction[581].

[577] Charles Sanders Peirce, p. 522, "Prolegomena to an Apology for Pragmaticism", The *Monist*, vol.XVI, no.4, Oct. 1906, pp. 492-546, reprinted in the *Collected Papers* vol 4, paragraphs 530–572, see paragraph 549.
[578] Charles Sanders Peirce, p. 522, "Prolegomena to an Apology for Pragmaticism", The *Monist*, vol.XVI, no.4, Oct. 1906, pp. 492-546, reprinted in the *Collected Papers* vol 4, paragraphs 530–572, see paragraph 549.
[579] Helmut Pape, "Abduction and the Topology of Human Cognition"
[580] Arnold Johanson, "Modern Topology and Peirce's Theory of the Continuum", Transactions of the Charles S. Peirce Society, Vol.37, No..1, Indiana University Press, 2001,pp.1-12
[581] **Hypostatic abstraction** in mathematical logic /also known as **hypostasis** or **subjectal abstraction/**, is a formal operation that transforms an assertion to a relation. **Peirce’s Hypostatic abstraction:** for example "Honey is sweet" is transformed into "Honey has sweetness". Hypostasis changes a propositional formula of the form *X is Y* to another one of the form *X has the property of being Y* or *X has Y-ness*. This abstraction process takes an element of information about the properties of a subject, and conceives it to consist in the relation between the original subject and the property seen as a second subject. The existence of the latter subject, here *Y-ness*, consists solely in the

Helmut Pape is the author of the wonderful essay "Abduction and the Topology of Human Cognition", devoted to Charles Sanders Peirce. The term “Abduction” used by Peirce refer to a form of inference /alongside deduction and induction/ by which we treat a signifier as an instance of a rule from a familiar code, and then infer what it signifies by applying that rule.

In his essay Helmut Pape establish that "Peirce developed in his post-1894 writings a topological way of viewing logical relations between representations with categorically different logical status: topological connectivity provided for him a comprehensive model of a space in which the individual logical sequences, e.g. those started by an abduction, could be embedded." As it is directed by Helmut Pape, "for a study that treats Peirce's whole logic from the point of view of his topology, see Burch [1991].)". Robert W. Burch's work - "A Peircean Reduction Thesis: The Foundations of Topological Logic" is probably the most comprehencive study on Topological Peirce.

According to Helmut Pape the "logical role of abduction in Peirce depends on the topological property of hypotheses as singularities in the space of logical relations." In part IV of his essay "Qualia Regained: The Topology of the Mind and the Representation of Qualitative Possibilties ", Pape explains the ground of his thesis giving an account of the timeline of topological turn in Peirce:

"let us first take a look at what happened to Peirce's logic and metaphysics between 1878 and 1903. During this interval Peirce discovered Listing's topology and Cantor's transfinite sets. Around 1895 he developed a two-dimensional graphical logic, the so called "Existential Graphs". On the development of this graphical logic Peirce spent most of his energy generating a series of logical systems that comprise a complete first order logic, some weak modal logics and some semantical models similar to possible world semantics. Later on, in 1905, he uses Listing's classification of topological forms and transfomations for completeness proofs for his graphical logic. But before this he developed an evolutionary

truth of those propositions that have the corresponding concrete term, here *Y*, as the predicate. The object of discussion or thought thus introduced may also be called a **hypostatic object**. This definition is adapted from the one given by Charles Sanders Peirce (CP 4.235, "The Simplest Mathematics" (1902), in *Collected Papers*, CP 4.227–323). - The way that Peirce describes it, the main thing about the formal operation of hypostatic abstraction, insofar as it can be observed to operate on formal linguistic expressions, is that it converts an adjective or some part of a predicate into an extra subject, upping the arity, also called the *adicity*, of the main predicate in the process.

metaphysics in the years 1884 - 1893 trying to explain the origin of the physical universe and the laws of physics as a transformation that changes the ontological modality and the dimensionality of a state of unrestricted indetermination, which Peirce described as being less than nothing.

Richter mentions these developments only in passing, if at all. When, in 1898, in notes for a lecture series titled "The Logic of Events" he applied Listing's topological framework to the "objective logic" of cosmological development, Peirce assumes that there is a structural similarity between the topological status I.) of a hypothesis as long as it is considered as inferred by an abductive inference, and, II.) of every more determinate state that results from the zero-state before the origin of the physical universe.First of all,  both the abductive hypothesis and the more determinate state are phase transitions from states where the transformation changes the ontological modality and the dimensionality from a 0- to a 1-dimensionality." As Pape asserts - "to put it in more traditional, though not quite adequate philosophical terms, both transformations lead from the potential to the actual. To describe this transformation in a logic of cosmological events the traditional logic of monotonic, deductive necessary inferences was neither possible nor adequate because it would have required giving the possibility of a logical connection between the state of unrestricted indeterminacy and any more determinate state."[582]

Let us see the background of the topological turn in Peirce.

Topology is the study of continuity and connectivity, it is concerned with properties that are preserved under continuous deformations of objects, such as deformations that involve stretching, but no tearing or gluing. Topology emerged through the development of concepts from geometry and set theory, such as space, dimension, and transformation.

Leonhard Euler's 1736 paper on the Seven Bridges of Koningsberg is regarded as one of the first academic treatises in modern topology. In 1847 Johan Benedict Listing first introduced the term "Topologie" in German with his Vorstudien zur Topologie. Listing had used the word topology for ten years in correspondence before its first appearance in print. "Topology," its English form, was first used in 1883 in Listing's obituary in the journal Nature to distinguish

[582] Helmut Pape, "Abduction and the Topology of Human Cognition"

"qualitative geometry from the ordinary geometry in which quantitative relations chiefly are treated". Johan Benedict Listing (independently) discovered the properties of the half-twisted strip at the same time as August Ferdinand Mobius. In 1895 Henri Poincare, published his Analysis situs introducing the concept of homotopy and homology, as part of topology.

And here is Helmut Pape account on the development of the Topological Peirce:

"Only late in his life Peirce become aware of what peculiar meta-logical and topological properties an adequate account of hypothetical inference has to deal with and the decisive experience was his formulation of a logic-based evolutionary metaphysics from 1890-93. By 1898, when Peirce planned to give a lecture-series at Harvard on his evolutionary metaphysics, the title "The Consequences of Mathematics" or "The Logic of Events" reflects a change in the style of reasoning: To some extent the biological, evolutionary, physiological and psychological concepts of the 1891 version of his evolutionary metaphysics were either replaced or interpreted by logical and mathematical notions. In particular he applied and developed topological concepts - adapted from early topology by Benedict Listing [1862] - to the cosmological development of an ontological phase transition necessary to initiate cosmology evolution."[583]

"This ontological transformation is not only described as a hypothetic inference but is described in the language of topological conditions for changes in and between dimensions of shape. Peirce describes singularities by the restriction in the type of movement in which a singular point or line can be reached or transformed. He constructs his description for the layout and the changes in the zero-state of cosmological development, which is a state of possible qualities, by drawing on topological notions. I will be arguing that for Peirce's notion of abduction the logical relation of singularities to qualities is crucial. Peirce tries to account for qualities in terms of one-dimensional continua with a hyperbolic shape. Of course, there may be singularities that break them, and so he has to explain when and how singularities occur. In doing so, he describes a tension between the consequences of his topological account and constraints that have to be added to the cosmological picture for external, non-topological reasons. This tension arrises, because "unless we suppose the continuum of possible quality to have topical singularities - which is quite inadmissible at the present stage

[583] Ibid.

of development, though they may be evolved later, - then we must admit that when a quality diminishes in intensity to zero it can then continue the same line of change and increase in intensity in a definite quality, a contrary quality" [NEM IV, p. 130]. Even worse, in this model the contrary quality would have its maximum exactly in the zero-point of its counterpart. There is no factual evidence for such a strange connection between qualities. But why are we allowed to treat contrary qualities as disjoint and how do we get singularities after all while still using the topological model? What Peirce appeals to is the fact that we do indeed know a case where continua of qualities and singularities are connected and this connection creates a psychological discontinuity "because we know that sensation as a limen which is a point of discontinuity.... This singularity cannot exist in the possible quality itself..." [Ibid., 131] But before we can explain and understand this suggestive move that defines the backdrop of Peirce's mature, topologically based concept of abduction in 1901 in more detail, let us take a look at some of the features and function of hypothetic inference and let us ask how he does account for the changes that took place."[584]

The Hegel's topology of the fold enveloped in the fourfold of infinities - /1/. the bad qualitative infinity; /2/. the good qualitative infinity; /3/. the bad quantitative infinity; /4/. the good quantitative infinity. [585] and the fourfold of the Qualitative quantity in my interpretation of - **quantitative quantity - quantitative quality - qualitative quantity - qualitative quality -** could be revieled in Peirce's Categories, or "Predicaments" in his claim of "predicates of predicates". For Peirce, meaningful predicates have both extension and intension, so predicates of predicates get their meanings from at least two sources of information, namely, the classes of relations and **the qualities of qualities** to which they refer. Considerations like these tend to generate hierarchies of subject matters, extending through what is traditionally called the *logic of second intentions*, or what is handled very roughly by second order logic in contemporary parlance, and continuing onward through higher intensions, or higher order logic and type theory.

[584] Ibid.

[585] See: Borislav Dimitrov, "The Topological notion of Qualitative quantity", 2009-2011: For Alain Badiou's disagreement with Hegel and what Badiou called "Hegelian hallucination".

There have been proposals by Donald Mertz, Herbert Schneider, Carl Hausman, and Carl Vaught to augment Peirce's threefold to fourfold; and one by Douglas Greenlee to reduce them to twofold. [586]

Peirce introduces his Categories and their theory in “On a New List of Categories” /1867/ [587], a work which is cast as a Kantian deduction and is short but dense and difficult to summarize. The following table is compiled from that and later works.

| Peirce's Categories (technical name: the cenopythagorean categories) | | | | | |
|---|---|---|---|---|---|
| **Name:** | **Typical characterizaton:** | **As universe of experience:** | **As quantity:** | **Technical definition:** | **Valence, "adicity":** |
| Firstness. | Quality of feeling. | Ideas, chance, possibility. | Vagueness, "some". | Reference to a ground (a ground is a pure abstraction of a quality). | Essentially monadic (the quale, in the sense of the *such*, which has the quality). |
| Secondness. | Reaction, resistance, (dyadic) relation. | Brute facts, actuality. | Singularity, discreteness, “this”. | Reference to a correlate (by its relate). | Essentially dyadic (the relate and the correlate). |
| Thirdness. | Representation, mediation. | Habits, laws, necessity. | Generality, continuity, "all". | Reference to an interpretant*. | Essentially triadic (sign, object, interpretant*). |

**Note:* An interpretant is an interpretation (human or otherwise) in the sense of the product of an interpretive process. (The context for interpretants is not psychology or sociology, but instead philosophical logic. In a sense, an interpretant is whatever can be understood as a conclusion of an inference. The context for the categories as categories is phenomenology, which Peirce also called phaneroscopy and categorics.) [588]

Same as the the Topological Hegel’s Qualitative quantity, Charles Sanders Peirce’s Contributions seems ignored by both the analytic philosophers and the so-called Continental philosophers. Rediscovering the topological fourfold of quality and quantity in Peirce could

[586] For references and discussion, see Burgess, Paul (circa 1988) "Why Triadic?: Challenges to the Structure of Peirce's Semiotic"; posted by Joseph M. Ransdell at *Arisbe*.

[587] Charles S. Peirce, On a New List of Categories, *Proceedings of the American Academy of Arts and Sciences* 7 (1868), pp. 287-298. Pesented to the Academy May 14, 1867, Specially Formatted for On-Line Presentation by Joseph Ransdell - http://www.cspeirce.com/menu/library/bycsp/newlist/nl-frame.htm

[588] http://en.wikipedia.org/wiki/Categories_(Peirce)#cite_note-cenopythagorean-7

be an promising direction for the 21st century in establishing a foundation for unifying and relating the insights of multiple branches of cognitive science (linguistics, philosophy, psychology, and artificial intelligence).

Yu, Chong Ho, in his research paper – "Abduction? Deductio? Induction? Is there a Logic of Exploratory Data Analysis?" [589] directed the philosophical notions introduced by Charles Sanders Peirce as helpful for researchers in understanding the nature of knowledge and reality:

"In Peircean logical system, the logic of abduction and deduction contribute to our conceptual understanding of a phenomenon, while the logic of induction adds quantitative details to our conceptual knowledge. Although Peirce justified the validity of induction as a self-corrective process, he asserted that neither induction nor deduction can help us to unveil the internal structure of meaning. As exploratory data analysis performs the function as a model builder for confirmatory data analysis, abduction plays a role of explorer of viable paths to further inquiry. Thus, the logic of abduction fits well into exploratory data analysis. At the stage of abduction, the goal is to explore the data, find a pattern, and suggest a plausible hypothesis; deduction is to refine the hypothesis based upon other plausible premises; and induction is the empirical substantiation." [590]

I will summarize paraphrasing the conclusion of Alex Yu /Yu Chong Ho/ that both qualitative and quantitative research methods have different merits and shortcomings such as "both deduction and induction have different merits and shortcomings", and Qualitative quantity's reasoner should apply abduction, deduction and induction altogether in order to achieve a comprehensive inquiry. Cultural Phenomenology of Qualitative quantity /Research/ should follow Peirce's topo-logic where, as Alex Yo concludes: "abduction and deduction are the conceptual understanding of a phenomena, and induction is the quantitative verification. At the stage of abduction, the goal is to explore the data, find out a pattern, and suggest a plausible hypothesis with the use of proper categories; deduction is to build a logical and testable hypothesis based upon other plausible premises; and induction is the approximation

[589] Yu, Chong Ho, "Abduction? Deductio? Induction? Is there a Logic of Exploratory Data Analysis?, presented at the Annual Meeting of American Educational Research Association, New Orleans, Louisiana, April, 1994 - http://www.creative-wisdom.com/pub/Peirce/Logic_of_EDA.html

[590] Ibid.

towards the truth in order to fix our beliefs for further inquiry. In short, abduction creates, deduction explicates, and induction verifies." [591]

## 4. Philosophical Topology and Epistemology: The new frontier of Topology as Epistemology

There is a research shift in progress for the last decades bringing the fusion between these two disciplines and giving the grounds to approach the relationship between topology and epistemology in different way of seeing them. **Oliver Schulte and Cory Juhl** regarded topology as epistemology, in their two papers from 1996 - **"Topology as Epistemology"**[592] and "**Epistemology, Reliable Inquiry and Topology"**[593]**.** Schulte and Juhl recommends the second paper as a friendly introduction to the connection between topology and epistemology, discussing some topological interpretations of Carl Popper's falsifiability criterion. Subject of Schulte and Juhl work is an interpretation of point-set topology as the theory of inquiry for logically omniscient agents with no limitations on memory capacity, a proposal that transferred topology into a powerful tool for epistemology. One of the distinguished researchers in the field of merger between topology and epistemology is professor **Kevin Kelly** (Carnegie Mellon) **with his remarkable work "Topological Epistemology".**[594] Kevin Kelly authored a tutorial on Topological Epistemology. According to Kelly, traditionally, discussions of inductive inference and empirical justification are framed within logic and probability theory, and he argued at length that the right framework is topology. The main initiators of the research in topological epistemology are **Rohit Parikh, Lawrence S. Moss, Chris Steinsvold (Topology and epistemic logic) exploring** applications of topological ideas in modal logic, especially in epistemic logic, applications of topological ideas in epistemic logic and offering a topological semantics and completeness proof for the logic of belief**.**[595] The mathematician, logician, and philosopher **Rohit Jivanlal Parikh** is one of the

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591 Ibid.

592 Oliver Schulte, Cory Juhl, Topology as Epistemology, (1996). The Monist vol. 79:1, 141-147. http://www.jstor.org/discover/10.2307/27903468?uid=3737608&uid=2129&uid=2&uid=70&uid=4&sid=21102214017207

593 Oliver Schulte, Cory Juhl, Epistemology, Reliable Inquiry and Topology, Department of Philosophy, Carnegie Mellon University, Pittsburgh, PA, Department of Philosophy, University of Texas at Austin, 1996: http://www.cs.sfu.ca/~oschulte/files/pubs/monist.pdf

594 Kevin Kelly, Topological Epistemology - http://www.illc.uva.nl/lgc/seminar/?p=1241and http://www.andrew.cmu.edu/user/kk3n/homepage/kelly.html

595 Rohit Parikh, Lawrence S. Moss, Chris Steinsvold, Topology and epistemic logic - applications of topological ideas in modal logic, especially in epistemic logic; applications of topological ideas in epistemic logic; a topological semantics and completeness proof for the logic of belief KD45: http://www.indiana.edu/~iulg/moss/TEL.pdf

authors of "Topological Reasoning and The Logic of Knowledge" (with Dabrowski and Moss) [596] An interesting approach, that illustrates the implementation of topology in epistemology, to the known Gettier problem, is offered by Chris Steinsvold in his work **"Topological Models of Belief Logics"**. Steinsvold suggests that "the topological semantic may be an interesting way to consider Gettier problem in epistemology." Discussing the theme of knowledge operator, Steinsvold claim that "there is a topological operator which acts like justified belief". (P.26 /1.11. "A note on knowledge and Gettier") According to Steinsvold - "The topological semantics seems to support the thesis that knowledge is true justified belief." As Steinvold establishes, "Up until 1963 this would not have been controversial. Since then Gettier's celebrated article has totally undermined this simple thesis." The arguments given by Steinsvold are that the modal logic S4 has a topological semantics and S4 is widely used as a basis of knowledge. In addition, Steinsvold establishes that there is an interior operator which obeys the S4 axioms. This interior operator acts knowledge and that helps to interpret the topological semantic on intuitive level. Steinsvold proposed the question – Is there a topological operator which obeys a logic of belief? The answer he gives is positive - Topological semantics seems to support the thesis that knowledge is true justified belief. Modal logic S4 has a topological semantics S4 is widely used as a logic of knowledge. There is a topological operator wich obeys a logic of belief and this topological operator acts as belief and helps to interpret the topological semantic on intuitive level. The interior operator acts like knowledge. In other words, S4 is topologically complete.

Topology of justification in new epistemological sense is subject of research works of Sergei Artemov and Elena Nogina.[597] Artemov and Nogina proposed topological implementation in epistemology based on the justification Logic as a family of epistemic logical systems obtained from modal logics of knowledge by adding a new type of formula constructed topologically, implementing the principal epistemic modal logic S4 which includes Alfred Tarski's well-known topological interpretation, according to which the modality $2X$ is read

[596] Parikh, Dabrowski, Moss, "Topological Reasoning and The Logic of Knowledge", Annals of Pure and Applied Logic 78 (1996) 73-110.

[597] Artemov, S., and E. Nogina, Topological Models for Justification Logic, TANCL 2007, Oxford University; Sergei Artemov and Elena Nogina, THE TOPOLOGY OF JUSTIFICATION: http://www.logika.umk.pl/llp/1712/21-1712zw.pdf; Artemov, S., J. Davoren, and A. Nerode, "Modal logics and topological semantics for hybrid systems", Technical Report MSI 97-05, Cornell University, 1997.; Artemov, S., and E. Nogina, "Logic of knowledge with justifications from the provability perspective", Technical Report TR-2004011, CUNY Ph.D. Program in Computer Science, 2004.; Artemov, S., and E. Nogina, "Introducing justification into epistemic logic, *Journal of Logic and Computation* 15, 6 (2005), 1059–1073.; Artemov, S., and E. Nogina, "On epistemic logic with justification", pages 279–294 in R. van der Meyden (ed.), *Theoretical Aspects of Rationality and Knowledge. Proceedings of the Tenth Conference (TARK 2005), June 10–12,2005*, Singapore, National University of Singapore, 2005.; Artemov, S., and E. Nogina, "On topological semantics of justification logic",*Algebraic and Topological Methods in Non-Classical Logics III (TANCL'07)Oxford, England, August 2007*.

*the Interior of X in a topological space* (the topological equivalent of the '*knowable part of X*'). Artemov and Nogina extends Tarski's topological interpretation from S4 to Justification Logic systems with both modality and justification assertions. The topological semantics interprets *t*:*X* as a reachable subset of *X* (the topological equivalent of '*test t confirms X*'). The authors etablished a number of soundness and completeness results with respect to Kripke topology and the real topology for S4-based systems of Justification Logic.

## 5. Topological ontology[598]

The topological ontology of qualitative quantity here is proposed in the sense and from the point of view of AI as an "explicit specification of conceptualization" and agreement aout shared conceptualization. The goal of this paper is to introduce the topological ontology of qualitative quantity in the conceptual frameworks of auditing research for modeling domain knowledge; content-specific protocols for communication among inter-operating agents; and agreements about the representation of particular domain theories. In the knowledge sharing context of auditing research, topological ontology of qualitative quantity deserve own place into the form of definitions of representational vocabulary.

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598 **Topological ontology** is quite foreign for philosophy but well at home in human-computer studies. The term "topological ontology" appears and can be found in the server of the KACTUS project at http://hcs.science.uva.nl/projects/NewKACTUS/library/lib/topology.html
KACTUS stands for modelling Knowledge About Complex Technical systems for multiple USe. It is an European ESPRIT project (ESPRIT Project 8145)aiming at the development of a methodology for the reuse of knowledge about technical systems during their life-cycle. This implies using the same knowledge base for design, diagnosis, operation, maintenance, redesign, instruction, et cetera. Reuse will be achieved by giving these knowledge bases an explicit structure (ontology).
KACTUS aim is to model knowledge about complex technical systems for multiple use. The project has started at January 1st 1994 and runs for 30 months. The prject consortium includes the partners as Gap Gemini Innovation, Integral Solution Limited, Cap Programator, Delos S.p.A, Fincarntieri, Iberdrola, Labein, Lloyd's Register, RPK University in Karlsuhe, Statiol, Sintef Automatic Control, SWI University of Amsterdam.
The strategic objectives of the KACTUS project are Efficiency via Method Integration, Competitive Advantage by Creation of an Ontological Basis and Interoperability by Modelling and Information Exchange.
Projects and Publications on the subject of topological ontology could are available on the ONTOLINGUA SERVER http://ontolingua.stanford.edu (Library of Ontologies Table of Contents)
with the Reference Documents, List of Ontologies (http://www-ksl.stanford.edu/knowledge-sharing/ontologies/html/index.html#theory-list)
Topological ontology is one of the main concerns of Human-Computer Studies Laboratory, a part of the Informatics Institute, which is a part of the Faculty of Science of the University of Amsterdam.

As we already mentioned above, the new area of interdependence between the quality and quantity is open now for rethinking and reconceptualization. In Qualitative Spatial Representation in the field of Qualitative Spatial Reasoning, the term “qualitative quantitative space” emerged and the concept of “qualitative quantity” is vitally accepted and utilized. [599] Qualitative quantity seen in Poincare’s topology and his development of the qualitative theory of differential equations is enhanced by the new science of Mereotopology, which began with theories A.N. Whitehead articulated in several books and articles he published between 1916 and 1929. Mereotopology is a branch of metaphysics, and ontological computer science, a first-order theory, embodying mereological and topological concepts, of the relations among wholes, parts, parts of parts, and the boundaries between parts. The Qualitative quantity is the base cathegory and phenomenom of Qualitative research /QR/, due to the aim of QR to utilise methods that seek to discern the quality — as opposed to the quantity — of its subject. Qualitative quantity is critical in Spatial-temporal reasoning applicable in computer science as Visual thinking and Visual music, Visual statistic /the series of works by David James Krus/.

Ontology in philosophy means theory of existence. Ontology tries to explain what is being and how the world is configured by introducing a system of critical categories to account things and their intrinsic relations. The origin of philosophical ontology is Aristotle’s metaphysics and his theory of categories, intelligible universals extending across all domains. According to Aristotle the world is organized via types/universals/categories which are hierarchically organized. The historical line goes from Aristotle trough the Medieval scholastics, such as Aquinas, Scotus, Ockham, from Descartes and Kant to Brentano’s rediscovery of Aristotle to Ednund Husserl, the inventor of formal ontology as a discipline distinct from formal logic, who showed how philosophy and science had become detached from the ‘life world’ of ordinary experience, claiming that the methods of philosophy and of science are one and the same. The historical line of ontology is continued by Wittgenstein, Vienna Circle (Schlick, Neurath, Gödel, Carnap, Gustav Bergmann), British Ordinary Language philosophy (Speech Act Theory of J. L. Austin), Quine’s ontological commitment, Analytical metaphysics of Chisholm, Lewis, Armstrong, Fine, Lowe. This line ends with the beginnings of a rediscovery of metaphysics as first philosophy.

[599] See: A. G. Cohn, Qualitative Spatial Representation and Reasoning Techniques, Division of Artificial Intelligence, School of Computer Studies, University of Leeds.- Note 1.’footnote’ about “qualitative quantity”.

## 5.1. Topological backgroung of Edmund Husserl’s Ontology – Topological, Mereological and Mereotopological Part-Whole Reasoning

Edmund Husserl’s formal ontology is the natural environment of topology and topological background of Husserl’s work. Husserl's “Logical Investigations” (1900/01) contain a formal theory of part, whole and dependence that is used by Husserl to provide a framework for the analysis of mind and language of just the sort that is presupposed in the idea of a topological foundation for cognitive science. [600]

One of the most sophisticated scientists and researcher who strongly support the idea of topological Husserl[601] is Barry Smith, Professor of Philosophy, Neurology and Computer Science and Director of the National Center for Ontological Research at the University at Buffalo.[602] Barry Smith and Pierre Grenon initiated the Basic Formal Ontology (BFO)[603] Project in 2002 with a series of publications.[604] The inspirations of Basic Formal Ontology are Aristotle, Husserl, Roman Ingarden, Ingvar Johansson, Wittgenstein’s Tractatus (picture theory of language).

In his paper “Mereotopology: A Theory of Parts and Boundaries”[605], remarkable contribution to formal ontology, Barry Smith use topological means in order to derive ontological laws pertaining to the boundaries and interiors of wholes, to relations of contact and connectedness, to the concepts of surface, point, neighbourhood. and so on. The basis of the theory is

[600] Barry Smith, Topological Foundations of Cognitive Science, a revised version of the introductory essay in C. Eschenbach, C. Habel and B. Smith (eds.), *Topological Foundations of Cognitive Science,* Hamburg: Graduiertenkolleg Kognitionswissenschaft, 1994, the text of a talk delivered at the First International Summer Institute in Cognitive Science in Buffalo in July 1994.: http://ontology.buffalo.edu/smith/articles/topo.html
And Barry Smith, (ed.) 1982 Parts and Moments. Studies in Logic and Formal Ontology, Munich: Philosophia.)

[601] Barry Smith and Kevin Mulligan, "Husserl's Logical Investigations," Grazer Philosophische Studien 27: 199-206 (1986).

[602] Barry Smith is also one of the principal scientists of the National Center for Biomedical Ontology (NCBO), an NIH Roadmap National Center for Biomedical Computing, a member oft he Scientific Advisory Board of the Gene Ontology Consortium, and a PI of the Protein Ontology and Infectious Disease Ontology projects. He is a Coordinating Editor of the OBO Foundry initiative, and plays a guiding role in Basic Formal Ontology and the Ontology for General Medical Science, the Environment Ontology, and the Plant Ontology initiatives.

[603] See: http://www.ifomis.org/bfo

[604] See: (http://www.ifomis.org/bfo/publications).

[605] Barry Smith, “Mereotopology: A Theory of Parts and Boundaries”, from: “Mereotopology: A Theory of Parts and Boundaries”, Data and Knowledge Engineering, 20 (1996), 287–303: http://ontology.buffalo.edu/smith/courses03/tb/SmithMereotopology1.pdf

mereology, the formal theory of part and whole, a theory which is shown to have a number of advantages for ontological purposes over standard treatments of topology.

The roots of topology in Husserl can be found in the forerunners of the school of **formal ontology** (founded by Husserl[606]) - **Bernard Bolzano** and **Franz Brentano[607],** within the origins of the main traditions of contemporary ontology: the Phenomenological, the Analytical, and the Austro-Polish (with the main exponents as Adolf Reinach, Roman Ingarden and Nicolai Hartmann).

During the period from 1884 through 1887, Husserl attended the lectures of Franz Brentano. Under the mentorship of Brentano, Husserl came to view philosophy as complementary to science. Husserl became concerned with linking mathematics and philosophy. Husserl's first text, *The Philosophy of Arithmetic*, was published in 1891 with a dedication to Brentano.

After Kant's rejection of the possibility of a general ontology[608], the first philosopher who contributed to the new ontological turn was Bernard Bolzano. In his 1837 "Theory of Science" (Wissenschaftslehre), Bolzano attempted to provide logical foundations for all sciences, building on abstractions like part-relation, abstract objects, attributes, sentence-shapes, ideas and propositions in themselves, sums and sets…The logical theory of Bolzano developed in his work "Theory of Science" has come to be acknowledged as ground-

[606] E. Husserl, Formal and Transcendental Logic (1929), English translation: The Hague: Martinus Nijhoff, 1969, 27, p. 86.): "The idea of a formal ontology makes its first literary appearance in Volume I of my Logische Untersuchungen (1900), [Chapter 11, The Idea of Pure Logic] in connection with the attempt to explicate systematically the idea of a pure logic -- but not yet does it appear there under the name of formal ontology, which was introduced by me only later. The Logische Untersuchungen as a whole and, above all, the investigations in Volume II ventured to take up in a new form the old idea of an apriori ontology -- so strongly interdicted by Kantianism and empiricism -- and attempted to establish it, in respect of concretely executed portions, as an idea necessary to philosophy."

[607] Brentano, Franz 1988 *Philosophical Investigations on Space, Time and the Continuum*, translated by Barry Smith, London/New York/Sydney: Croom Helm.

[608] I. Kant, Critique of Pure Reason (A247/B304), Cambridge: Cambridge University Press, 1998, pp. 358-359 : "The Transcendental Analytic accordingly has this important result: That the understanding can never accomplish a priori anything more than to anticipate the form of a possible experience in general, and, since that which is not appearance cannot be an object of experience, it can never overstep the limits of sensibility, within which alone objects are given to us. Its principles are merely principles e of the exposition of appearances, and the proud name of an ontology, which presumes to offer synthetic a priori cognition of things in general in a systematic doctrine (e.g., the principle of causality), must give way to the modest one of a mere analytic of the pure understanding."

breaking. Bolzano was mainly concerned with three realms: (1) The realm of language, consisting in words and sentences; (2) The realm of thought, consisting in subjective ideas and judgements; (3) The realm of logic, consisting in objective ideas (or ideas in themselves) and propositions in themselves. The distinction between parts and wholes play a prominent role in Bolzano's system. Bolzano's work "The Paradoxes of the Infinite" was greatly admired by Charles Sanders Peirce, Georg Cantor and Richard Dedekind.

Bolzano's works was rediscovered by Edmund Husserl[609] and the Polish philosopher and logician Kazimierz Twardowski [610], both students of Franz Brentano. Through Husserl and Twardowski , Bolzano became a formative influence on both phenomenology and analytic philosophy. The influence of Bolzano on Heidegger is witnessed by Heidegger himself. [611]
The first work of Brentano On the Several Senses of Being in Aristotle (1862) and the Logical Investigations (1900) by Husserl were at the origin of the interest in philosophy of the most authoritative exponent of Continental ontology, Martin Heidegger.

If the *place* of ontology in modern philosophy is naturally topological, same is valid for the critical categories of ontology – quality and quantity.

Ontology is the theory of objects and **their ties**. Topology, as a branch of mathematics, can be formally defined as "the study of qualitative properties of certain objects (called topological spaces) that are invariant under a certain kind of transformation (called a continuous map), especially those properties that are invariant under a certain kind of equivalence (called homeomorphism)." To put it more simply, topology is the study of continuity and connectivity. Topology is concerned only with the geometric **relationship between** the

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609 Hermes Scholz, Concise History of Logic (1931), English translation: New York: Philosophical Library, 1961, p. 47. : "With such illogicality did things happen in the history of logic which we are pursuing here that this great, born logician fell prey to a fate which beats the fate of Joachim Jungius. For the latter at least was read, and read by a Leibniz; but that cannot even be said of Bolzano. Hence we cannot even maintain in his case that he was forgotten. All the greater is the merit of Edmund Husserl who discovered Bolzano."

610 the founder of the Lvov-Warsaw School of logic, together with Alfred Tarski and Jan Lukasiewicz, Kazimierz Twardowski formed the *troika* which made the University of Warsaw, during that period, the most important research center in the world for formal logic.

611 Martin Heidegger, Preface to: William Richardson, Heidegger. Through Phenomenology to Thought, The Hague: Martinus Nijhoff, 1963. p. X.: "The first philosophical text through which I worked my way, again and again from 1907 on, was Franz Brentano's dissertation: On the Manifold Sense of Being in Aristotle."

objects (figures) or elements of a network, not with the kind of objects or elements themselves.

Ontology provides criteria for distinguishing different types of objects (concrete and abstract, existent and nonexistent, real and ideal, independent and dependent) and their ties (relations, dependencies and predication). We can distinguish: a) formal, b) descriptive and c) formalized ontologies.

Formal ontology was introduced by Edmund Husserl in his Logical Investigations [612]. According to Husserl, the object of formal ontology is the study of the genera of being, the leading regional concepts, the categories; the true method - the eidetic reduction coupled with the method of categorial intuition. The phenomenological ontology is divided into two: (I) Formal, and (II) Regional, or Material, Ontologies. The former investigates the problem of truth on three basic levels: (a) Formal Apophantics, or formal logic of judgments, where the a priori conditions for the possibility of the doxic certainty of reason are to be sought, along with (b) the synthetic forms for the possibility of the axiological, and (c) "practical" truths. In other words it is divided into formal logic, formal axiology, and formal praxis.

According to Barri Smith, "In contemporary philosophy, formal ontology has been developed in two principal ways. The first approach has been to study formal ontology as a part of ontology, and to analyze it using the tools and approach of formal logic: from this point of view formal ontology examines the logical features of predication and of the various theories of universals. The use of the specific paradigm of the set theory applied to predication, moreover, conditions its interpretation.[613]

[612] E. Husserl, Formal and Transcendental Logic (1929), English translation: The Hague: Martinus Nijhoff, 1969, 27, p. 86.: "To the best of my knowledge, the idea of a formal ontology makes its first literary appearance in Volume I of my Logische Untersuchungen (1900), [Chapter 11, The Idea of Pure Logic] in connexion with the attempt to explicate systematically the idea of a pure logic -- but not yet does it appear there under the name of formal ontology, which was introduced by me only later. The Logische Untersuchungen as a whole and, above all, the investigations in Volume II ventured to take up in a new form the old idea of an apriori ontology -- so strongly interdicted by Kantianism and empiricism -- and attempted to establish it, in respect of concretely executed portions, as an idea necessary to philosophy."

[613] Formal Ontology, in: Barry Smith, Hans Burkhardt (eds.), Handbook of Metaphysics and Ontology, Munich: Philosophia Verlag, 1991 p. 640.

The second line of development of the phenomenological ontology or the Regional, or Material, Ontologies, returns to its Husserlian origins and analyses with the the fundamental categories of object, state of affairs, part, whole, and so forth, as well as the relations between parts and the whole and their laws of dependence -- once all material concepts have been replaced by their correlative form concepts relative to the pure 'something'. This kind of analysis does not deal with the problem of the relationship between formal ontology and material ontology."

In philosophy and mathematical logic, **mereology** (from the Greek μέρος, root: μερε(σ)-, "part" and the suffix -logy "study, discussion, science") treats parts and the wholes they form.
The informal part-whole reasoning is rooted in Plato's metaphysics and ontology (in particular, in the second half of the Parmenides). It appears around 1910 in the set theory of George Cantor nut the first to reason consciously and at length about parts and wholes was Edmund Husserl in his 1901 *Logical Investigations* (Husserl 1970 is the English translation). The word "mereology" is absent from Husserl's writings, and he employed no symbolism even though his doctorate was in mathematics. There are reasons today to establish link between mereology and Husserl's theory of part and whole. (see Barri Smith's chapter "**7. Husserl's Mereotopology" from his** "Topological Foundations of Cognitive Science").

The term "mereology" was coined in 1927 by Stanislaw Lesniewski, from the Greek word μέρος (*méros*, "part"), to refer to a formal theory of part-whole he devised in a series of highly technical papers published between 1916 and 1931, and translated in Leśniewski (1992) [614].
Leśniewski's student Alfred Tarski simplified Leśniewski's formalism. Other students (and students of students) of Lesniewski elaborated this "Polish mereology" over the course of the 20th century.

The marriage of topological notions of boundaries and connection, and mereology results in mereotopology. The informal mereotopology is presented by Whitehead in his 1929's work "Process and Reality".

[614] Stanislaw Lesniewski, 1992. *Collected Works*. Surma, S.J., Srzednicki, J.T., Barnett, D.I., and Rickey, V.F., editors and translators. Kluwer.

According to Barry Smith, "..Husserl was at least implicitly aware of the topological aspect of his ideas, even if not under this name, is unsurprising given that he was a student of the mathematician Weierstrass in Berlin, and that it was Cantor (George Cantor), Husserl's friend and colleague in Halle during the period when the *Logical Investigations* were being written, who first defined the fundamental topological notions of open, closed, dense, perfect set, boundary of a set, accumulation point, and so on. Husserl consciously employed Cantor's topological ideas, not least in his writings on the general theory of (extensive and intensive) magnitudes which make up one preliminary stage on the road to the third Investigation. (See Husserl 1983, pp. 83f, 95, 413; 1900/01, "Prolegomena", §§ 22 and 70.)[615]

According to Barry Smith "it is worth pointing out that the early development of topology on the part of Cantor and others was part of a wider project on the part of both mathematicians and philosophers in the nineteenth century to produce a *general theory of space* - to find ways of constructing fruitful generalizations of such notions as extension, dimension, separation, neighbourhood, distance, proximity, continuity, and boundary. Husserl participated in this project with Stumpf and other students of Brentano such as Meinong. [616] Significantly, the 1906 paper on "The Origins of the Concept of Space"[617], in which Riesz first formulated the closure axioms at the heart of topology is in fact a contribution to formal phenomenology, a study of the structures of *spatial presentations*, in which the attempt is made to specify the additional topological properties which must be possessed by a mathematical continuum if it is adequately to characterize the continuity and order properties of our experience of space."[618]

Husserl's *Logical Investigations* (1900/01) contain a formal theory of part, whole and dependence that is used by Husserl to provide a framework for the analysis of mind and

[615] Husserl, Edmund 1983 Studien zur Arithmetik und Geometrie. Texte aus dem Nachlass, 1886-1901, The Hague: Nijhoff (Husserliana XXI).

[616] Husserl, Edmund 1983 Studien zur Arithmetik und Geometrie. Texte aus dem Nachlass, 1886-1901, The Hague: Nijhoff (Husserliana XXI). pp. 275-300, 402-410; Stumpf, C. 1873 Über den psychologischen Ursprung der Raumvorstellung, Leipzig: Hirzel.; Meinong, Alexius von 1903 "Bermerkungen über die Farbenkörper und das Mischungsgesetz", Zeitschrift für Psychologie und Physiologie der Sinnesorgane, 33, 1-80, and in Meinong, Gesamtausgabe, vol. I, 497- 576. esp. §2: "Farbengeometrie und Farbenpsychologie", and §5: "Der Farbenraum und seine Dimensionen".

[617] Riesz, F. 1906 "Die Genesis des Raumbegriffs", *Mathematische und naturwissenschaftliche Berichte aus Ungarn*, 24, 309-53.

[618] Barry Smith, Topological Foundations of Cognitive Science

language of just the sort that is presupposed in the idea of a topological foundation for cognitive science. [619] makes itself felt already in his theory of dependence.[620] (Fine 1995.)

The title of the third of Husserl's Logical Investigations is "On the Theory of Wholes and Parts" and it divides into two chapters: "The Difference between Independent and Dependent Objects" and "Thoughts Towards a Theory of the Pure Forms of Wholes and Parts". Husserl's theory is concerned also with the horizontal relations between the different parts within a single whole, relations which serve to give unity or integrity to the wholes in question. According to Barry Smith, “To put the matter simply: some parts of a whole exist merely side by side, they can be destroyed or removed from the whole without detriment to the residue. A whole all of whose parts manifest exclusively such side-by-sideness relations with each other is called a heap or aggregate or, more technically, a purely summative whole. In many wholes, however, and one might say in *all* wholes manifesting any kind of unity, certain parts stand to each other in relations of what Husserl called *necessary dependence* (which is sometimes, but not always, necessary *interdependence*). Such parts, for example the individual instances of hue, saturation and brightness involved in a given instance of colour, cannot, as a matter of necessity, exist, except in association with their complementary parts in a whole of the given type. There is a huge variety of such lateral dependence relations giving rise to correspondingly huge variety of different types of whole which more standard approaches of 'extensional mereology' are unable to distinguish.”

Husserl's topological ideas on parts, wholes and categories from the *Logical Investigations* were applied by Stanislaw Lesniewski, the founder of mereology, and the linguist Roman Jakobson in different branches of linguistics, in the early development of categorial grammar.
**The topological background of Edmund Husserl’s work is presented in his formal ontology.** The term ‘formal ontology’ was first used by in his *Logical Investigations* (1900/01) to signify the study of those formal structures and relations – above all relations of part and whole – which are exemplified in the subject-matters of the different material sciences. We follow Husserl in presenting the basic concepts of formal ontology as falling into three groups:

[619] Smith, Barry (ed.) 1982 Parts and Moments. Studies in Logic and Formal Ontology, Munich: Philosophia.
[620] Fine, K. 1995 "Part-Whole", in B. Smith and D.W. Smith (eds.), *The Cambridge Companion to Husserl*, Cambridge: Cambridge University Press, 463-485.

**- the theory of part and whole,**

**- the theory of dependence,**

**- and the theory of boundary, continuity and contact.**

In his *Logical Investigations* (1900/01), Edmund Husserl, draws a distinction between *formal logic*, on the one hand, and *formal ontology,* on the other.

Formal logic deals with the interconnections of truths (or of propositional meanings in general) – with inference relations, with consistency and validity.

Formal ontology deals with the interconnections of things, with objects and properties, parts and wholes, relations and collectives.

As formal logic deals with inference relations which are formal in the sense that they apply to inferences in virtue of their form alone, so formal ontology deals with structures and relations which are formal in the sense that they are exemplified, in principle, by all matters, or in other words by objects in all material spheres or domains of reality.

Qualitative quantity seen in Poincare's topology and his development of the qualitative theory of differential equations is enhanced by the new science of **Mereotopology**. If Topology is perhaps the most elemental aspect of space, and must be form a fundamental aspect of qualitative spatial reasoning since it certainly can only make qualitative distinctions, Mereotopology is a branch of metaphysics, and ontologigal computer science. Mereotopology is a first-order theory, embodying mereological and topological concepts, of the relations among wholes, parts, parts of parts, and the boundaries between parts. Mereotopology begins with theories A.N. Whitehead articulated in several books and articles he published between 1916 and 1929.

The Qualitative quantity is the base cathegory and phenomenom of Qualitative research /QR/. Qualitative Research utilizes methods that seek to discern the quality — as opposed to the quantity — of its subject. QR is more often concerned with explaining the *why* and *how* of a phenomenon rather than the *what*, *when* and *where*. Qualitative research methods are most often utilized in fields such as anthropology, the humanities and sociology, although each of these fields can be studied through quantitative methods as well. Since qualitative research is exploratory and focuses on discerning the *why* of things, such as human behavior, rather than the *what* of the natural world, it is often criticized for being too subjective. Many make the counter-argument, however, that since qualitative methods are hypothesis generating, they are not only just as valuable as quantitative methods but necessary for the production of theoretical models which come to inform the direction of quantitative research methods. Data collection and analysis is another way that quantitative and qualitative research differ. In qualitative research, data samples are usually not collected through random selection but rather *purposive reasoning*, which is to say they are chosen for how well they typify the characteristics of a certain class. For example, a qualitative research study on racial inequality will not likely concern itself with affluent minorities or the entire population of a minority, but rather, it might focus on depressed areas where minorities are most prevalent. This approach is chosen because qualitative researchers are not concerned with discerning the quantity of people in a minority class, but rather the quality of life for minorities who are affected by inequality. Qualitative research is thought especially valuable in circumstances where quantitative data does not account for a particular phenomenon. For example, while economics frequently concerns itself with collecting concrete information, like statistics and financial data, it can be said to be flawed because it ignores the humanistic and psychological aspects of the people that are a key component. This human component requires a qualitative understanding. An important variable to consider when analyzing the dependability of qualitative research is validity. It is important to consider how a conclusion was reached, and whether it really represents a dependable and realistic interpretation of its subject. It may or may not be pertinent to ask whether or not a conclusion is reproducible, or whether it was affected by bias. One should also consider whether data from qualitative research is well reasoned and the extent to which it accounts for a substantial majority of the available data.

**Spatial-temporal reasoning is applicable in ontology**. The implication of the concept of qualitative quantity could be found in the work “Spatial Reasoning and Ontology: Parts, Wholes, and Locations”, authored by Achille C. Varzi, Columbia University /Published in M. Aiello, I. Pratt-Hartmann, and J. van Benthem (eds.), Handbook of Spatial, Logics, Berlin: Springer-Verlag, 2007, pp. 945-1038/.

The notion of the Quality of the quantity is related with **Qualitative Spatial Change**. In his study "Qualitative Spatial Change : Space-Time Histories and Continuity", Submitted in accordance with the requirements for the degree of Doctor of Philosophy, The University of Leeds, School of Computing, January 2005, Shyamanta M. Hazarika provides proofs and examples how quality of the quantity is implemented in **Topology, Mereotopology and Qualitative Continuity.** Continuity of change is the perception of being seamless and is dependent on the granularity. What seems as continuous at some level of granularity may be discontinuous at a finer level. Nevertheless, continuity may be thought of as the intuitive idea of a gradual variation with no abrupt jumps or gaps. A formal characterization of such an intuitive notion of continuity for a qualitative theory of motion is what Shyamanta M. Hazarika refers to as qualitative continuity.

## 5.2. The Appearance of Topological Qualitative quantity in The Upper Ontology of YAMATO: Yet Another More Advanced Top-level Ontology

The relationships between quality and quantity are in the core of the **formal ontology and the well known** "upper ontologies" in Ontological engineering and Ontology design.

YAMATO (HOZO) upper ontology is build for the onto editor Hozo.[621] One of the main important characteristic of this ontology is the definition of the concepts as "basic concepts", "role", and "role holder". Some possible examples are for the basic concept 'human', for the role – woman, man.

Professor Richiro Mizoguchi (The Institute of Scientific and Industrial Research at Osaka University, Japan), has analysed the well-known "upper ontologies" (such as DOLCE [622](Guarino 2010), BFO[623] (Smith 2010), GFO (Herre 2010), SUMO[624] (Pie 2010), CYC (Lenat 2010)) and finds them lacking in three areas of great importance to e-research and interoperability: quality, representation, and process/event. Riichiro Mizoguchi presented an

[621] Hozo - Ontology Editor: http://conf.infosoc.ru/thesis

[622] DOLCE : a Descriptive Ontology for Linguistic and Cognitive Engineering – see: http://www.loa.istc.cnr.it/DOLCE.html from The Laboratory for Applied Ontology (LOA)

[623] Basic Formal Ontology (BFO) – see: http://www.ifomis.org/bfo - the BFO project was initiated in 2002. The theory behind BFO was developed initially by Barry Smith and Pierre Grenon and presented in a series of publications (http://www.ifomis.org/bfo/publications).

[624] Suggested Upper Merged Ontology (SUMO) – see: http://www.ontologyportal.org/

alternative ontology, called YAMAMOTO (**YAMATO: Yet Another More Advanced Top-level Ontology)**[625] at AOW 2010.

The YAMATO upper ontology has been built and maintained by Riichiro Mizoguchi, first introduced in 1999. Since then, it has been refined and revised several times. Rough introduction to YAMATO is available in Mizoguchi's article "YAMATO: Yet Another More Advanced Top-level Ontology"[626] One of the three main features of YAMATO is the theory on Quality and quantity. Mizoguchi claims that the existing upper ontologies fail to explain satisfactorily the theory of quality and quantity. Many of the existing upper ontologies, according to Mizoguchi, are "too simple to explain the reality and to guide domain people to build their ontologies. What they need are not only distinction between objects and qualities but also that between quality and quantity and that between quality and description of quality, not only that between objects and representation but also that between a copy of book and book and that between a novel and a musical score, and not only that between process and event but also that between a pulse and a sequence of pulses and that between to grow and to cut, etc."

The three theories of YAMATO are on:

- (1) Quality and quantity
- (2) Representations(content-bearing things)
- (3) Objects, processes and events

[625] YAMATO : Yet Another More Advanced Top-level Ontology Riichiro Mizoguchi *ISIR,Osaka University*http://www.ei.sanken.osaka-u.ac.jp/hozo/onto_library/upperOnto.htm
With the References:
[Galton 2009] Galton, A. and R. Mizoguchi: The water falls but the waterfall does not fall -- New perspectives on objects, processes and events, Journal of Applied Ontology, Vol.4, No.2, pp.71-107, 2009.
[Kitamura 2006] Yoshinobu Kitamura, Yusuke Koji and Riichiro Mizoguchi: An Ontological Model of Device Function: Industrial Deployment and Lessons Learned, Journal of Applied Ontology (Special issue on "Formal Ontology Meets Industry"), Vol. 1, No. 3-4, pp. 237-262, 2006.
[Kou 08] Hiroko Kou, et al.: A Fundamental Consideration toward Development of Medical Ontology, Proc. of the 22nd Annual Conference of the Japanese Society for Artificial Intelligence, 2E3-01, 2008(in Japanese).
[Masuya 2009] Hiroshi Masuya and Riichiro Mizoguchi: Toward integration of mouse phenotype information, The 2nd Interdisciplinary Ontology Conference, Tokyo, Japan, pp.35-44, 2009
[Mizoguchi 2004] Mizoguchi, R.: Tutorial on ontological engineering - Part 3: Advanced course of ontological engineering, New Generation Computing, OhmSha & Springer, Vol.22, No.2, pp.198-220, 2004.
[Mizoguchi 2007] Mizoguchi R., Sunagawa E., Kozaki K. and Kitamura Y.: A Model of Roles within an Ontology Development Tool: Hozo, J. of Applied Ontology, Vol.2, No.2, pp.159-179. Sep. 2007.
[626] http://www.ei.sanken.osaka-u.ac.jp/hozo/onto_library/YAMATO101216.pdf

According to Riichiro Mizoguchi "careful examination of them (the well-known "upper ontologies") reveals some room for improvement in three respects: (1) Quality and quantity, (2) Ontology of representation, and (3) Distinction between processes and events."[627] As Mizoguchi establishes "… quality and quantity need more careful investigation. One of the most sophisticated property ontologies is found in DOLCE. It specifies value space for each quality type under the name of Quality space(region). A remarkable feature of DOLCE's conceptualization about it is the clear distinction between the quality an object possesses and the quality value(quale/qualia) itself."

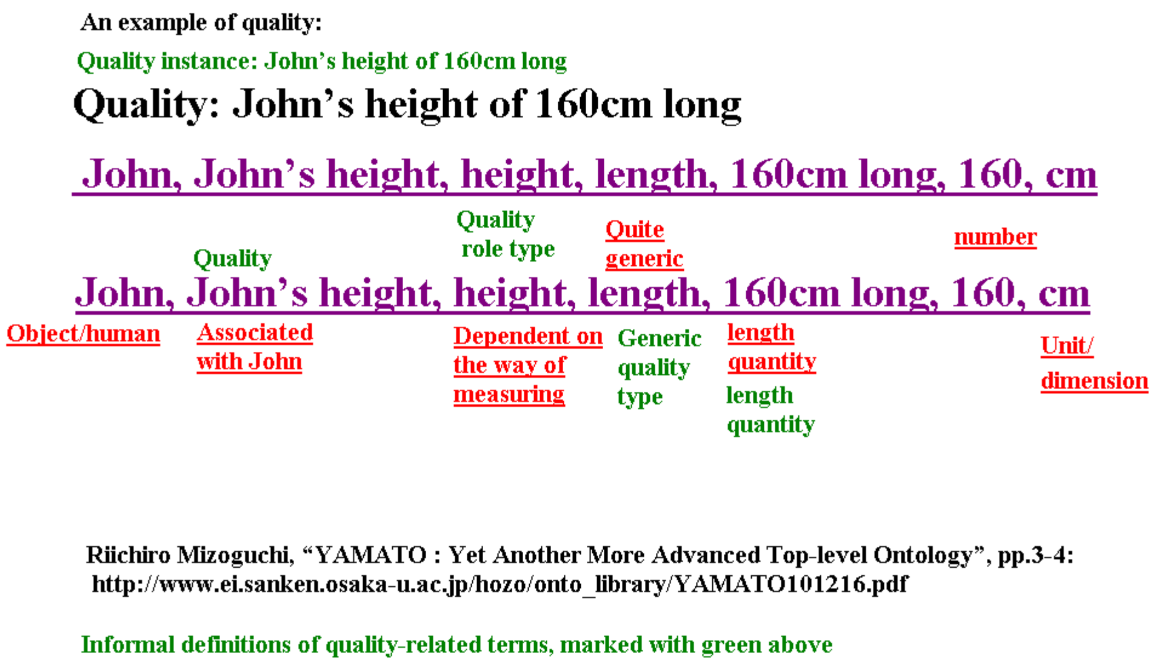
For Mizoguchi "no existing upper ontology distinguishes between quality in reality and quality description, which causes some confusion when talking about quality and quantity." Mizoguchi believes that "quality ontology should contribute to facilitation of data interoperability as well as capturing quality ontologically."

The topological notion of quality and quantity is evident in the understanding and definition of "quality" in ontology YAMATO. The definition of quality follows the threefold of concept in YAMATO - "basic concepts", "role", and "role holder". Quality is dependent attribute of the basic concept, which *preserved* its <u>identity</u> unchanged during *the occurrence of changes* of its <u>meaning</u> with the time flow. Topology is concerned with properties or qualities that are *preserved* under continuous deformations of objects. *The topological notion of quality and quantity in this ontology is demonstrated by the simultaneous of these two processes, first by the preservation of identity and second by the contiuous change of the meaning.*

In YAMATO the notion of quality is divided in *quality role* and in *quality role type.* Richiro Mizoguchi illustrate this notions of quality with the example of the statement "John's height of 160 im long". Here this division of *quality role* and *quality role type* is presented in the parts of the statement, where <u>John</u> is the *role holder* (object/human), element <u>John's height</u> is the *quality role* (particul dependent attribute) of height associated with particular role holder (John), and element <u>height</u> is the *quality role type,* not John's hight but just the <u>hight</u> (dependent on the way of measuring). After measuring the *hight* will be <u>length</u> *(generic*

627 http://www.ei.sanken.osaka-u.ac.jp/hozo/onto_library/YAMATO101216.pdf

*quality type),* in our example the length quantity is "160 cm long", where "160" is number and "cm" is unit/dimension.




YAMATO distinguishes between **quality in reality** and **quality description** and has a type hierarchy which reflects existing notions used in quality descriptions. There are clear distinction between attribute and attribute values. YAMATO is ontology build for each of attribute (*generic quality*) and attribute value (quantity) intended to describe things in terms of the combination of those dependent entities.

According to Richiro Mizoguchi, there are two ontological definitions for the meaning of "the quality", first is ca*tegorical* and the second is *quantitative*. Thus there are two divisions of quality in YAMATO - *categorical quality* and *qualitative quantity*. Mizoguchi defines two types of quantities – quantitative and qualitative or *quantitative quantity* и *qualitative quantity*.

Quantitative quantities are divided as dependent from attribute (attribute-specific) and independent from attribute (attribute-neutral). Qualitative quantities are also dependent from the attribute.[628]

[628] For the above discussion on "qualitative quantity" and "quantitative quantity" in YAMATO see the overview provided by Л.М. Пивоварова, "Онтология верхного уровня (Обзор)", Санкт-Петербургский государственный университет, Санкт-Петербург: http://conf.infosoc.ru/2010/thesis/PDF/51-61_Pivovarova.pdf

In our quest for Qualitative quantity, intriguing result will be find in part 3..3. Quantity, 3.3.1. What is quantity?" from the paper "YAMATO : Yet Another More Advanced Top-level Ontology ", [629] where Richiro Mizoguchi claims that: "quantity is instance of quality". According to Mizoguchi, "the facts that quantity must have its own subtype structure to represent quantized quantity for qualitative values which should be independent of quality role type and that quantity has something to say about their instances independently of their corresponding quality, e.g., *"large* is larger than *small"* and "green is the complement of red"(Dolce) show it is not appropriate. Quality and quantity must have their own hierarchies to represent their mutual independence. At the same time, their tight relationship must be modeled appropriately."

And Mizoguchi goes:

"Now, what is quantity? Philosophically speaking, quantity is generically-dependent entity in the sense that while it is not specifically-dependent on any physical entity, it cannot exist if all physical objects disappear from the world. Practically, it is what is denoted by its measurement. We never know the true quantity of anything because measurement is always incomplete. What is correct is that there are two kinds of quantity: true quantity and its measurement in which the latter is an approximation of the former. True quantity is unit-independent. A length quantity is independent of how long 1m is. We will come back to this issue later."

[629] YAMATO : Yet Another More Advanced Top-level Ontology, Richiro Mizoguchi *ISIR,Osaka University*http://www.ei.sanken.osaka-u.ac.jp/hozo/onto_library/upperOnto.htm

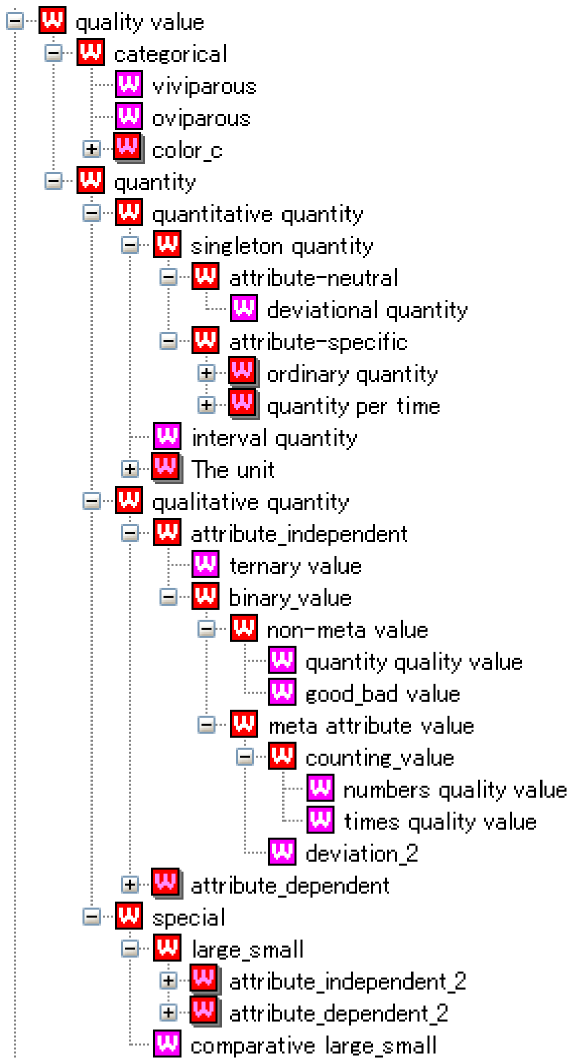



Fig. 8 Quantity
(from http://www.ei.sanken.osaka-u.ac.jp/hozo/onto_library/YAMATO101216.pdf)

The notion of qualitative quantity (and quantitative quantity) is ecplicite in Mizoguchi's hierarchy of quantity ("**3.3.2 is-a hierarchy of quantity"), where "**Quantity has its own hierarchical structure as shown in Fig. 8. Major features of this *quality value* hierarchy include separation of *categorical* and *quantity*, and introduction of qualitative values. By *categorical*, I mean genuine categorical quality value such as viviparous/oviparous, colors such as red, blue, green, etc. Note here that categorical value and qualitative (quantized) value ***(qualitative quantity)*** are intrinsically different. While categories have typicality, qualitative values do not. For example, red as a category has the typical redness at a certain frequency with peripheral redness at higher and lower frequencies. On the other hand, quantized qualitative values do not have such typical thing but still have, like quantity, ordinal relationship between values which categories do not. *Quantity* is further divided into *quantitative quantity* and *qualitative quantity*. The former is divided into *attribute-neutral* and *attribute-specific,* and the latter includes *"large", "high", "big", etc*. Examples of *attribute-dependent qualitative quantity* include *long, heavy, expensive, etc.*

***Quantitative quantity*** includes ordinary values such as *length quantity,weight quantity*, etc. ***Large_small*** is special in the ontology of quantity. It is introduced to make powerful qualitative values. Examples include high/low, big/small, etc. One of the well-known issues related to these values is how to realize qualitative values that are compliant with "A small elephant is bigger than a big ant" while guaranteeing "big" is larger than "small" in each context. ***Large_small*** realizes such qualitative values in a sophisticated way. Details are not discussed here. Those who are interested in it are encouraged to read the ontology. This is one of the most remarkable advantages of YAMATO over existing upper ontologies which cover only higher level types which are nowadays becoming a bit obvious."[630]

In the search machine of Hozo: Ontology Viewer[631], one can intervent with the word "qualitative quantity" and the result will show the following:

- List of Classes which referring to this class
    - quantitative generic quality
    - extrinsically qualitative
    - numbers_2
    - times_2
    - large_small
    - E_A_V
    - qualitative property
    - equivalence
    - comparative large_small

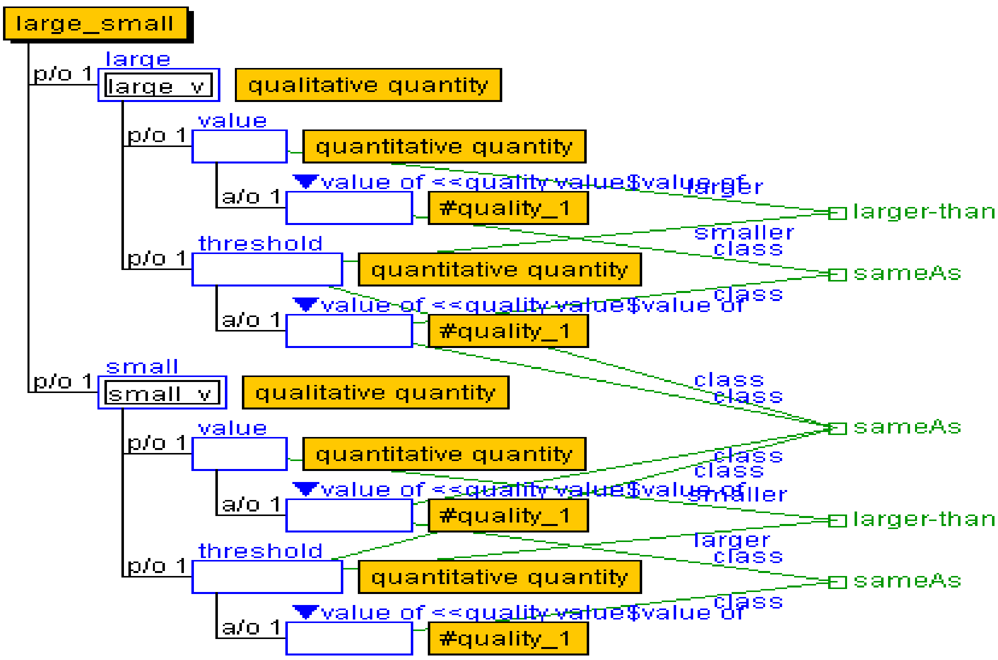
- List of Super-classes
    - Any
        - Particular
            - dependent entity

[630] http://www.ei.sanken.osaka-u.ac.jp/hozo/onto_library/YAMATO101216.pdf)

[631] http://hozoviewer.ei.sanken.osaka-u.ac.jp/HozoWebXML/?file_name=YAMATO20120714.xml

            - generically dependent
                - quality value
                    - quantity
                        - qualitative quantity

- List of Sub-classes
    - qualitative quantity
        - attribute_independent
            - meta attribute value
                - counting_value
                    - numbers quality value
                    - times quality value
                - deviation_2
            - non-meta value
                - binary_value
                    - quantity quality value
                    - good_bad value
                - ternary value
        - attribute_dependent
            - length quality value
            - weight quality value
            - volume quality value
            - controllability value
            - rationality value
            - autonomy value
            - beautiful_ugly value
            - diarrhea value



The search can be performed for the “quantitative quantity” as well.

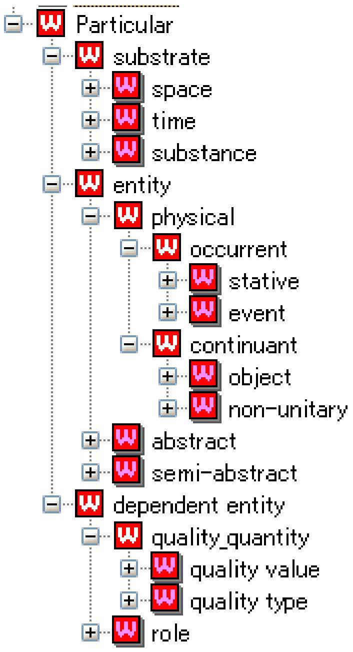



Fig. 1 Top-level categories.

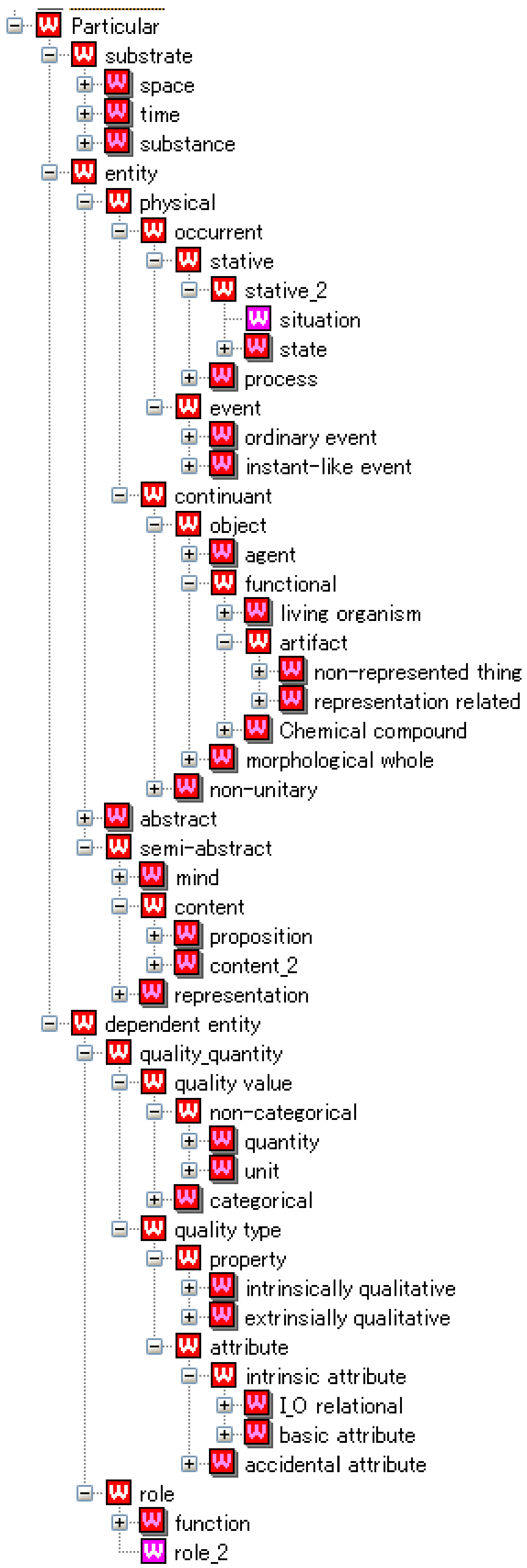
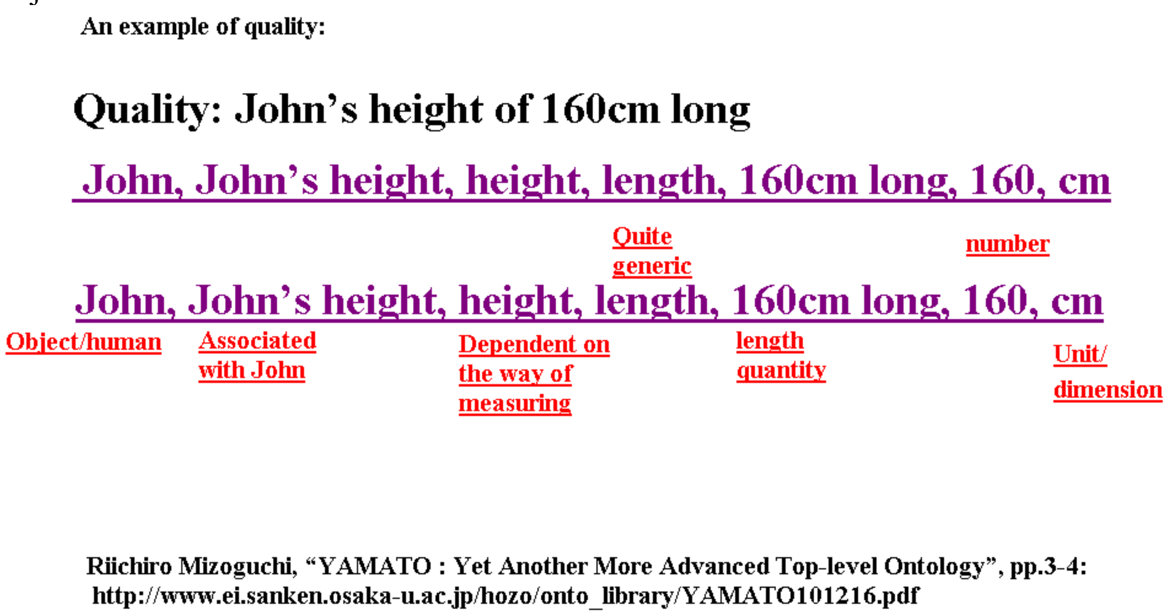



Fig. 2 Detail version of top-level

The link between Qualitative quantity and Topology (in particular Topological Data Analysis), and the proposition of the Qualitative quantity method as tool for the Topological Data Analisys, is encouraged by Richiro Mizoguchi's interpretation offered in part "3. Quality and Quantity, 3.1 Background" from his **"**YAMATO : Yet Another More Advanced Top-level Ontology" – "e-Science needs data exchange world-wide. Especially, in biology and bio-informatics, data exchange has been intensively conducted through global collaboration in the daily activities. In order to make it smooth to exchange scientific data, the way of description must be compatible with each other. Description of data/objects is usually done by determining attribute values. However, quality description is not well-understood by practitioners. Even professionals seem not to agree on the common way of quality description of objects." [632]




According to Riichiro Mizoguchi, "There are at least three ways of quality description recommended by BFO, DOLCE and Galen(Alan 2010). BFO recommends <Entity, Property> (e.g., <John, tall>) formalism, while DOLCE <Entity, Attribute, Value>(e.g., <John, height, 180cm>) and Galen <Entity, Property, Value>(e.g., <John, tallness, large>5). The problem is two fold: (a) one way of representation is not enough for real world problems because there exist multiple kinds of quality descriptions in the real data and (b) there is no explicit model of how these three are different from and interrelated with each other. The ontology of Quality and Quantity in YAMATO tries to give fundamental conceptualization of those existing descriptions in one ontology. To do so, YAMATO distinguishes between **quality in reality**

[632] Riichiro Mizoguchi, **"**YAMATO : Yet Another More Advanced Top-level Ontology", pp.3-4: http://www.ei.sanken.osaka-u.ac.jp/hozo/onto_library/YAMATO101216.pdf

and **quality description** and has a type hierarchy which reflects existing notions used in quality descriptions."

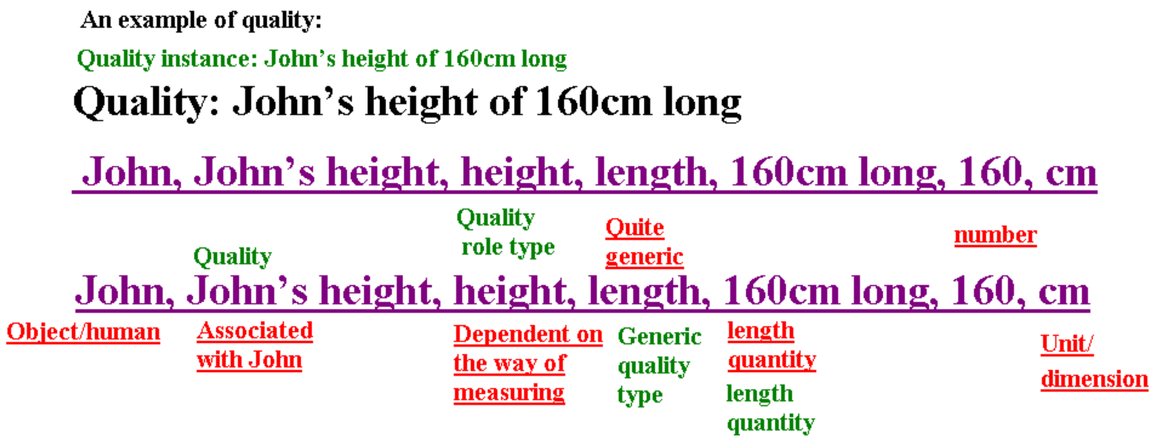
**Investigating "the underlying philosophy" of Quality, in part "3.2 Quality - 3.2.1 The underlying philosophy",** Riichiro Mizoguchi establishes that "one of the contributions of our ideas is clear distinction between attribute and attribute values. As far as I know, there are three dependent entities related to the term *attribute*: (1) attribute as a type that plays as dimension, (2) attribute as an instance and (3) attribute value that might correspond to quantity. However, some ontologies do not distinguish between (2) and (3). In this respect, DOLCE nicely differentiates instance of attribute from attribute value by introducing quality for attribute as an instance and quale/qualia for attribute value. This differentiation enables us to capture a thing, holding a quantity(qualia), that can change keeping its identity as we discuss it below. In other words, any quantity exist only one in the world independently of how many things have those quantities as their values. We build an ontology for each of **attribute (*generic quality*)** and **attribute value(quantity)** intended to describe things in terms of the combination of those dependent entities. We discuss each of the two in turn."

The figure above (marked as Fig.3 in Mizoguchi's work) shows an example of quality. According to Mizoguchi, "the issue here is how to wisely model this quality. ..Fig. 3 also shows types and individuals underlying the quality. Among them, "John", "160" and "cm" have no issue to discuss. Let us briefly investigate characteristics of the others. Firstly, "John's height" is something associated with John and exists at the instance level. "Height" is generic in the sense that the entity which it is associated with is unspecified, but is more specific than "length" in the sense that it is dependent on what to measure and how. This suggests the notion of "quality role" played by length, which is an interesting topic, but it will be discussed later. The "length" is quite generic because it can be height, depth, distance, etc. according to the context and exists at the class level. Finally, "160cm long" seems to be quantity and exists at the instance level.

In summary, issues include (1) if "160cm long" is a quality or a quantity, (2) if an instance of a quality is a quantity or not, (3) what is "John's height"?, (4) what is "height"?, and (5) how is "length" different from "height"? The following subsections are devoted to answer these questions."

Mizoguchi discusses the example of quality on change, illustrating with the "a boy named John who grows", where the John's height was 160cm in 2008 and 170cm in 2009. For Mizoguchi, "ontologically, in order to talk about *change* properly, we need a non-changing thing keeping its identity during the change process, otherwise, there cannot exist a change but just a difference. In the case of growing John, John and his height must keep their identities. That is, "John's height" must have the same identity at the times of 2008 and 2009. The values, on the other hand, are 160cm in 2008 and 170cm in 2009, respectively, and they have their own different identity. This analysis reveals that "John's height" has something having its identity and it is generic in the sense that its possible values are indefinite."
Later one Mizoguchi discusseses the "Quality role":

"We here discuss the notion of *quality role.* Many of the qualities people know are not basic quality (This term is temporary and will be renamed as *generic quality type* later.- Mizoguchi) but *quality role*. Let us take an example of height. It is a quality role whose basic quality is length. In the role theory (Mizoguchi 07), it can be said that "height role is played by length in the context of a human body". Height is what is measured in the direction roughly along the vertical axis from the ground level to the top. Similarly, depth, width and distance are quality roles played by length as well. It is just like a man plays a husband role when he has got married and like a human plays a teacher role in the context of a school. Let us call quality such as length as *basic quality* temporarily. Examples of basic qualities include length, area, mass, temperature, flow rate, voltage, etc. Basic qualities are context-independent and intrinsically represent kinds of quality, and roughly equal to physical dimensions. Quality roles include height, depth, input flow rate, maximum weight, area of cross section, etc."




Riichiro Mizoguchi, "YAMATO : Yet Another More Advanced Top-level Ontology", pp.3-4:
http://www.ei.sanken.osaka-u.ac.jp/hozo/onto_library/YAMATO101216.pdf

Informal definitions of quality-related terms, marked with green above

Mizoguchi's definition of "quality" is given in part "**3.2.5 Summary of the discussion on quality"** – "A quality must be a certain dependent property possessed by an entity and keeps its identity while its value changes."

In the case of growing John, this understanding suggests that not "John's height of 160cm long" but "John's height" should be taken as a ***quality*** as shown in Fig. 5. As is already clear, quality roles are necessarily associated with a particular entity which is the context of them at the instance level. The fact satisfies the requirement of quality. **Therefore, *quality roles* are what should be called *quality*.**

Then, "John's height of 160cm long" would be called ***quality instance (***Although *quality realization* might be a better name, I – Mizoguchi - use *quality instance* to minimize peculiarity). because it is a realization of quality role "John's height" at a certain time. In terms of role theory, "John's height of 160cm long" is called a *role holder*, which will be explained later, which means a length playing the "John's height" role. Then, "height" is called ***quality role type*** which has "John's height" as its instance and "length" should be called ***generic quality type*** because it is not associated with any entity yet and specifies what kind of quality there exist. It was called basic quality in the above. The results are summarized in Fig. 5.

Definitions of these basic terms are shown in the following:

1. ***Generic quality type:*** The most generic property which is not yet associated with any particular context and can play *quality role*. It represents kinds of property at the class level and its instances represent concrete values corresponding to quantity to play *quality role*: e.g., length, weight, mass, color, area, flow rate, etc.

2. ***Quality role type:*** It is a type of property associated with a context, and hence is equal to *quality role* at the class level. Its instance is played by an instance of *generic quality type*: e.g., height, depth, (something's) length, (someone's) weight, mass of cross section, input voltage, etc.

3.***Quality:*** It is a property associated with an individual entity and is a generic name to denote each instance of ***quality role type***: e.g., John's height, Tom's weight, area of the cross section of this pipe, length of this pen, etc.

4.***Quality instance:*** A realization of ***quality***: e.g., John's height of 160cm long, Tom's weight of 50kg, etc."

According to Richiro Mizoguchi, the existing similar terms such as *property, attribute, quality, quality type, quality role type and quantity* are dependent entities and cannot exist without an entity with which they are associated. It is almost impossible to define these terms without discussing deep issues related to them. Because of this difficulty, terms such as ***attribute***and ***property*** used thus far should not be taken as technical terms but as common words which are vaguely defined."

Mizoguchi establishes that:
"… now it is the time to differentiate them based on our discussion thus far. *Quality, quality role type and quantity* are already defined with examples. So, the problem is the rest two. Therefore, these terms appearing below should be taken as defined here.
*Property:* As a common term, property is almost a synonym of quality in our context. As a technical term, however, they are different. The major difference is that while a *property* can be used as any predicate to any individual like human(x) or animal(x), *quality* is used only for what an entity has. However, considering that our problem in this paper is to talk about how

to describe/characterize entities, property and quality become synonym again because there is no necessity for talking about “human as a property” when describing “human” entity. This definition is only valid in the following sections in this paper, that is, the term *property* appearing in the above should not be taken as defined here. Because we defined ***quality*** differently from what is defined in BFO, by *property*, I mean *“quality”* used in BFO, that is, value combined with its variable or the variable taking a value that can be possessed by an entity. I think I need to defend my decision on this. ***Quality*** must be something associated with an entity and can change with keeping its identity. These characteristics necessarily derive that ***quality*** cannot be something combined with its value, otherwise, it cannot keep its identity when its value has changed. Therefore, the notion of *quality* employed in BFO is not appropriate in this respect. As was already discussed, an instance of BFO-defined quality such as “John’s height of 160cm long” is called *quality instance* and any length whose value is 160cm is represented as *property* in YAMATO, though *property* is defined mainly for *qualitative property* such as *being red*, *being natural, being artificial, etc*..

*Attribute:* Originally, it is a binary predicate which relates an entity to its *quality instance*, e.g., length(pen_1, 10cm). In AI, <E, A, V> is often used to describe entities. In such cases, “A” stands for *quality role type* and/or *generic quality type*. Following the use of this term in AI, we could define *attribute* as a synonym of ***generic quality type***. However, we intentionally leave this term undefined for flexible use in text. So, the term *attribute* should be taken as a common term in this paper. In YAMATO implemented in Hozo, the term *property* is used to mean BFO’s *quality*. Because the role hierarchy is implicit in Hozo, hierarchies of ***quality*** and ***quality role type*** are invisible unless Role hierarchy mode of Hozo is used.”

The problem is two fold: (a) one way of representation is not enough for real world problems because there exist multiple kinds of quality descriptions in the real data and (b) there is no explicit model of how these three are different from and interrelated with each other.

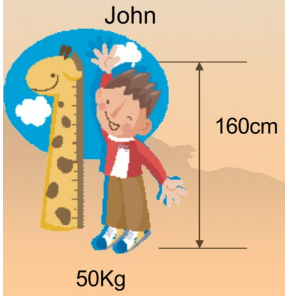
The ontology of Quality and Quantity in YAMATO tries to give fundamental conceptualization of those existing descriptions in one ontology. To do so, YAMATO distinguishes between quality in reality and quality description and has a type hierarchy which reflects existing notions used in quality descriptions.”

# Reality of quality/quantity

**Quality**: John's height of 160cm long ← This is the only reality we share!?
The issue is how to wisely model it

What exist here: John, John's height, height, length, 160cm long, 160, cm

Object/human — Associated with John — Dependent on the way of measuring — Quite generic — length quantity — number — Unit/dimension



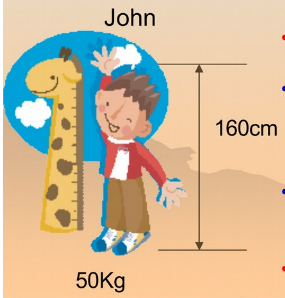

- **Quality type:** length, weight, etc. as a kind of qualities
  Then, what is height?
  Is 160cm an instance of length? BFO: <160cm *instance-of* quality type>
- **Quality dimension:** 160cm is a value rather than an instance
- **Quality:** John's height, John's weight, etc.,
  independently of how big and when *as an identity holder*
- **Quality as role:** height, depth, etc. are roles played by length
- **Quality must be something associated with an entity**
  So, <160cm long> can't be a quality
- **Quantity:** 160cm, 50Kg, etc.
  Quantity is generic. 160cm could be height of John and Tom, distance between A and B, etc.
  Quantity as representation: 1m=100cm=1000mm=0.001Km
- **Property:** a pair of quality type and quantity: <height, 160cm>
  This is compliant with "state"; hungry = <hunger state, hungry>
  tall = <height, high or long>

# Reality of quality/quantity 2

**Quality instance**: John's height of 160cm long ← This is the only reality we share!
The issue is how to wisely model it

What exist here: John, John's height, height, length, 160cm long, 160, cm

Object/human — **Quality** — **Quality type** — **Generic Quality type** — length quantity — number — Unit/dimension




- **Generic quality type:** length, weight, etc. as a kind of qualities
  *Height, width, or distance are not included.*
  *160cm is an instance not of length but of length quantity.*
- **Quality dimension:** 160cm is a value rather than an instance
- **Quality:** John's height, John's weight, etc.,
  independently of how big and when ***as an identity holder***
- **Quality type (Quality as role):** height, depth, etc. played by length
  So, <160cm long> is a player of quality as role.
- **Quantity:** 160cm, 50Kg, etc.
  Quantity is generic. 160cm could be height of John and Tom, distance between A and B, etc.
  Quantity as representation: 1m=100cm=1000mm=0.001Km
- **Property (quality):** a pair of quality type and quantity: <height, 160cm>
  which is compliant with "state"; hungry = <hunger state, hungry>
  tall = <height, high or long>
- **Quality must be something associated with an entity**

# References:

**References:**


1. Allen J (2011a) Topological twists: Power's shifting geographies.Dialogues in Human Geography 1(3): 283–298.

2. Allen J (2011b) Powerful assemblages? Area 43: 154–157.

3. Aristotle (1983) Aristotle's Physics Books III and IV, trans. Edward Hussey, Oxford: Clarendon Press.

4. Artemov, S., and E. Nogina. (2007), Topological Models for Justification Logic, TANCL 2007, Oxford University; Sergei Artemov and Elena Nogina, THE TOPOLOGY OF JUSTIFICATION: http://www.logika.umk.pl/llp/1712/21-1712zw.pdf;

5. Artemov, S., J. Davoren, and A. Nerode. (1997), "Modal logics and topological semantics for hybrid systems", Technical Report MSI 97-05, Cornell University, 1997.;

6. Artemov, S., and E. Nogina. (2004), "Logic of knowledge with justifications from the provability perspective", Technical Report TR-2004011, CUNY Ph.D. Program in Computer Science, 2004.;

7. Artemov, S., and E. Nogina. (2005), "Introducing justification into epistemic logic, Journal of Logic and Computation 15, 6 (2005), 1059–1073.;

8. Artemov, S., and E. Nogina. (2005), "On epistemic logic with justification", pages 279–294 in R. van der Meyden (ed.), Theoretical Aspects of Rationality and Knowledge. Proceedings of the Tenth Conference (TARK 2005), June 10–12,2005, Singapore, National University of Singapore, 2005.;

9. Artemov, S., and E. Nogina. (2007), "On topological semantics of justification logic",Algebraic and Topological Methods in Non-Classical Logics III (TANCL'07)Oxford, England, August 2007.

10. A Topological Approach to Cultural Dynamics (ATACD) http://www.atacd.net/index.php %3Foption=com_content&task=view&id=107&Itemid=42.html

11. Aubin, David, (2004), Forms of explanation in the catastrophe of Rene Thom: topology, morphogenesis, structuralism, in *Growing Explanations: Historical Perspective on the Sciences of Complexity*, ed. M. N. Wise, Durham: Duke University Press, 2004, 95-130.

12. Badiou, Alain. (2000), "Deleuze: the Clamor of Being", translated by Louise Burchill, 2000, The University of Mianesota Press.

13. Badiou, Alain. (2007), Being and Event, Oliver Feltham (tr.), Continuum, 2006 (Badiou A. 2007)

14. Badiou, Alain. (2009),, Logic of Words: Being and Event, Volume 2, transl. by Alberto Toscano, New York: Continuum, 2009 (Badiou A. 2009)

15. Badiou, Alain. (2009), Theory of the Subject, transl. By Bruno Bosteels, New York: Continuum, 2009

16. Badiou, Alain, Bellassen, Joël and Mossot , Louis. ( ) "The Rational Kernel of the Hegelian Dialectic", See: (b) On the interior and the exterior. - Hegelian topology

17. Baer, Reinhold (1932). Hegel und die Mathematik. In Veröffentlichungen des Internationalen Hegelbundes, Verhandlungen des Zweiten Hegelkongresses vom 18. bis 21. Oktober 1931. Berlin & Tübingen: s.n., 104-120.

18. Back, Kurt W. (1992) "This business of topology." Journal of Social Issues 48(2): 51-66.
http://onlinelibrary.wiley.com/doi/10.1111/j.1540-4560.1992.tb00883.x/abstract

19. Barnett, Stuart, Ed. (19989/2001) Hegel after Derrida, Taylor & Francis, Warwick studies in European philosophy (Stuart Barnett S, Ed., (1989/2001)

20. Baumann, B. (2010), On Centrism and Dualism - A Critical Reassessment of Structural Anthropology's Contribution to the Study of Southeast Asian Societies, Südostasien Working Papers No. 40, Berlin 2010

21. Beaudry, Pierre. (2012), Analysis Situs and the principle of reciprocity, . . . .

22. Bell, John L (2005), The Continuous and the Infinitesimal in Mathematics and Philosophy. Polimetrica, 2005

23. Bergson, Henri (1910): Time and Free Will, New York, MacMillan Co., p.123. (French original, entitled 'Essai sur les données immediates de la conscience', Paris 1889)

24. Belcher O, Martin L, Secor A, et al. (2008) Everywhere and nowhere: The exception and the topological challenge to geography. Antipode 40(4): 499–503.

25. Berto, Francesco. (2007), Hegel's Dialectics as a Semantic Theory: An Analytic Reading, Blackwell Publishing Ltd. 2007:24

26. Blackwell, Brent M. (2004), Cultural Topology: an Introduction to Postmodern Mathematics http://reconstruction.eserver.org/044/blackwell.htm

27. Bullinger, E. W., D.D. Entry for 'Metalepsis; or double metonymy'. Bullinger's Figures of Speech Used in the Bible. http://www.studylight.org/lexicons/fos/view.cgi?n=132

28. Burbridge, John W. (1981), On Hegel's Logic: Fragments of the Comentary, 1981

29. Burbridge, John W. (1996), Real Process: How Logic and chemistry combine in Hegel's philosophy of nature, 1996.

30. Bookstein, Fred L. (1982), Foundation of Morphometrics, Annual Reviw of Ecology and Systematics, Vol.13, (1982), pp.451-470

31. Bookstein, Fred L. (1991), Morphometric Tools for Landmark Data: Geometry and Biology, Cambridge, England: Cambridge University Press, 1991.

32. Bookstein, Fred L. (1996), "Biometrics, Biomathematics and the Morphometric synthesis …. The quantitative study of biological shape variation" Bulletin of Marhemancal Biology, Vol. 58, No. 2, pp. 313-365, 1996, Elsevier Science Inc., Society for Mathematical Biology.

33. Bookstein, Fred L. (1980), When One Form is Between Two Others: An Application of Biorthogonal Analysis, Amer. Zool. (1980) 20 (4): 627-641.

34. Boyadhiev, Tsocho. (1984), "The Unwritten Doctrine of Plato", Sofia, Bulgaria, 1984.

35. Botnan, Magnus Bakke, (2011) Three Approaches in Computational Geometry and Topology - Persistent Homology, Discrete Differential Geometry and Discrete Morse Theory", 2011

36. Brandom, R. B. (1994), Making It Explicit. Cambridge, MA: Harvard University Press.

37. Brandom, R. B. (1999), 'Some Pragmatist Themes in Hegel's Idealism: Negotiation and Administration in Hegel's Account of the Structure and Content of Conceptual Norms', European Journal of Philosophy, 7: 164–89.

38. Brandom, R. B. (2000), Articulating Reasons. Cambridge, MA: Harvard University Press.

39. Brandom, R. B. (2001), 'Holism and Idealism in Hegel's Phenomenology', Hegel-Studien, 36: 57–92.

40. Brandom, R. B. (2002), Tales of the Mighty Dead: Historical Essays on the Metaphysics of Intentionality. Cambridge, MA: Harvard University Press.

41. Brentano, F. 1988. Philosophical Investigations on Space, Time and the Continuum, translated by Barry Smith, London/New York/Sydney: Croom Helm.

42. Brown, Steven D. (2012) Memory and mathesis: For a topological approach to psychology., Theory, Culture & Society. (Brown S, 2012)

43. Bruce, B. and D.N. Mond, eds. (1999), Singularity Theory, Cambridge, England: Cambridge U.Press, 1999

45. Cantillo, Giuseppe .(2013), The concept of space in Hegel: The Early Jena Years, Lexicon Philosophicum, 1, 2013, http://lexicon.cnr.it/, p.46 (Cantillo, 2013:29)

46. Carlson, David G. (2000). Hegel's Theory of Quality. Public Law Research Paper, 17. New York: Cardozo Law School. Consulted version: http://ssrn.com/abstract=241950.

47. Carlson, David G. (2002). Hegel's Theory of Quantity. Cardozo Law Review 24, 6. Consulted
version: http://ssrn.com/abstract=326822.188

48. Carlson, David G. (2003a). Hegel's Theory of Measure. Public Law Research Paper, 66. NewYork: Cardozo Law School. Consulted version: http://ssrn.com/abstract=413602. DOI: 10.2139/ssrn.413602

49. Carlson, David G. (2003b). The Antepenultimacy of the Beginning in Hegel's Science of Logic. Public Law Research Paper, 74. New York: Cardozo Law School. Consulted version: http://ssrn.com/abstract=425122
Carlson, David G. (2005), Why Are There Four Hegelian Judgments?, p.114:125, in Hegel's theory of the subjects, David G. Carlson, ed. 2005, Palgrave Macmillan 2005

50. Casati, R. (2000), Topology and cognition, Encyclopedia of cognitive science, Macmillan Reference, CNRS Institut Nicod, Paris (Casati R, 2000)

51. Carlson, G. 2009. Topology and data//Bull. Amer. Math. Soc. 46: 255-308 http://www.ams.org/journals/bull/2009-46-02/S0273-0979-09-01249-X/S0273-0979-09-01249-X.pdf

52. Chan JM, Carlsson G, Rabadan R, 2013. Topology of viral evolution,// Proc Natl Acad Sci USA 2013, Nov 12;110(46):18566-71.

53. Chen, L. (1982), Topological structure in visual perception, Science. 218, 699-700 (Chen L, 1982)

54. Chen, L. (2005), The topological approach to perceptual prganization, Visual Cognition, 12, 553-637 (Chen L, 2005)

55. Chen, L. Zhang, S.W. & Srinivasan, M. V. (2003) Global perception in small brains: Topological Pattern recognition in honeybees. (Chen L, Zhang S W, & Srinivasan M V, 2003)

56. Coltman, R. (1998) The Language of Hermeneutics, Gadamer and Heidegger in Dialogue, 1998, State University of New York (Caltman A, 1998)

57. Connor, Steven (2004) Topologies: Michel Serres and the Shapes of Thought, Anglistik, 15 (2004): 105-117, also http://www.stevenconnor.com/topologies/

58. Connor, S. (2004). Topologies: Michel Serres and the Shapes of Thought. Anglistik , 15, 105-107.

59. Collier, S. (2011, March 23). Interview with Stephen J. Collier on Foucault, Assemblages and Topology. Retrieved September 26, 2013 from Theory, Culture & Society: http://theoryculturesociety.blogspot.fr/2011/03/interview-with-stephen-j-collier-on.html

60. Coombs, Nathan. (2013), Politics of the event after Hegel, PhD dissertation, University of London, Department of Politics and International Relations.

61. Crowell, Steven..(2011), Is Transcendental Topology Phenomenological?, International Journal of Philosophical Studies, 19 (2): 267-276(2011) 201

62. Crockett, Clayton (2013), Deleuze Beyond Badiou: Ontology,. Multiplicity, and Event. New York: Columbia University Press,. 2013

63. Cipra, Barry A. (2007), Invasion of the Shapeshifters, published in *SIAM News*, Volume 40, Number 2, March 2007

64. Curtius, Ernst. (2013), European Literature and the Latin Middle Ages, Princeton University Press

65. Damsma, Dirk, (2006), On the Dialectical Foundations of Mathematics.

66. Damsma, Dirk,, (2008) On the Dialectical foundations of mathematics http://www1.fee.uva.nl/pp/bin/286fulltext.pdf

67. Damsma, Dirk. (2010). Qualitative and quantitative analysis in systematic dialectics: Marx vs. Hegel and Arthur vs. Smith. (Preprints). Amsterdam: University of Amsterdam, School of Economics.[go to publisher's site]

68. Damsma, Dirk. (2011). Set theory and geometry in Hegel. In A. Arndt, P. Cruysberghs & A. Przylebski (Eds.), Geist? - Tl. 2 Vol. 2011. Hegel-Jahrbuch (pp. 54-58). Berlin: Akademie Verlag.

69. Damsma, Dirk. (2011). On the dialectical foundations of mathematics. (Preprints). Amsterdam: University of Amsterdam. Available from: http://www.researchgate.net/publication/254914726_On_the_dialectical_foundations_of_mathematics [accessed Jul 21, 2015].

70. Damsma, Dirk, (2011), Set Theory and Geometry in Hegel (2011), In Hegel Gesellschaft, Hegel-Jahrbuch. Berlin: Akademie Verlag, http://www1.fee.uva.nl/pp/bin/311fulltext.pdf;

71. Damsma, Dirk. (2015, January 09). On the articulation of systematic-dialectical methodology and mathematics. Universiteit van Amsterdam (viii, 201 pag.). Supervisor(s): prof.dr. J.B. Davis & dr. G.A.T.M. Reuten.

72. Darke, Ian . (1982), A Review of Research Related to the Topological Primacy Thesis, Educational Studies in Mathematics, Vol. 13, No. 2 (May, 1982), pp. 119-142.

73. Dauben, J. (1979). Georg Cantor: His Mathematics and Philosophy of the Infinite. Harvard University Press.

74. Dawkins, R. (1989). The Extended Phenotype. Oxford University Press

75. Deleuze, Gilles. (1993) The Fold: Leibniz and the Baroque. Translated by Tom Conley. London: The Athlone Press, London (Deleuze G, 1993)

76. Deleuze G (1990) The Logic of Sense. New York: Columbia University Press.

77. Deleuze G (1994) Difference and Repetition. New York: Columbia University Press.

78. Deleuze G and Guattari F (1987) A Thousand Plateaus: Capitalism and Schizophrenia. Minneapolis, MN:University of Minnesota Press.

79. Deleuze G. (1962), Nietzsche and philosophy, Originaly published in France in 1962

80. Derrida, Jacques. (1982) The Pit and the Pyramid: Introduction to Hegel's Semiology, in Margins of Philosophy, trans. A. Bass, Chicago: University of Chicago Press, p. 88. (Derrida J, 1982)

81. Dimitrov, Borislav. (2014), Philosophical topology and Topological philosophy as the mode of thinking of Evolution of Hierarchical Systems: The Role of Heterarchy and Heteronomy in Evolution, Conference Edition: Evolution of Hierarchical Systems, Sofia, Faber Publishing House, September 2014, p.285-318

82. Dimitrov, Borislav. (2014), Auditor Independence within An Auditing Analysis Situs: Topology of Places as factor for enhancing auditor independence, competence, and audit quality, between the global and local: Topological Approach to Audit Dynamics, Focused on Auditor Independence, Competence and Audit Quality through Qualitative quantity methodology and Topological Data Analysis, Philosophy of Science for Social Science, Lund University, Faculty of Social Science.

83. Dimitrov, Borislav. (2014), The Struggle of Cultural Identity between the Dichotomies of Society and Community, between Liberalism and Communitarianism: Dialogue or becoming Topological? Philosophical topology of intercultural (identity) relationships, Study presented at the International Conference 'The Individual and Society: Challenges of Social Change", April 5th, 2014, Sofia, Bulgaria (Bulgarian Academy of Science and Arts, Serbian Royal Academy of Science and Arts, European Center of Business, Education and Science, published in the Conference edition collection 'The Individual and Society: Challenges of Social Change' (ISBN 978-954-411-151-9), 2014, p. 266-296.

84. Dimitrov, Borislav. (2014), Hegel's Analysis Situs: Topological Notions of Multiplicity in Hegel's Fourfold of Infinities, pending publication, Sophia Philosophical Review, 2014

85. Dimitrov, Borislav. (2013), Topological Ontology and Logic of Qualitative quantity, https://www.academia.edu/3237237/Topological_Ontology_and_Logic_of_Qualitative_quantity

86. Dimitrov, Borislav. (2012), A Topological Approach to 'The Hospital of the Future': Topological Model based on the qualitative quantity research method, Amazon.

87. Dimitrov, Borislav. (2012), The Topological Approach of Qualitative quantity Implemented in Autopoietic Law and Audit: The Cultural Phenomenology of Qualitative quantity and … The Cultural Phenomenology of Law and Auditing as Autopoiesis", „Ariadne

– Topology and Cultural Dynamics – Institute for Cultural Phenomenology of Qualitative quantity, http://ariadnetopology.org/3.html;

88. Dimitrov, Borislav. (2012), Cultural Phenomenology of Law and Topological Approach to Law, A Series of papers presenting the essentials of Topological Approach to Law : Qualitative quantity - The Cultural Phenomenology of Literature and … The Cultural Phenomenology of Law; Law and Literature Movement; Cognitive Science and The Law - Topological Approach To Law; Phenomenology of Law; Law and Social Choice: Qualitative quantity, Topological Social Choice and Topological Approaches to Law; The proposition of Qualitative quantity mode of Inquiry in the Classic Debate – Qualitative vs Quantitative research; The Topological approach of Charles Sanders Peirce's qualitative-ness and The Topological Qualitative quantity; The philosophy of Émile Boutroux – a profound influence on Henri Poincaré and Charles Peirce. „Ariadne – Topology and Cultural Dynamics – Poincaré and Charles Peirce. „Ariadne – Topology and Cultural Dynamics – Institute for Cultural Phenomenology of Qualitative quantity", http://ariadnetopology.org/3.html;

89. Dimitrov, Borislav. (2011), The Relevance of Topological Approach, based on Qualitative quantity research method, to Audit Dynamics and Auditing Research – Cultural Phenomenology of Audit and Auditing research, „Ariadne – Topology and Cultural Dynamics – Institute for Cultural Phenomenology of Qualitative quantity", http://ariadnetopology.org/Cultural_Phenomenology_of_Audit_Dynamics_and_Auditing_Research_web.pdf

90. Dimitrov, Borislav, (1989a) "Time and Simultaneousness. Quality of the Quantitative – an interpretation on Henri Poincare's theory of the Simultaneousness", based on G.F.W.Hegel's logic of Qualitative quantity", under the academic research supervision of Sava Petrov,PhD, Institute for Philosophical Research, Bulgarian Academy of Science. (Dimitrov B, 1989)

91. Dimitrov, Borislav, Quality of quantity, "Philosophic Thought Magazine", March, 1989, the journal edition of Institute of Philosophical Sciences, Bulgarian Academy of Science. (Dimitrov B, 1989)
https://www.academia.edu/657086/Quality_of_the_Quantity

92. Dimitrov, B. (1990)."Quality and Time", presented at the conference "The Fundamental Knowledge between Ontology Dilemma and Cognitive Problems", published in 1990, by The Institute for Philosophical Research at the Bulgarian Academy of Science. https://www.academia.edu/657102/Quality_and_Time (
(Dimitrov B, 1990)

93. Dimitrov, B. (2013). Topological notions of Bracketing Difference: Husserl's phenomenological reduction and Derrida's deconstruction. available at (Dimitrov B, 2013) https://www.academia.edu/3790694/Topological_notions_of_Bracketing_Difference_Husserls_phenomenological_reduction_and_Derridas_deconstruction

94. Donnelly, N. (2005), The topological approach to perceptual organization :wholes to parts, Visual Cognition, 12 (4), 648-652 (Donnelly N, 2005)

95. Donnelly, N. (2005), The topological approach to perceptual organization:wholes to parts, Visual Cognition, 12 (4), 648-652 (Donnelly N, 2005)

96. Eldred, Michael. (2010), Digital Dissolution of Being, Published in Left Curve no. 34 (2010)

97. Ellsworth de Slade, H. (1994). Das wahrhafte Unendliche und die Unendlichkeit der Mathematik; Eine Studie zu den Entsprechungen zwischen Hegels Bestimmung des wahrhaften Unendlichen in der "Wissenschaft der Logik" und seiner Auffassung der Infinitisimal-Mathematik. Heidelberg: Philosophisch-Historischen Fakultät der Ruprecht-Karls-Universität Heidelberg.

98. Epple, Moritz (1998), Topology, Matter, and Space, I: Topological Notions in 19th-Century Natural Philosophy, Arch. Hist. Exact Sci. 52 (1998) 297–392.c Springer-Verlag 1998 (Epple, 1998)

99. Fell, Joseph P. (1983), Heidegger and Sartre: An Essay on Being and Place (New York: Columbia University Press, 1983)

100. Ferrarin, A., 2007, Hegel and Aristotle, Cambridge University Press, 2007
Fuller, Buckminster. (1975/1979). "Synergetics – Explorations in the Geometry of thinking", Macmillan Publishing Co. Inc. 1975, 1979

101. Ferrini, Cinzia. (1998), Framing Hypotheses: Numbers in Nature and the Logic of Measure in the Development of Hegel's System, in Hegel, and the Philosophy of Nature, 283 (Stephen Houlgate ed., 1998)

102. Flach, W. (1964), 'Hegels dialektische Methode', Hegel-Studien, 1: 55–64.

103. Fulda, H. F. (1973), 'Unzula¨ngliche Bemerkungen zur Dialektik', in R. Heede and J. Ritter (eds.) Hegel-Bilanz. Frankfurt a.M.: Klostermann, repr. in R.-P- Horstmann (ed.) Seminar: Dialektik in der Philosophie Hegels. Frankfurt a.M.: Suhrkamp.

104. Gadamer, Hans-George. (1971) "Die Idee der Hegelschen Logik (1971) in Hans-George Gadamer, "Hegel's Dialectic: Five Hermeneutical Studies", translated into English by P. Christopher Smith and collected in (New Haven: Yale University Press, 1976). These five essays are "Hegel and Heidegger,"; "Hegel's Dialectic of Self-consciousness"; "Hegel and the Dialectic of the Ancient Philosophers,"; "Hegel's 'Inverted World,'; and "The Idea of Hegel's Logic," 75-99

105. Garcia, Angel Lopez, (1990), Introduction to Topological Linguistics - Annexa-LynX. Valencia-Minnesota, 1990.

106. Gelley, Alexander. (1974), Ernst Robert Curtius: Topology and Critical Method. In Velocities of Change. Ed. Richard Mackey. Baltimore: The Johns Hopkins University Press (Gelley A, 1974)

107. Genette, G. (1972, 2004) Narrative Discourse: An Essay in Method, tr. J.E. Lewin (Ithaca 1980; orig. publ. as Discours du récit, Paris 1972), 234-37, and Métalepse: de la figure à la fiction (Paris 2004). (Genette G, 1972, 2004)

108. Genette, G.([1983] 1988). Narrative Discourse Revisited. Ithaca: Cornell University Press. (Genette G, 1983, 1988)

109. Garrett HE (1939) Lewin's 'topological' psychology: An evaluation. Psychological Review 46: 517–524.

110. Gies, M.(1983), Einführung in Hegels Naturphilosophie. Lecture Notes Vol. 3339-9-01-S1, Fern-University of Hagen, 1983

111. Giovanelli, Marco. (2011), Reality and Negation - Kant's Principle of Anticipations of Perception. Dordrecht: Springer.

112. Giovanni, Boniolo and Valentini Silvio. (2008), Vagueness, Kant and Topology: A Study of Formal Epistemology, Journal of Philosophical Logic, Vol. 37, No. 2 (April 2008), pp. 141-168, Springer.

113. Gotthard Günther. (1964), "Cybernetics and the Dialectic Materialism of Marx and Lenin", published at http://www.thinkartlab.com as an enlarged representation of a lecture Gotthard Günther did deliver at the University of Cologne (Köln, Germany) July 17, 1964. The paper was prepared under the Sponsorship of the Air Force Office of Scientific Research, Directorate of Information Sciences, Grant AF - AFOSR - 8 - 63 and 480-64.

114. Gotthard Günther, "Number and Logos", published at http://www.thinkartlab.com

115. Groome, Robert (2009), "Formalization of Hegel's Phenomenology of the Spirit" http://www.lacanlosangelespsychoanalysis.com/classes/course/info.php?id=21

116. Gruner, Stefan; Bartelmann, Matthias, (2015), The notion of 'Aether': Hegel versus contemporary physics, Cosmos & History; 2015, Vol. 11 Issue 1, p 41-68

117. Ghrist, Robert. (2007), Barcodes: The Persistent Topology of Data, 2007, Bulletin of American Mathematical Society, Volume 45, Number 1, January 2008, Pages 61–75

118. Ghrist, Robert., Vin de Silva Homological Sensor Networks, http://www.math.upenn.edu/~ghrist/preprints/noticesdraft.pdf

119. Gould, Stephen Jay. (1971), D' Arcy Thompson and the Science of Form, New Literary History Vol. 2, No. 2, Form and Its Alternatives (Winter, 1971), pp. 229-258, Published by The John Hopkins University Press

120. Gehring, W.J., (1998), *Master control genes in developmental evolution.* New Haven and London: Yale University Press.

121. Grenander, Ulf; Miller, Michael (2007). Pattern Theory: From Representation to Inference. Oxford University Press.

122. Grenander, Ulf. (2004), The Patterns in Growth", source *www.dam.brown.edu/ptg/REPORTS/GRID_TMI_07.pdf*

123. Grenander, Ulf, Srivastava, Anuj, Saini, Sanjay, (2007), A Pattern – Theoretic Characterization of Biological Growth, IEEE TRANSACTIONS ON MEDICAL IMAGING, VOL. 26, NO. 2, MAY 2007, 657-658 pp.

124. Goldstein, H. and Johnston, F.E. (1978), A Method for studying shape change in children, Annals of Human Biology, 1978, Vol. 5, No. 1 , Pages 33-39

125. Haas, A. (2000), Hegel and the Problem of Multiplicity. SPEP Studies in Historical Philosophy. Evanston: Northwestern University Press (Haas A, 2000)

126. Haas, A. (2008) Gewalt and Metalepsis: On Heidegger and the Greeks Bulletin d'Analyse Phénoménologique.

127. Haas, A. (2013) Hegel and the Art of Negation: Negativity, Creativity and Contemporary Thought, I.B. Tauris.

128. Haas, A. (2008) Gewalt and Metalepsis: On Heidegger and the Greeks Bulletin d'Analyse Phénoménologique.

129. Haken, Hermann. (1983), "Synergetics: Introduction and Advanced Topics", Springer, 1983
Heidegger, Martin. (1962), Preface to: William Richardson, Heidegger. Through Phenomenology to Thought, The Hague: Martinus Nijhoff, 1963.

130. Haldar, Hiralal. (1920), Space and Time in Hegel's Philosophy, The Monist., Volume 42, Issue 4, October 1932. Devoted to the Philosophy of Science. Pages 520-532.

131. Harris, Errol E.. (1983), An Interpretation of the Logic of Hegel, 1983

132. Harvey P (2012) The topological quality of infrastructural relation: An ethnographic approach. Theory, Culture and Society 29(4/5): 76–92.

133. Hegel, G.W.F. (1969) Science of Logic, trans. by A.V. Miller, New York: Humanity Books.

134. Hegel, G.W.F. (1892) The Logic of Hegel, trans. by William Wallace, Oxford: Clarendon Press.

135. Hegel, G.W.F. (2010a). Encyclopedia of the Philosophical Sciences in Basic Outline, Part 1: Science of Logic [1817]. Trans. Klaus Brinkmann and Daniel O. Dahlstrom. New York: Cambridge University Press. (Hegel, 2010a).

136. Hegel, G.W.F. (2010b). The Science of Logic[1812–1816].Trans. George Di Giovanni. New York: Cambridge University Press.

137. Hegel, G. (1961). Science of Logic. Tr. Johnston & Struthers. Allen & Unwin., p.211

138. Hegel's Philosophy of Nature, translation of Hegel's Naturphilosophie by A. V. Miller, Clarendon Press, Oxford, 1970.

139. Hegel: Geometrische Studien (GS), by G. W. F. Hegel, pp. 288-300 of: Dokumente zu Hegels Entwicklung, herausgegeben von Johannes Hoffmeister, Frommann, Stuttgart, 1936. (This early work of Hegel contains fragents of what remains of a more extensive work on geometry which Hegel wrote in his Jena years. A detailed discussion of it is given in the present writer's paper: Hegel's Early Geometry, Hegel Studien 39/40, 2004-2005, 61-124. A translation of GS, together with an Introduction (both by the present writer) is given in: G. W. F. Hegel: Geometrical Studies - translated with Introduction and Notes, Bulletin of the Hegel Society of Great Britain 57/58, 2008, 118-153 (ITGS).))

140. Hegel, The Difference between the Fichtean and Schellingian Systems of Philosophy, translated by J. P. Surber, Ridgeview Publishers, 1978

150. Hess-Lüttich, Ernest W. B. (2012) Spatial turn: On the Concept of Space in Cultural Geography and Literary Theory, (Vol. 5; 2012) Journal for Theoretical Cartography (Ernest W. B. Hess-Lüttich, 2012)

151. Houlgate, Stephen., 2014, Hegel on the Category of Quantity, Hegel Bulletin, Vol 35, Issue 01, pp 16-32

152. Hyppolite, Jean (1997). Logic and Existence [1952]. Trans. Leonard Lawlor and Amit Sen. Albany, N.Y.: State University of New York Press.

153. Hill, Brian, (2006), The (topo)logic of vagueness, paper presented at the IHPST (May 2006), ENFA (June 2006), Mind-Aristotelian Society Meetings (July 2006) and The Prague International Colloquium (September 2006). The author would like to thank all audiences for comments.

154. Hughes, Glenn, (1993) Mystery and Myth in the Philosophy of Eric Voegelin, University of Missouri Press, Columbia, Missouri. (Hughes G, 1993)

155. Husserl, E. (1929), Formal and Transcendental Logic (1929), English translation: The Hague: Martinus Nijhoff, 1969

156. Inwood, Michael (1992). *A Hegel Dictionary*. Oxford: Blackwell. John (unknown date). The Gospel according to Saint John. In Moses, Peter, Luke *et. al. The Holy Bible: American Standard Version.*

157. Jauhiainen, Ilmari. (2011), Constructions and situations: a constructivist reading of Hegel’s System, Academic dissertation, University of Helsinki, 2011

158. Kalevi Kull, “Structuralism and semiotic of organic form”, Presentation, 2010, Seminar SEMIOTICS OF ORGANIC FORM in the Department of Semiotics, Tiigi St. 78-119, University of Tartu, Tartu, Estonia

159. Kant, I. (2003) Critique of Pure Reason, trans. by Norman Kemp Smith, Hampshire: Palgrave Macmillan.

160. Kay, Lily E. (1993), The Molecular Vision of Life: Caltech, The Rockefeller Foundation and the Rise of the New Biology (New York: Oxford University Press). 1993.

161. Kelly, Kevin. Topological Epistemology - http://www.illc.uva.nl/lgc/seminar/?p=1241and http://www.andrew.cmu.edu/user/kk3n/homepage/kelly.html

162. Kneller, Jane. (2003), Novalis's "Fichte Studies" Cambridge University Press

163. Krämer, Hans Joachim . (1959), "Arete in Plato and Aristotle" /Arete bei Platon und Aristoteles/, Heidelberg 1959

164. Kosok, Michael (1972). The Formalization of Hegel's Dialectical Logic; Its Formal Structure, Logical Interpretation and Intuitive Foundation. In MacIntyre, Alasdair C. (Ed.), Hegel; a Collection of Critical Essays. London: University of Notre Dame Press, 237-287.

165. Klebanoff, A., Rickert, J. 1998. Studying the Cantor Dust at the Edge of Feigenbaum Diagrams// *The College Mathematics Journal*, Vol. 29, No. 3, (1998), pp. 189-198:

166. Kosykhin, Vitaly. (2013), Losev's Eidetic Dialectic: The Structuring of Being and Anagogical Cognition, source: http://stasisjournal.net/all-issues/12-1-politics-of-negativity-october-2013/6-losev-s-eidetic-dialectic-the-structuring-of-being-and-anagogical-cognition (Kosykhin 2013)

http://www.maa.org/sites/default/files/pdf/upload_library/22/Polya/07468342.di020782.02p0399a.pdf

167. Kunze, D. (2013), Metalepsis of the site of exception, Available at: http://art3idea.psu.edu/metalepsis/texts/AAPP.pdf [Accessed: 4 September 2014]. (Kunze D, 2013)

The term 'topology of metalepsis' is used by Donald Kunze (Kunze, D. 2013, endnote 18: "This 'entanglement' is the topology of metalepsis, the delay of the detached reflection."

168. Kunze, D. (2013), Metalepsis of the site of exception, Available at: http://art3idea.psu.edu/metalepsis/texts/AAPP.pdf [Accessed: 4 September 2014].

169. Kunze, D. (1990), Skiagraphy and the Ipsum of Architecture, Architecture and Shadow, Via XI, p.62-75. (Kunze D, 1990)

170. Lacroix, Alain (2000). The Mathematical Infinite in Hegel. The Philosophical Forum, 3-4, XXXI, 298-327. DOI: 10.1111/0031-806X.00043

171. Lacan, J. (1962-1963). Séminaire X: L'angoisse. Retrieved September 26, 2013 from http://www.ecole-lacanienne.net/seminaireX.php

172. Lacan, J. (1964-1965). Séminaire XII: Problèmes cruciaux pour la psychanalyse. Retrieved September 26, 2013 from http://www.ecole-lacanienne.net/seminaireXII.php

173. Lacan, J. (1961). Séminaire IX: L'identification. Retrieved September 26, 2013 from http://www.ecole-lacanienne.net/seminaireIX.php

174. Lafont, J. (2004). Topology and Efficiency. In D. Milovanovic, & E. Ragland-Sullivan (Eds.), Lacan: Topologically Speaking (pp. 3-27). New York: Other Press.

175. Lamb, D. (1979), Language and Perception in Hegel and Wittgenstein. Avebury.

176. Larvor, Brendan. (2010), Albert Lautman: Dialectics in mathematics, Philosophiques 37 (1):75-94 (2010)

177. Lash S (2012) Deforming the figure: Topology and the social imaginary. Theory, Culture and Society 29(4/5): 261–287.

178. Latham A (2002) Re-theorizing the scale of globalization: topologies, actor-networks, and cosmopolitianism. In: Herod A and Wright M (eds) Geographies of Power: Making Scale. Oxford: Blackwell, 115–144.

179. Latham A, 2011, ``Topologies and the multiplicities of space ^ time'' Dialogues in Human Geography 1 312 ^ 315

180. Law J and Mol A (2001) Situating technoscience: An inquiry into spatialities. Environment and Planning D: Society and Space 19: 609–621.

181. Law J and Urry J (2004) Enacting the social. Economy and Society 33(3): 390–410.

182. Lawvere William, (1996), Grassmann's Dialectics and Category Theory, in Hermann Günther Graßmann (1809–1877): Visionary Mathematician, Scientist and Neohumanist Scholar, Boston Studies in the Philosophy of Science Volume 187, 1996, pp 255-264

183. Lawvere William, (1995), A new branch of mathematics, "The Ausdehnungslehre of 1844," and other works. Open Court (1995), Translated by Lloyd C. Kannenberg, with foreword by Albert C. Lewis, Historia Mathematica Volume 32, Issue 1, February 2005, Pages 99–106

184. Lawvere William, (1991), Some Thoughts on the Future of Category Theory in A. Carboni, M. Pedicchio, G. Rosolini, Category Theory , Proceedings of the International Conference held in Como, Lecture Notes in Mathematics 1488, Springer (1991) (Lawvere 1991);

185. Lawvere William, (1992), Categories of space and quantity in J. Echeverria et al (eds.), The Space of mathematics , de Gruyter, Berlin, New York, pages 14-30, 1992. (Lawvere 1992);

186. Lawvere William, (1994), Tools for the advancement of objective logic: closed categories and toposes, in J. Macnamara and Gonzalo Reyes (Eds.), The Logical Foundations of Cognition, Oxford University Press 1993 (Proceedings of the Febr. 1991 Vancouver Conference "Logic and Cognition"), pages 43-56, 1994. (Lawvere 1994);

187. Lawvere William, (1995) A new branch of mathematics, "The Ausdehnungslehre of 1844," and other works. Open Court (1995), Translated by Lloyd C. Kannenberg, with foreword by Albert C. Lewis, Historia Mathematica Volume 32, Issue 1, February 2005, Pages 99–106 (Lawvere 1995)

189. Lawvere William, (1996), Unity and identity of opposites in calculus and physics, Applied Categorical Structures, June 1996, Volume 4, Issue 2, pp 167-174 (Lawvere 1996);

190. Lawvere William, (1997), Toposes of laws of motion , transcript of a talk in Montreal, Sept. 1997 (pdf) (Lawvere 1997)

191. Lima, Carlos Cirne. (2006), "Beyond Hegel – A Critical Reconstruction of the Neoplatonic System", 2006 /http://www.cirnelima.org/Beyond-book.doc/ - See also: Carlos R. V. Cirne-Lima, Antonio C. K. Soares, "Being, Nothing, Becoming - Hegel and Us – A Formalization", Translatated into English by Luís M. Sander, Niura Fontana and Beatriz Fontana.

192. Lemke, Jay L., "Topological Semiosis and the Evolution of Meaning" – http://www-personal.umich.edu/~jaylemke/webs/wess/index.htm

193. Lemke, Jay L.,, Typological vs. Topological Semiosis – .http://www-personal.umich.edu/~jaylemke/webs/wess/tsld002.htm

194. Lemke, Jay L., Mathematics in the middle: measure, picture, gesture, sign, and word, and Opening Up Closure: Semiotics Across Scales. http://www-personal.umich.edu/~jaylemke/webs/wess/tsld002.htm

195. Lefebvre H (1991) The Production of Space, translated by Nicholson-Smith D. Malden, MA: Blackwell.

196. Lewin, Kurt, (1936), Principles of Topological Psychology (1936) (Lewin K, 1936)

197. Leiber, Justin. (2001), Turing and the fragility and insubstantiality of evolutionary explanations: a puzzle about the unity of Alan Turing's work with some larger implication, Philosophical Psychology, Vol.14, NO.1, 2001

198. Lorenz, K. (2007) The anatomy of metalepsis: visuality around on the late fifth century pots, in Debating the Athenian Cultural Revolution: Art, Literature, Philosophy, and Politics 430–380, Ed. Robin Osborne, Cambridge University Press, p. 116-144 (Lorenz K, 2007)

199. Losev, Aleksei F., (1927), Ancient Cosmos and Modern Science, 1927

200. Losev, Aleksei F., (1990). "Filosofiia imeni" [Philosophy of the name]. In A.F. Losev, Iz rannikh proizvedenii [From the early works]. Moscow: Pravda. (Losev 1990).

201. Losev, Aleksei F., (1993). Ocherki antichnogo simvolizma i mifologii [Essays on ancient symbolism and mythology]. Moscow: Mysl'. (Losev 1993).

202. Losev, Aleksei F., (2003). The Dialectics of Myth. Trans. Vladimir Marchenkov. London: Routledge.

203. Louie, A. H., and Kercel, S.W. (2007). Topology and Life Redux: Robert Rosen's Relational Diagrams of Living Systems. *Axiomathes* 17: 109–136

204. Lind, L.R. (1951), "Rev. of Curtius, Europäische Literatur und lateinisches Mittelalter". The Classical Weekly 44 (14): 220–21.)

205. Lury C., Parisi L. and Terranova T. (2012) Introduction: The becoming topological of culture. Theory, Culture & Society 29(4–5): 3–35. (Lury C., Parisi L. and Terranova T., 2012)

206. MacLeod, Norman, (2010) Paleo-Math 101, Shape Models II: The Thin Plate Spline http://www.palass.org/modules.php?name=palaeo_math&page=26

207. Malina, Debra (2002). Breaking the Frame: Metalepsis and the Construction of the Subject. The Ohio State University (Malina D, 2002)

208. Malpas, Jeff. (), Self, Other, Thing, http://philevents.org/event/show/13584

209. Malpas, Jeff. (), The Place of Topology: Responding to Crowell, de Beistegui, and Young Jeff Malpas https://www.academia.edu/18545068/The_Place_of_Topology_Responding_to_Crowell_Beistegui_and_Young

210. Malpas, Jeff. (), Place and Hermeneutics: Towards a Topology of Understanding, . . . . . https://www.academia.edu/12073967/Place_and_Hermeneutics_Towards_a_Topology_of_Understanding

211. Malpas, Jeff. (2007),, Heidegger and the Thinking of Place: Explorations in the Topology of Being, MIT Press, 2007

212. Malpas, Jeff. (2011), "The Place of Topology: Responding to Crowell, Beistegui, and Young", University of Tasmania and La Trobe University, Australia, 2011

213. Malpas, Jeff. Ed. (2015), In 'Place and Situation', in Routledge Companion to Philosophical Hermeneutics, edited by Jeff Malpas and Hans-Helmuth Gander (Abingdon: Routledge, 2015), pp.354-366

214. Malpas, Jeff. (2010), 'The Beginning of Understanding: Event, Place, Truth', in Jeff Malpas and Santiago Zabala (eds), Consequences of Hermeneutics (Chicago: Northwestern University Press, 2010), pp.261-280

215. Massumi, Brian. (2001). "Strange Horizon: Buildings, Biograms and the Body Topologic." In Architecture and Science, edited by Giuseppe Di Christina, 190-197. London: Wiley-Academy, 2001. (Massumi B, 2001)

216. Massumi B (2002) Parables for the Virtual. Durham, NC: Duke University Press.

217. Massumi, Brian (2002) Strange Horizon: Buildings, Biograms and Body Topologic, in Parables of the Virtual: Movement, Affect, Sensation, Durham: Duke University Press, also http://www.brianmassumi.com/textes/Strange%20Horizon.pdf (Massumi B, 2002)

218. Martin, Lauren, and Secor, Anna J., Towards a post-mathematical topology, Progress in Human Geography, 2014, Vol.38(3) 420 – 438

219. Maybee, Julie E. (2009), Picturing Hegel: An Illustrated Guide to Hegel's Encyclopaedia Logic, Lexington Books, 2009, 34

220. McCulloch, Warren. (1945), "A Heterarchy of Values Determined by the Topology of Nervous Nets" /In: Bulletin of Mathematical Biophysics, 7, 1945, 89–93.

221. McNeil, Donald H. (1995), "What's going on with the topology of recursion?", Science and Art: The Red Book of `Einstein Meets Magritte': The Red Book Vol 2 (Einstein Meets Magritte: An Interdisciplinary Reflection on Science, Nature, Art, Human Action and Society), Kluwer, 1999 - Flett, R. Ian, and Donald H. McNeil. 1995).

222. Mormann, T. (1995), "Trope Sheaves. A Topological Ontology of Tropes", in Logic and Logical Philosophy: 3, 129–150.

223. Mormann, T. (1996), "Similarity and Countinous Quality Distributions", in The Monist: 79, 76–88.

224. Mormann, T. (1997), "Topological Aspects of Combinatorial Possibility", in Logic and Logical Philosophy: 5, 75–92.

225. Mormann, T. (2000), "Topological Representation of Mereological Systems", in Poznan Studie in the Philosophy of the Sciences and the Humanities: 76, 463–486.

226. Mormann, T. (2013), "Topology as an Issue for History of Philosophy of Science", in New Challenges to Philosophy of Science, The Philosophy of Science in a European Perspective 4, edited by H. Andersen et al., Dordrecht, 423–434. (Mormann T 2013).

227. Mormann, T. (2013), "Topology as an Issue for History of Philosophy of Science", in New Challenges to Philosophy of Science, The Philosophy of Science in a European Perspective 4, edited by H. Andersen et al., Dordrecht, 423–434.

228. Mormann, T. (1997), "Topological Aspects of Combinatorial Possibility", in Logic and Logical Philosophy: 5, 75–92.

229. Moretti F, Signs Taken for Wonders (1983), Modern Epic (1995) and Atlas of the European Novel 1800-1900 (1998).

230. Murphy, Peter. (2014), Topeme: Truth. Topology. Cartography, Analogy., The Hydra Dialogues, May 22-23 2014, The Royal Danish Academy of Fine Arts, School of Architecture, Design and Conversation. (Murphy P, 2014)

231. Mol A and Law J (1994) Regions, networks, and fluids: Anaemia and social topology. Social Studies of Science
24: 641–670.

232. Mezzadra S and Nielson B (2012) Between inclusion and exclusion: On the topology of global space and borders. Theory, Culture and Society 29(4/5): 58–75.

233. Nuzzo, Angelica. (2011), The Problem: Perspective on Method, Or, How to Approach Being, (111-139), p. 112, in Stephan Holgate, Michel Baur, ed., 2011, A Companion to Hegel, Blackwell Companion, Willey- Blackwell.Angelica Nuzzo (Thinking Being: Method in Hegel's Logic of Being).

234. Nuzzo, Angelica. (2010), Vagueness and meaning variance in Hegel's logic, in Nuzzo, A. ed., 2010, Hegel and the Analytic Tradition, London: Continuum, 2010, pp.208

235. Nunziante, Antonio M., (2015), Infinite vs. Singularity. Between Leibniz and Hegel, University of Padova http://revista.hegelbrasil.org/iii-nunziante-singularity-vs-infinite-print/

236. Ostheeren, Klaus (1998). Ernst Robert Curtius (1886 – 1956). In Helen Damico Ed., *Medieval Scholarship: Literature and Philology*. Taylor & Francis. pp. 365–80. (Ostheeren K, 1998)

237. O'Regan, (1994), The Heterodox Hegel. SUNY Series in Hegelian Studies. State University of New York Press, Albany, p. 47.

238. Parikh, Dabrowski, Moss, Steinsvold, Topology and epistemic logic - applications of topological ideas in modal logic, especially in epistemic logic; applications of topological ideas in epistemic logic; a topological semantics and completeness proof for the logic of belief KD45: http://www.indiana.edu/~iulg/moss/TEL.pdf

239. Parikh, Dabrowski, Moss, "Topological Reasoning and The Logic of Knowledge", Annals of Pure and Applied Logic 78 (1996) 73-110.
Paasi A (2011a) Geography, space, and the reemergence of topological thinking. Dialogues in Human Geography 1(3): 299–303.

240. Paasi A (2011b) From region to space, Part II. In: Agnew JA and Duncan JS (eds) The Wiley-Blackwell Companion to Human Geography, first edition. Chichester: Wiley-Blackwell, 161–175.

241. Paterson, Alan. (2008), G. W. F. Hegel: Geometrical Studies - translated with Introduction and Notes, Bulletin of the Hegel Society of Great Britain 57/58, 2008, 118-153.;

242. Paterson, Alan. (2007), The Hegelian Concept and set theory, 15 pages, 2007.;

243. Paterson, Alan. (2004/2005), A modern Hegelian Philosophy of Special Relativity, 32 pages, 2006.; Hegel's Early Geometry, Hegel Studien 39/40, 2004/2005, 61-124.; Does Hegel have anything to say to modern mathematical philosophy?, Idealistic Studies 32:2, 2002, 143-158.;

244. Paterson, Alan. (2000), The Successor Function and Induction Principle in a Hegelian Philosophy of Number, Idealistic Studies 30 (1) 2000, 25-61.;

245. Paterson, Alan. (1997), Frege and Hegel on concepts and number, 22 pages.; Self-reference and the natural numbers as the logic of Dasein, Hegel Studien 32(1997), 93- 121.;

246. Paterson, Alan. (1997), Towards a Hegelian philosophy of mathematics, Idealistic Studies, 27(1997), 1-10.

247. Paterson, Alan. (2002), Does Hegel have anything to say to modern mathematical philosophy?, Idealistic Studies 32:2, 2002, 143-158.

248. Paterson, Alan. (1997) Towards a Hegelian philosophy of mathematics, Idealistic Studies, 27 (1997)

249. Paterson, Alan. (2006), A modern Hegelian Philosophy of Special Relativity, 32 pages, 2006. https://sites.google.com/site/apat1erson/ (Paterson 2006)

250. Paterson, Alan L. T. (1994). The Concept of the Propositional Calculus. Preprint. Consulted version: http://home.olemiss.edu/~mmap/HPROPC.pdf.; (Paterson A, 1994).

251. Paterson, Alan L. T. (1997a). Towards a Hegelian philosophy of mathematics. Idealistic Studies, 27, 1-10. Consulted version:http://home.olemiss.edu/~mmap/TOHEGFF.pdf.; (Paterson A, 1997a).

252. Paterson, Alan L. T. (1997b). Self-reference and the natural numbers as the logic of Dasein. Hegel Studien, 32, 93-121. Consulted version:http://home.olemiss.edu/~mmap/HEGSTUFR.pdf.; (Paterson A, 1997b).

253. Paterson, Alan L. T. (1999) Frege and Hegel on concepts and number. Preprint.Consulted version: http://home.olemiss.edu/~mmap/nnfregh.doc.; (Paterson A, 1999).

254. Paterson, Alan L. T. (2000). The Successor Function and Induction Principle in a Hegelian Philosophy of Number. Idealistic Studies 30 (1), 25-61.Consulted version: http://home.olemiss.edu/~mmap/inductio.doc.; (Paterson A, 2000).

255. Paterson, Alan L. T. (2002). Does Hegel have anything to say to modern mathematical philosophy? Idealistic Studies, 32, 143-158. Consulted version: http://home.olemiss.edu/~mmap/sjp6.doc.; (Paterson A, 2002).

256. Paterson, Alan L. T. (2004/2005). Hegel's Early Geometry. Hegel Studien 39/40, 61-124. Consulted version: http://home.olemiss.edu/~mmap/Heggeom7.doc. (Paterson A, 2004, 2005).

257. On Paterson's website (http://sites.google.com/site/apat1erson/) we will find five papers on the merits of a Hegelian philosophy of mathematics. Three of these (Paterson 1997b; 1999; 2000) are about the philosophy of Number, one is about the concept of the propositional calculus (Paterson 1994) and one is about the Hegelian philosophy of mathematics in general (Paterson 2002). In each of these, Hegelian philosophy is proposed as a solution to the problems 'which arise out of the existence in mathematics of self-referential, nonconstructive concepts (such as class)' (Paterson 2002: 143).

258. Paterson, Mark A. (2011) Galileo's Muse: Renaissance Mathematics and the Arts, Harward University Press, 2011)

259. Piaget, J., and Inhelder, B., (1956). The Child's Conception of Space. London: Routledge & Kegan Paul.

260. Piaget J., Inhalder, B. (1958). The Growth of Logical Thinking from Childhood to Adolscence. New York: Basic Books. New York (Piaget J., Inhalder B, 1958)

261. Pinkard, T. (1994), Hegel's Phenomenology. The Sociality of Reason. Cambridge: Cambridge
University Press.

262. Phillips, J John WP. (2013), On Topology, Theory, Culture & Society, 30(5) 122–152

263. Pippin, R. B. (1988), Hegel's Idealism. The Satisfactions of Self-Consciousness. Cambridge:
Cambridge University Press

264. Piper, Andrew and Algee-Hewitt, Mark. (2014), "The Werther Effct I: Goethe Topologically." Distant Readings: Topologies of German Culture in the Long Nineteenth Century. Ed. Matt Erlin and Lynn Tatlock (Rochester, NY: Camden House, 2014) 155-184. (Piper A & Algee-Hewitt M, 2014)

265. Piper, Andrew, (2013) A Theory of Topological Reading : http://txtlab.org/?p=188 (Piper A, 2013)

266. Plotnitsky, Arkady. (2012), Badiou's later Experimenting with ontologies: sets, spaces, and topoi with Badiou and Grothendieck, Environment and Planning D: Society and Space 2012, volume 30, pages 351 – 368.

267. Plotnitsky, Arkady. (2009), Bernhard Riemann's Conceptual Mathematics and the Idea of Space, Configuration, Volume 17, no. 1, pp. 105-130 (Plotnitsky A, 2009)

268. Plotnitsky, Arkady., The Spaces of the Baroque (with Leibniz, Riemann, and Deleuze)

269. Plotnitsky, A. (2003). Algebras, Geometries and Topologies of the Fold: Deleuze, Derrida and Quasi-Mathematical Thinking (with Leibniz and Mallarmé). In P. Patton, & J. Protevi (Eds.), Between Deleuze and Derrida (pp. 98-119). New York: Continuum. New York Piper, Andrew. (2013) "Reading's Refrain: From Bibliography to Topology." ELH. Special Issue on Reading. Ed. Joseph Slaughter (Summer 2013): 373-399. http://piperlab.mcgill.ca/pdfs/Piper_ReadingsRefrain.pdf (Piper A, 2013)

270. Phillips, John WP. (2013), On Topology, Theory, Culture and Society, 9/2013; 30(5):122-152: http://www.researchgate.net/profile/John_Phillips20/publications [accessed Mar 21, 2015].

271. Poincaré, Henri (1885). L'Équilibre d'une masse fluide animée d'un mouvement de rotation, Acta Mathematica, t.7, pp. 259-380, sept 1885.

272. Pont, Jean-Claude. (1974), Topologie Algébrique, des origines àPoincaré, (Algebraic Topology, from the origins to Poincaré), Presses Universitaires de France, 1974

273. Пунчев, И. 2011. Увод в системата на диалектическата логика - Част II: Класическа теория на мистичната диалектическа логика// http://gs-research.org/wp-content/uploads/2013/02/chast1.pdf

274. Пунчев И.П. 2006. От класическа към некласическа теория на диалектическата логика. Част Първа: От класическа формална логика към класическа диалектическа логика. // Сп. „Философски алтернативи" бр. 1 2006 г.

275. Пунчев И.П. 2006. От класическа към некласическа теория на диалектическата логика. Част Втора: От математическа „формална логика" към математическа „диалектическа логика" // Сп. „Философски алтернативи", бр. 2, 2006.

276. Ragland, Ellie and Milavanovic, Dragan (eds) (2004) Lacan: Topologically Speaking, New York: Other Press (Ragland E, Milavanovic D,eds, 2004)

277. Ramírez, J.L. (1995) Skapande Mening: En begreppsgenealogisk undersökning om rationalitet, vetenskap och planering [Creative Meaning: A Contribution to a Human-Scientific Theory of Action]. Stockholm: NORDPLAN.

278. Rämö, Hans. (1999), An Aristotelian Human Time-Space Manifold. From chronochora to kairotopos. In Time & Society VOL 8(2) 309-328 Sage 1999.

279. Redding, P. (1996), Hegel's Hermeneutics. Ithaca & London: Cornell University Press.

280. Rees, J.M. (2010) Geometry and Rhetoric: Thinking about Thinking in Picture, Nexus Network Journal, Vol .12, No.3: 507-526. (Rees J.M, 2010)

281. Rosen, Stephen M (2004) Topology, in Dimensions of Apeiron: A Topological Phenomenology of Space, Time, and Individuation, Amsterdam-New York: Editions Rodopi B.V. pp 168 -210 (Rosen S, 2004)

282. Rosen, Stephen M (2006) Topologies of the Flesh: A Multidimensional Exploration of the Lifeworld, Athens: Ohio University Press (Rosen S, 2006)
Rosen, R. (1962), The derivation of D'Arcy Thompson' theory of transformations from the theory of optimal design, Buletin of Mathematical Biophysics, Volume 24, Number 3, 279-290, 1962

283. Rosen, R. (…), Dynamical similarity and the theory of biological transformations, Buletin of Mathematical Biophysics, Volume 40, Number 5, 549-579, 19…

284. Rosen, R. (1989), Similitude, similarity, and scaling, Landscape Ecology vol. 3 nos. 3/4 pp 207-216 (1989), SPB Academic Publishing bv, The Hague.

285. Richeson, David S (2008) Euler's Gem: The Polyhedron Formula and the Birth of Topology, Princeton: Princeton University Press (Richeson D, 2008)
http://mathworld.wolfram.com/Topology.html
Ryan Sean, The Topology of Being, Parrhesia, Number 11, 2011, 56-61

286. Ryan Sean, Heidegger's Topology: Being, Place, World, 2009, Australasian Journal of Philosophy 87, 1, 169-171

287. Secor A (2008) Zizek's dialectics of difference and the problem of space. Environment and Planning A 40(11): 2623–2630.

288. Secor A (2013) 2012 Urban Geography Plenary Lecture: Topological city. Urban Geography 34(4): 430–444.

289. Sandoz, Ellis, (1981), The Voegelinian Revolution: a Biographical Introduction, LSU Press, Baton Rouge. (Sandoz E, 1981)

290. Scholz, Hermes. (1961), Concise History of Logic (1931), English translation: New York: Philosophical Library, 1961

291. Schürmann, Reiner. (1987), Heidegger on Being and Acting: From Principles to Anarchy, trans. Christine-Marie Gros (Bloomington: Indiana University Press, 1987.

292. Schulte, Oliver , Juhl, Cory . (1996), Topology as Epistemology, (1996). The Monist vol. 79:1, 141-147. http://www.jstor.org/discover/10.2307/27903468?uid=3737608&uid=2129&uid=2&uid=70&uid=4&sid=21102214017207

293. Skowron, Bartłomiej. (2014), The Forms of Extension, Substantiality and Causality, M. Szatkowski, M. Rosiak (eds.), Walter de Gruyter Verlag, Philosophical Analysis (Book 60), Berlin/Boston 2014, pp. 175-187.

294. Steele, Donald A. Tr. (1950) Bolzano's Paradoxes of the Infinite, London: Routledge and Kegan Paul, 1950.

295. Smith, Barry. (1994), Topological Foundations of Cognitive Science, a revised version of the introductory essay in C. Eschenbach, C. Habel and B. Smith (eds.), Topological Foundations of Cognitive Science, Hamburg: Graduiertenkolleg Kognitionswissenschaft, 1994, the text of a talk delivered at the First International Summer Institute in Cognitive Science in Buffalo in July 1994.: http://ontology.buffalo.edu/smith/articles/topo.html

296. Smith, Barry, Ed., (1982), Parts and Moments. Studies in Logic and Formal Ontology, Munich: Philosophia.)

297. Smith, Paul (2011), Pictorial Grammar: Chomsky, John Willats, and the Rules of Representation. Art History, 34: 562–593. doi: 10.1111/j.1467-8365.2011.00835.x (Smith P, 2011)

298. Smith, P. C. tr. (1976). Hegel's Dialectic: Five Hermeneutical Studies (Gadamer, H-G. 1971.). New Haven: Yale University Press, 1976.

299. Sleeman Brian D., 2003, review on "Mathematics in Nature: Modeling Patterns in the Natural World", John A. Adam, Princeton University Press, 2003, Reviewed by Brian D. Sleeman

300. Stewart, Ian. (1989), Does God Play Dice?: The Mathematics of Chaos, Oxford: Blackwell, 1989.

301. Suppes, P.C., (1988). "Representation theory and the analysis of structure." Philosophia Naturalis 25: 254-268.

302. Sutherland, W. A. (1975). Introduction to Metric and Topological Spaces. OUP, Oxford.

303. Tarnopolsky Y. and Grenander U., (1989-2003), History as Points and Lines, 1989-2003

304. Thompson, D. W., (1917), On Growth and Form. Cambridge University Press.

305. Thom, R. (2001), Structural Stability and Morphogenesis, Westview Press; New Ed R. Thom (2001).

306. Thom, R., 1973: Langage et catastrophes: elements pour une semantique topologique, in: Peixoto, M. M. (ed.) *Dynamical Systems.* Proceedings of the Symposium at Salvador, Brazil, 619—654.

307. Tomarev, S., Callaerts, P., Kos, L., Zinovieva, R., Halder, G., Gehring, H. & Piatigorsky, J., (1997). Squid Pax-6 and eye development. Proceedings of the National Academy of Sciences, 94,2421-2426.

308. Toscano, Alberto. (2006), The Bourgeois and the Islamist, or, The Ot her Subjects of Politics, Cosmos and History: The Journal of Natural and Social Philosophy, vol. 2, no. 1-2, 2006, source www.cosmosandhistory.org

309. Tóth, Imre. (1972). Die nicht-euklidische Geometrie in der Phänomenologie des Geistes; Wissenschaftstheoretische Betrachtungen zur Entwicklungsgeschichte der Mathematik. Frankfurt am Main: Horst Heiderhoff Verlag.

310. Thiher, Allen. (1997), The power of tautology: The roots of leterary theory, Associated University Press, 1997

311. Tsatsanis, Peter. (2012), On Rene Thom Sinificance for Mathematics and Philosophy, Scripta Philosophicae Naturalis 2:213-229 (2012), p.223-224.

312. Tyson, Gofton B. (2013), Analysis, Systematicity and the Transcendenal in Herman Cohen,
www.tysongofton.com/s/Gofton_B_Tyson_2013_PhD_thesis.pdf:

313. Tze-Wan Kwan, (2005), Hegelian and Heidegerian Tautologies, Analecta Husserlliana, The yearbook of Phenomenological research, Logos of Phenomenology and Phenomenology of Logos, Volume LXXXVIII, 2005, The World Institute for Advanced Phenomenological Research and Learning

314. Turing, Alan M. (1952), The chemical basis for morphogenesis, Philos. Trans. Roy. Soc. London Ser. B 237 (1952), 37–72.

315. Turing, A..M., Saunders, P. T. Ed., (1953/1992), Collected works of A. M. Turing: Morphogenesis. Amsterdam: North-Holland. and Wardlaw, C. W. (1953/1992).

316. Verene, Donald Phillip. (2007), Hegel’s Absolure: An Introduction to Reading the Phenomenology of Spirit, State University of New York Press

317. Verene, Donald Phillip. (2009), Speculative philosophy, Lexington Books, 2009

318. Voegelin, Eric., (1990) Reason: The Classic Experience,” in Voegelin, Published Essays, 1966-1985, vol. 12 of The Collected Works of Eric Voegelin, ed.Ellis Sandoz, Baton Rouge: Louisiana State University Press, 1990), 289-90. (Voegelin E, 1990)

319. Voegelin, Eric., (2000) Order and History, vol. IV: The Ecumenic Age, vol. 17, The Collected Works of Eric

320. Voegelin, ed. Michael Franz, Columbia, University of Missouri Press, 408. (Voegelin E, 2000)

321. Voegelin, E. (2004), The drama of humanity and other miscellaneous papers 1939-1985. University of Missouri Press, Missouri. (Voegelin E, 20040)

322. Weber, Z. and Colyvan, M. (2010), ‘A topological sorites’, The Journal of Philosophy, 107: 311–325

323. Weinberger, Shmuel. (), “Persistent Homology”

324. White, Michael J. (1988/2007), On continuity: Aristotle versus topology?, History and Philosophy of Logic,Volume 9, issue 1, 1988

325. Wilder, R. L. (1978), Evolution of the Topological Concept of ‘Connected’, American Mathematical Monthly, 85 (1978), pp. 720-26

326. Wildgen, W., & Brandt, P. A. (2010). Semiosis and catastrophes: René Thom's semiotic heritage. Bern: Peter Lang.

327. Williamson, Roland. (2009), Hegel Among the Quantum Physicists, International Journal of Žižek Studies, 3 (1) (2009).

328. Winfield, Richard Dien. (2012), Hegel's Science of Logic: A Critical Rethinking in Thirty Lectures, Rowman & Littlefield Publishers, Inc.

329. Wolff, Michael (1979). Über das Verhältnis zwischen logischem und dialektischem Widerspruch. In: Hegel Gesellschaft, Hegel Jahrbuch. München: Hegel Gesellschaft, 340-348. In Horstmann, Rolf-Peter & Petry, Michael J. (Eds.), Hegels Philosophie der Natur: Beziehungen zwischen empirischer und spekulativer Naturerkenntnis. Stuttgart: Klett-Cotta, (1986: 197-263).

330. Wolff, Michael (1986). Hegel und Cauchy; Eine Untersuchung zur Philosophie und Geschichte der Mathematik. In Horstmann, Rolf-Peter & Petry, Michael J. (Eds.), Hegels Philosophie der Natur: Beziehungen zwischen empirischer und spekulativer Naturerkenntnis. Stuttgart: Klett-Cotta, 197-263. . In Horstmann, Rolf-Peter & Petry, Michael J. (Eds.), Hegels Philosophie der Natur: Beziehungen zwischen empirischer und spekulativer Naturerkenntnis. Stuttgart: Klett-Cotta, (1986: 197-263).

331. Wickramasekara, S. (2003), A Note on the Topology of Space-time in Special Relativity, Class. Quantum Grav., 18 (2001) 5353

332. Willats, John (1997). Art and Representation: New Principles in the Analysis of Pictures. Princeton University Press, Princeton (Willats J, 1997)

333. Willats, John (2005), Making Sense of Children's Drawings, Mahwah, NJ, (Willats J, 2005)

334. Willats, John and Durand, Frédo (2005) Defining pictorial style: Lessons from linguistics and computer graphics', Axiomathes, 15: 3, September 2005 (Willats J, & Durand F, 2005)

335. Wildgen, Wolfgang, (2004), Time, Motion, Force, and the semantic of natural languages, Antwerp Papers in Linguistics, University of Bremen

336. Willats, John (1997). Art and Representation: New Principles in the Analysis of Pictures. Princeton University Press, Princeton

337. Webb, Eugene, (1981) Eric Voegelin: Philosopher of History, University of Washington Press, Seattle, Washington. (Webb E, 1981)

338. Yeomans, C., Mind, Volume 119, Issue 475, pp. 783-786, Oxford Journals, Art and Humanities and Social Sciences http://mind.oxfordjournals.org/content/119/475/783.extract#

339. Yovel, Y. (1981), 'Hegel's Dictum that the Rational is Actual and the Actual is Rational. Its Ontological Content and Its Function in Discourse', in W. Becker and W. K. Essler (eds.)

340. Zimmerman, Dean W. (1996), Indivisible parts and extended objects: Some Philosophical Episodes from Topology's Prehistory, The Monist, Vol. 79, No. 1 (January 1996)

341. Žižek, Slavoj. (2012), Less than Nothing: Hegel and the Shadow of Dialectical Materialism. Verso, New York and London. (Žižek S, 2012)

342. Zwick, Martin, Dialectics and Catastrophe, Proceedings of the Fourth International Congress of Cybernetics & Systems 21–25 August, 1978 Amsterdam, The Netherlands